

A dynamical approach to quantitative precipitation forecast

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(Received 22 June 1984)

सारांश — परिमाणत्मक वर्षण पूर्वानुमान के लिए एक विधि सुझाई गई है। ऐसा अनुमान है कि वर्षण दर अत्यधिक ऊर्ध्वाधर गति के साथ अनुक्रमानुपाती है। अनुपातिकता का स्थिरांक तापमान, वायुदाब, आर्द्रता या विशेष रूप से शुष्क तथा सांद्र रुद्रोष्म ह्रास दरों के बीच अन्तर का एक फलन है। इस विधि का उपयोग करके एक विशिष्ट सिनॉप्टिक स्थिति की वर्षण दर अभिकलित की गई है तथा प्रक्षिप्त वर्षाजल से उसकी तुलना की गई है।

ABSTRACT. A method for quantitative precipitation forecast has been suggested. The precipitation rates are assumed to be directly proportional to large-scale vertical velocity. The constant of proportionality is a function of temperature, pressure and moisture contents or more specifically of difference between dry and saturated adiabatic lapse rates. Using the method, precipitation rates have been calculated for a specific synoptic situation and compared with the observed rainfall.

1. Introduction

The Quantitative Precipitation Forecast (QPF) refers to the quantity of rainfall expected at a point or in a catchment during the specified duration, say 12 hours or 24 hours. It is also related to the questions of when, where and how long a rainstorm will occur. The physical process governing the transportation of moisture aloft and their eventual release as rainfall are far too complex to be fully understood. The present network of upper air stations is also inadequate for the computation of wind, temperature and moisture fields accurately over a limited area to determine relationships between precipitation and atmospheric parameters. In spite of these limitations, some efforts have been made in past 50 years to compute the rate of precipitation. The techniques used may be broadly classified in three categories :

(i) Dynamical, (ii) Synoptic and (iii) Statistical. Some authors have also evolved the techniques of forecasting precipitation quantitatively by combining two or more of these techniques.

Recently the cloud mapping and computing vertical motion using satellite data have provided new concept in the estimation of precipitation rates. Empirical relationships between precipitation quantities and height of cloud tops or cloud top temperatures are being attempted, for an example, under convective conditions 24 hours precipitation (R in mm) may be given as :

$$R = 8 e^{0.2 H}$$

where, H is the height of cloud top in km.

For a dynamical approach to the problem of QPF the following two parameters appear to be the most important :

- (i) The distribution of moisture in the air column and
- (ii) The distribution of vertical velocity at different levels.

In the present study, a technique has been developed for the computation of rainfall intensity in a depression field. It is basically a dynamical approach which involves lifting of an air parcel with the vertical velocities presumed to be generated. The procedure has been illustrated by an actual case of depression centred over central India on 5 August 1968.

2. Data

Case of depression centred near Seoni (M.P.) at 0300 GMT of 5 Aug 1968 is considered which caused widespread rain in south Rajasthan, west M.P., Gujarat and adjoining Maharashtra. Surface weather chart with grid is given in Fig. 1. A grid with 5×5 points with square base ABCD, where A ($17.5^\circ\text{N}, 73^\circ\text{E}$), B ($17.5^\circ\text{N}, 85^\circ\text{E}$), C ($29.5^\circ\text{N}, 85^\circ\text{E}$), D ($29.5^\circ\text{N}, 73^\circ\text{E}$) are the corner points of the square. Besides sea level 12 isobar surface at 950, 900, 850, 800, 750, 700, 650, 600, 550, 450 & 400 mb levels have been considered along p-coordinate. Sea level pressure, temperature and dew point temperature have been interpolated at each grid point by actual analysis of surface and upper air charts.

