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A numerical method for computing stream function and velocity potential from observed wind field

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सार — क्षैतिज पवन क्षेत्र की भ्रमिलता एवं अपसरण से प्रवाही फलन एवं वेग विभव गणना के लिए एक संख्यात्मक विधि को स्पष्ट किया गया है । उसमें एक मिश्रित ढंग की परिसीमा दशा का सुझाव दिया गया है । अनेक प्रचलित विधियों और इस अध्ययन नें प्रस्तावित विधि के आधार पर गणनाएं की गई हैं। इस नई विधि में प्रेक्षित पवन क्षेत्र को आधारभूत आंगत माना गया है और क्षोभमण्डलीय पट्टी (3.75°- 30° उ० एवं 45°-105° पू०) के ऊपर एक गोलाकार ग्रिड का अध्ययन किया गया है ।

क्षैतिज पवन क्षेत्रों को प्रवाही फलन एवं वेग विभव की गणना से प्राप्त क्षेत्रों से पुनः निर्मित किया है । फिर उनकी प्रेक्षित पवन क्षेत्रों से तुलना की है । अनेक विधियों से गुणात्मक एवं मात्रात्मक दोनों तरह की तुलनाओं के परिणाम यह प्रकट करते है कि इस अध्ययन में बताई विधि से पुनः निर्मित कुल पवन मौलिक प्रेक्षित पवन का सर्वोत्तम आकल है ।

ABSTRACT. A numerical procedure is illustrated for computing stream function and velocity potential fields from the vorticity and divergence of the horizontal wind field. A mixed type of boundary condition is suggested in the present work. The computations are performed with a number of existing methods and the method proposed in this study, on a spherical grid over the tropical belt (3.75N-30N and 45E-105E) with observed wind field as basic input.

Horizontal wind fields are reconstructed from the computed fields of stream function and velocity potential and compared with the observed wind field. The results of both qualitative and quantitative intercomparison among the various methods show that the reconstructed total wind obtained by the method suggested in the present study is the best representation of the original observed wind.

1. Introduction

Importance of stream function field calculated from the horizontal wind field can hardly be over emphasized (Bedient and Vederman 1964, Shukla and Saha 1974, Mohanty 1978). It is widely accepted that in the low latitudes the wind field is more reliable than the geopotential field for various diagnostic and prognostic studies, related to tropical weather systems (Yanai and Nitta 1967; Gorden *et al.* 1972). The stream function ψ and velocity potential X which resolve the observed wind field into its non-divergent and irrotational parts are now widely used for diagnostic (Krishnamurthy and Ramanathan 1980, Sumi and Murakami 1981) and prognostic (Vanderman 1962; Sikka 1975; Kivganov and Mohanty 1979) studies. Further, in global spectral models the scalar ψ and X fields are preferred to the vector wind field to avoid the problems related to singular points. The present scheme is developed for its use in diagnostic studies which require analysis based on wind observations and also for the reconstruction of balanced fields of wind and geopotential height, used in numerical simulation and prediction problems as initial data. The main objective of this study is to compute more reliable non-divergent stream function and irrotational velocity potential fields which can represent accurately the observed wind field as the outcome of all meteorological studies are sensitive to initial data sets.

The basic problem of computation of stream function ψ and velocity potential X lies in solving the following equations, derived from Helmholtz theorem :

$$\nabla_H^2 X = \nabla_H \cdot \mathbf{V}_H = D \qquad (1)$$

$$\nabla_H^2 \psi = \mathbf{k} \cdot \nabla_H \times \mathbf{V}_H = \zeta \qquad (2)$$

where,

$\mathbf{V}_H$ — horizontal wind vector,

$\mathbf{k}$ — unit vector normal to the earth's surface,

∇_H — horizontal gradient operator,

D — divergence of horizontal wind and

ζ — vertical component of vorticity.

For a limited domain, the main problem in solving these Poisson's partial differential equations is to specify the values of ψ and X at the boundary points. A number of recent studies have been concerned with numerical computation of ψ and X fields from the wind field and also with the specification of suitable boundary conditions (Phillips 1958, Sangster 1960,

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Brown and Neilon 1961, Bedient and Vederman 1964, Hawkins and Rosenthal 1965, Tangri 1966, Mancuso 1967, Yania and Nitta 1967, Shukla and Saha 1974, Kivganov and Mohanty 1978). Since no information of the values of ψ and X at the boundary points is available from the physics of the problem (Miyakoda 1962), it is difficult to arrive at a definite conclusion on perfect boundary conditions in spite of these large number of studies. In general, three types of boundary conditions are used for the solution of Poisson equations, namely, Dirichlet's conditions, Neumann's conditions and mixed boundary conditions. Though ψ and X have no physical significance of their own, the gradient of these scalar fields represent the wind components and, therefore, the following boundary conditons are widely in use :

$$\mathbf{n} \cdot \mathbf{V}_H = V_n = \mathbf{n} \cdot (\mathbf{k} \times \nabla_H \psi + \nabla_H X)$$
$$= -\frac{\partial \psi}{\partial s} + \frac{\partial x}{\partial n} \tag{3}$$

$$\mathbf{s} \cdot \mathbf{V}_H = V_s = \mathbf{s} \cdot (\mathbf{k} \times \nabla_H \psi + \nabla_H X)$$
$$= \frac{\partial \psi}{\partial n} + \frac{\partial X}{\partial s} \tag{4}$$

where,

- **n** — unit vector normal to the boundary and directed outward.
- **s** — unit vector along the boundary, reckoned positive in the counter-clockwise direction.