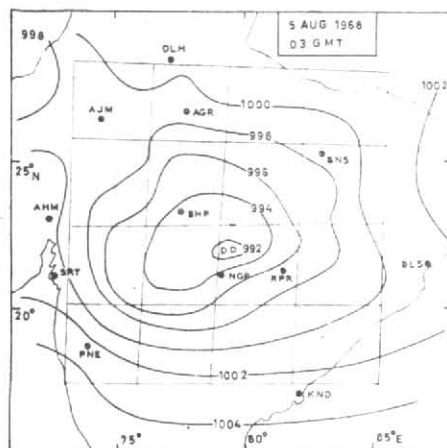


Fig. 1. Isobaric synoptic chart of 03 GMT on 5 August 1968

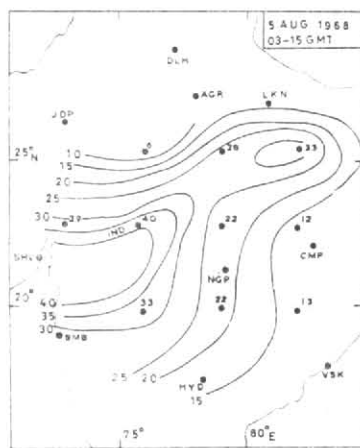


Fig. 2. Rainfall (mm) forecast 03-15 GMT on 5 August 1968

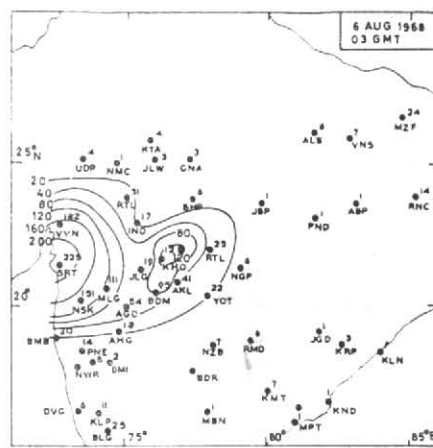


Fig. 3. Rainfall (mm) past 24 hr at 03 GMT on 6 August 1968

3. A dynamical approach

Let us consider that a saturated air column of thickness dp standing over a unit area is ascending with a vertical velocity $\omega = dp/dt$. The rate of precipitation (R) released by this column varies directly as ωdp or

$$R = C \omega dp$$

The coefficient C is a function of temperature, pressure and moisture content of the layer as it represents rate of precipitation released by the layer of unit thickness (1mb) ascending with unit velocity (-1 mb/sec).

From the first law of thermodynamics

$$dQ = c_p dT + g dZ \quad (1)$$

The terms carry their usual meaning. If dq_s is the change in saturated specific humidity in time interval dt the latent heat of condensation released will be $-L dq_s$.

$$\therefore -L dq_s = c_p dT + g dZ \quad (2)$$

The rate of precipitation,

$$R = -\frac{1}{g} \int \frac{dq_s}{dt} dp$$

$$\begin{aligned} \text{From (2)} \quad -L \frac{dq_s}{dt} &= c_p \frac{dT}{dZ} \cdot \frac{dZ}{dt} + g \cdot \frac{dZ}{dt} \\ &= c_p \frac{dZ}{dt} \left(\frac{dT}{dZ} + \frac{g}{c_p} \right) \\ &= c_p W (\gamma_d - \gamma_s) \end{aligned}$$

$$R = \frac{c_p}{g} \int \frac{\gamma_d - \gamma_s}{L} W dp$$

Taking $\omega = 1$ mb sec^{-1} , where we assume $\omega = -g\rho W$ we get :

$$C = \frac{c_p}{g} \int \frac{\gamma_d - \gamma_s}{L} \cdot \frac{1}{-g\rho} dp \quad (3)$$

which can be evaluated numerically.

The S.A.L.R. (γ_s) may be computed by

$$\gamma_s = \frac{g}{c_p} \left(\frac{1 + Lq_s/R_d T}{1 + \epsilon L^2 q_s / c_p R_v T^2} \right) \quad (4)$$

The integrand of (3) is thus expressed as a function of temp. (T), pressure (p) and the specific humidity (q).

$$\therefore C = \int \phi (T, p, q) dp \quad (5)$$

4. Scheme of computation

(1) The actual weather charts (surface and 950, 900,, 400 mb levels) were manually analysed and temp, dew point, u , v components of wind and sea level pressure for surface chart were interpolated for all the grid points.

(2) The constants (in cgs units) used are given as

$$c_p = 1.0048 \times 10^7 \text{ ergs}$$

$$g = 980.665 \text{ cm} \cdot \text{sec}^{-2}; \quad \gamma_d = g/c_p$$

$$L = (734.0 - 0.51T) 4.816 \times 10^7, \text{ where } T \text{ is temp. } (^\circ\text{K})$$

$$R_d = 2.8705 \times 10^6, \text{ erg } \text{gr}^{-1} \text{K}^{-1}$$

$$R_v = 1.609 R_d, \quad \epsilon = .622$$

$$q_s = \epsilon \frac{e_s}{p}, \text{ where } e_s \text{ is saturation vapour pressure given}$$

TABLE 2

Unit : $C(T,p)$ (mm/hr), Pressure in mb

T	1000	950	900	850	800	750	700	650	600	550	500	450	400
-40	0.01	0.01	0.02	0.02	0.02	0.02	0.03	0.03	0.04	0.04	0.05	0.06	0.08
-35	0.02	0.02	0.03	0.03	0.03	0.04	0.04	0.05	0.06	0.07	0.08	0.10	0.13
-30	0.03	0.04	0.04	0.05	0.05	0.06	0.07	0.08	0.09	0.11	0.13	0.16	0.19
-25	0.05	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.13	0.16	0.19	0.23	0.24
-20	0.07	0.08	0.09	0.10	0.11	0.13	0.14	0.17	0.19	0.23	0.27	0.33	0.40
-15	0.10	0.11	0.13	0.14	0.16	0.19	0.20	0.23	0.27	0.31	0.37	0.44	0.55
-10	0.14	0.16	0.17	0.19	0.21	0.24	0.27	0.31	0.36	0.41	0.49	0.58	0.71
-05	0.19	0.21	0.23	0.25	0.28	0.31	0.35	0.40	0.45	0.53	0.52	0.73	0.89
00	0.25	0.27	0.29	0.32	0.36	0.40	0.45	0.50	0.57	0.66	0.76	0.89	1.07
05	0.31	0.33	0.37	0.40	0.44	0.49	0.54	0.61	0.69	0.79	0.90	1.05	1.25
10	0.37	0.40	0.44	0.48	0.53	0.58	0.64	0.72	0.81	0.91	1.04	1.21	1.42
15	0.44	0.47	0.51	0.56	0.61	0.67	0.74	0.82	0.92	1.03	1.17	1.35	1.57
20	0.51	0.54	0.59	0.64	0.69	0.76	0.83	0.92	1.02	1.14	1.29	1.48	1.72
25	0.57	0.61	0.66	0.71	0.77	0.84	0.91	1.01	1.12	1.25	1.40	1.60	1.85
30	0.63	0.67	0.72	0.77	0.84	0.91	0.99	1.09	1.20	1.34	1.50	1.71	1.97
35	0.68	0.73	0.78	0.84	0.90	0.98	1.07	1.17	1.28	1.43	1.60	1.81	2.03
40	0.73	0.78	0.83	0.84	0.96	1.04	1.13	1.24	1.36	1.51	1.69	1.91	2.19

by $e_s = 6.11 \times 10^E$, $E = 7.5 T_d / (T_d + 237.3)$,

T_d : Dew point temp. ($^{\circ}\text{C}$).

(3) For each of the 9 interval grid points (T, p, q) were computed at each level $p_0, 950, \dots, 400$ mb using expression (4) and the values provided above. The integral (3) is evaluated by using Simpson's rule given as :

$$\int_{\text{level 1}}^{\text{level 13}} \phi dp = \frac{dp}{3} \left[\phi_1 + \phi_{13} + 4(\phi_2 + \phi_4 + \dots + \phi_{12}) + 2(\phi_3 + \phi_5 + \dots + \phi_{11}) \right] \quad (6)$$

(4) The values C for various standard levels are computed and provided in Table 1 for ready reference in the use of expression (1).

5. Computation of ω

The vertical velocity ω may be assumed to be zero at tops of the atmosphere and also at each grid point on the side walls of the parallelopiped. It is not justified to make the ω vanished at the bottom of the atmosphere, which is 900 mb level in the present case. At the level, vertical motion depends mainly on the divergence in the frictional layer (100 to 1000 m above ground) and can be computed with the help of Ekman spiral.

As suggested by Das (1969) vertical velocity at the top of frictional layer may be given by :

$$\omega_0 = \frac{-g\rho_0}{f^2} \sqrt{\frac{Kf}{2}} \sin(2\delta) \cdot \nabla^2 \phi$$

where K is the coefficient of eddy viscosity and δ represents the angle between actual and geostrophic winds.