Root mean square vector error between the observed wind field and the derived wind fields from ψ and X is used by various authors (Hawkins and Rosenthal 1965, Bedient and Vederman 1964, Shukla and Saha 1974) as a criterion of determining the goodness of a particular method. In any case if closeness of the derived winds to the observed wind field is the main requirement; besides above quantitative estimate, qualitative comparison of these two analysed fields which provide spatial characteristics of the flow pattern should also be given due weightage.

This paper describes a numerical iterative procedure for solving Eqns. (1) and (2) with a new method of specifying the boundary conditions. This scheme is so designed that it allows the boundary values of ψ and X the freedom to vary with and adjust to the interior grid point values at each iteration. The results of this new method are compared with those of some of the existing schemes and it is found that the root mean square vector errors between the observed and the reconstructed total and non-divergent wind fields are minimum in the present scheme.

2. Review of earlier works

Shukla and Saha (1974) and Mohanty (1978) gave a detail review of the earlier works on the computation of ψ and X from horizontal wind field. For the sake of completeness, we will describe only those methods which are considered in this study for the purpose of comparison.

One of the approaches for solving the Poisson Eqn. (2) for ψ is to prescribe a value of ψ at the boundary and keep it constant during the process of iteration. Such type of boundary conditions are of first kind and are known as Dirichlet type of boundary conditions. The simplest one is to solve Eqn. (2) with boundary value of ψ as zero (Tangri 1966). Now onwards this approach will be referred to as method I.

Another approach of Dirichlet type is to obtain ψ value at the boundary by using the observed values of V_n and integrating the equation :

$$\frac{\partial \psi}{\partial s} = -V_n \tag{5}$$

This type of approach is used by Phillips (1958), Brown and Neilon (1961), Bedient and Vederman (1964), Sumi and Murakami (1981) and others. The integration of Eqn. (5) over a closed domain gives rise to two different values (starting point and end point values) of ψ at the same point. In order to avoid this discrepancy in integration the difference is distributed uniformly among all the boundary points. This technique will be referred to as method II.

Sangster (1960) suggested for the first time a scheme which takes into account the effect of irrotational velocity potential component in deriving non-divergent stream function at the boundary. In this scheme first Eqn. (1) was solved for X with $X = 0$ at the boundary. Subsequently ψ at the boundary was derived by using Eqn. (3) and derived X field. These boundary values of ψ were kept constant during the process of solving Eqn. (2) for ψ at the interior points. This scheme was extensively used by Hawkins and Rosenthal (1965). This method will, hereafter, be referred to as method-III.

More recently Shukla and Saha (1974) proposed an iterative scheme for computation of ψ and X which is essentially an extension of the Sangster method. In this approach the values of ψ and X obtained in the Sangster method are taken only as the first approximation values at the end of first iteration and as ψ was obtained at the boundary by integrating Eqn. (3) by Sangster, similarly X at the boundary was also obtained by integrating Eqn. (4) along the boundary in the second step of iteration using the ψ values of the previous step and the new X field was computed by relaxing Eqn.(1). With these new values of X, the ψ values at the boundary were obtained using Eqn. (3) for solving Eqn. (2). This completes second iteration to obtain new values of ψ and X. This procedure was repeated for a number of times N for which the root mean square vector error between the observed wind and reconstructed total wind was minimum. It was found by these authors that a value of $N=4$ satisfied this criterion. This approach will, hereafter be referred to as method IV. It may be noted that method IV with $N=1$ corresponds to method III.

3. Proposed method of computation of ψ and X ψ

The method described in the present study is based on the idea of Sangster (1960) to satisfy the Eqns. (3) and (4) at the boundaries. However, the type of boundary conditions and the numerical technique followed in the present method are quite different from that of the Sangster method and the method of Shukla and Saha (1974), the latter being essentially an extention of the former. In solving Eqns. (1) and (2) Sangster (1960) and Shukla and Saha (1974) have chosen Dirichlet type of boundary conditions. In the present method, an attempt has been made to use Neumann's boundary condition in solving Poisson's Eqns. (1) and (2) which allows the boundary values of stream functions and velocity potentials the freedom to vary with and to adjust to changing interior grid points values during the process of iterative convergence of the scheme. However like the approach of Shukla and Saha (1974), at the first stage in order to obtain the X field, Sangster's approach is adopted, *i.e.*, Eqn. (1) is solved with $X=0$ at the boundary points (Dirichlet's boundary condition). After the evaluation of $\partial X/\partial n$ from the calculated values of X, ψ is obtained at the boundary points by integrating Eqn. (3). The solution of a Poisson partial differential equation for ψ with normal derivative boundary conditions leads to :

$$\frac{\partial \psi}{\partial n} = G(x, y) \tag{6}$$

where $G(x, y)$ is an unknown function of the horizontal coordinates.