For $g = 980 \text{ cm/sec}^2$, $\rho_0 = 10^{-3} \text{ gm/cm}^3$, $f = 10^{-4} \text{ sec}^{-1}$, $K = 10^4 \text{ cm}^2/\text{sec}$ and $\delta = 15^{\circ}$,

the vertical velocity at $p_0 = 900$ mb level becomes

$$\omega = -35 + 10^6 \nabla^2 \phi,$$

which may be taken as the initial values at the bottom grid points of the parallelopiped. Since the present aim is to illustrate the technique evolved in section 3, for the sake of convenience ω has been worked out by stepwise integration of horizontal convergence as given below :

$$\omega = - \int_p \nabla_H \cdot \mathbf{V} dp$$

instead of by solving ω equation (Haltiner 1971).

6. Results and discussion

(1) Table 1 provides the values of C (in mm/hr) representing rate of precipitation at 1000-400 mb level for $\omega = -1 \text{ mb/sec}$, $\Delta p = 1 \text{ mb}$.

Thus for a saturated air parcel C ranges from .013 mm/hr at -40°C to 0.732 at $+40^{\circ}\text{C}$ at 1000 mb level.

(2) As it proceeds to upper atmosphere a gradual increase in corresponding values of C is observed. This may be attributed to the exponential increase in the thickness of layer for 1 mb interval.

(3) The precipitation may occur even when the entire column of air may not be fully saturated. From the observed values of T and T_d in respect of a particular layer the values of C can be estimated using the technique described in sections 3 and 4. This takes into account the problem of overestimation under the presumption of saturated air as pointed out by Fulk (1935) and others.

(4) The use of these tables for the computation of rate of precipitation is illustrated below in respect of a depression of 5 August 1968 :

Pressure layer	C	ΔP	$\omega(\times 10^{-3})$ (mb/sec)	R (mm/hr)
Surface-950	.77	43	-6.6	.025
950-900	.80	50	-22.3	.089
900-850	.84	50	-26.4	.111
850-800	.89	50	-22.7	.110
800-750	.92	50	-25.2	.116
750-700	.95	50	-33.4	.159
700-650	.98	50	-39.8	.193
650-600	1.00	50	-42.7	.223
600-550	1.01	50	-46.6	.235
550-500	1.01	50	-54.2	.228
500-450	1.02	50	-44.2	.226
450-400	1.04	50	-43.3	.225
Total=				1.94 mm/hr.

(5) The predicted rainfall pattern for 12 hr (03-15 GMT) is presented in Fig. 2. As the self recording rainfall data for many stations are not available, a direct comparison of observed and predicted rainfall is not feasible. However, 24 hourly rainfall recorded at 03 GMT on 6 August 1968 are provided in Fig. 3 which reveals that a general pattern of rainfall distribution is in fair degree of agreement with the predicted one.

Limitations

(1) The reliability factor will decrease very fast as the forecast range exceeds a span of 3 hours because the

various input parameters are presumed to be constant in time. This is obviously unrealistic. If this technique is to be applied for longer duration, there is a need to introduce the temporal variability factors.

(2) Due to scanty upper air network we are bound to keep a grid length of 2° to 3° latitude which is not desirable when we are interested for computing the rainfall intensities.

(3) The theory assumes that the rate of precipitation is equal to the rate of change of saturated specific humidity. In the complex atmospheric process all the condensed material may not fall as rain. The evaporation, as the material passed to lower levels should also be considered.

(4) The vertical velocity is generated under many influences such as vorticity advection, temperature advection, diabatic heating and orographic lifting. There has to be a suitable technique which takes care of these factors in right proportions. The quasigeostrophic ω -equation described by Haltiner (1971) and others does consider the first three factors, but it is well known that in practice it generally gives erroneous results although the equations are theoretically sound. The errors in vertical velocity calculations keep accumulating with height unless some corrective measures are adopted.

(5) The Eqn. (5) does not take into account the details of large scale precipitation or cumulus convection. As such the results of point precipitation should be used with caution for isohyetal analysis.

Acknowledgements

The authors are grateful to Shri S.K. Das, Director General of Meteorology for permitting to present this paper. Authors are also grateful to Dr. R.P. Sarker, ADGM and Shri S.R. Puri, Director for the encouragement and keen interest in this study.

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