As shown by Miyakoda (1962), in this case, Eqn. (6) should agree with the Eqn. (2), *i.e.*, should satisfy the Stoke's theorem, *i.e.*,

$$\iint \zeta \, dx\, dy = \iint \nabla^2 \psi \, dx\, dy = \oint \frac{\partial \psi}{\partial n} dc$$

$$\text{or} \quad \iint \zeta \, dx\, dy = \oint G(x, y)\, dc \tag{7}$$

From Eqns. (7) and (4), we have :

for east and west boundaries,

$$\frac{1}{a\cos\phi}\frac{\partial \psi}{\partial \lambda} = v - \frac{1}{a}\frac{\partial X}{\partial \phi} \tag{8}$$

and for north and south boundaries,

$$\frac{1}{a}\frac{\partial \psi}{\partial \phi} = -u + \frac{1}{a\cos\phi}\frac{\partial X}{\partial \lambda} \tag{9}$$

Similarly for the solution of Eqn. (1), we have the following boundary conditions :

for east and west boundaries,

$$\frac{1}{a\cos\phi}\frac{\partial X}{\partial \lambda} = u + \frac{1}{a}\frac{\partial \psi}{\partial \phi} \tag{10}$$

and for north and south boundaries,

$$\frac{1}{a}\frac{\partial X}{\partial \phi} = v - \frac{1}{a\cos\phi}\frac{\partial \psi}{\partial \lambda} \tag{11}$$

where $\triangle\phi$ and $\triangle\lambda$: latitudinal and longitudinal grid increments on a spherical domain respectively,

a : radius of earth and

ϕ : latitude.

This implies that the gradient of stream function normal to the boundary is specified by the non-divergent wind component tangent to the boundary. And the gradient of velocity potential normal to the boundary is given by irrotational wind component normal to the boundary.

These new ψ values at the boundary and the boundary conditions (8) and (9) may be used for solving the Eqn. (2) to get a new distribution of ψ field over the entire domain. This completes the first step to obtain ψ and X fields. In the next step, if desired, the previously obtained values of ψ are used to evaluate $\partial\psi/\partial n$ and new boundary values of X are obtained from the Eqn. (4). At this second step instead of Dirichlet's conditions normal derivative boundary conditions (10) and (11) are used to obtain X field by iterative solution of Eqn. (1) in a manner similar to that used for ψ. The new values of X may be used then to evaluate $\partial X/\partial n$ in Eqn. (3) which may be integrated again to obtain new ψ values at the boundary. These values of ψ at the boundary may be further used to obtain new values of ψ over the entire domain by a procedure similar to the first step. Thus we get new values of ψ and X in the second iteration. A number of such iterative steps may be performed to get values of ψ and X. An attempt is made to compute the root mean square vector error (r.m.s.v.e.) between the wind fields reconstructed from the ψ and X fields and the original wind field at each step so as to find optimum number of iterative steps (N) for which the r. m. s. v. e. is minimum. Such an iterative scheme was followed by Shukla and Saha (1974). However, in our computations both with the present scheme and the method of Shukla and Saha, the optimum number (N) of iterative steps is not constant and varies between 1 & 5. The present method will, hereafter, be referred to as method V.

4. Numerical solution

Computations are performed over a spherical grid. In a spherical coordinate system the position of a particle may be given in terms of latitude ϕ, longitude λ and radial distance r from the earth's centre. It is assumed that r=constant=a, where a=radius of earth. In such a system of coordinates, the expressions for divergence, vorticity and Laplacian operator ∇^2 are given as :

$$\nabla_H \cdot \mathbf{V}_H = \frac{1}{a\cos\phi}\frac{\partial u}{\partial \lambda} + \frac{1}{a}\frac{\partial v}{\partial \phi} - \frac{v}{a}\tan\phi \tag{12}$$

$$\mathbf{k}\cdot\nabla_H \times \mathbf{V}_H = \frac{1}{a\cos\phi}\frac{\partial v}{\partial \lambda} - \frac{1}{a}\frac{\partial u}{\partial \phi} + \frac{u}{a}\tan\theta \tag{13}$$

$$\nabla_H^2 = \frac{1}{a^2\cos^2\phi}\frac{\partial^2}{\partial \lambda^2} + \frac{1}{a^2}\frac{\partial^2}{\partial \phi^2} - \frac{\tan\phi}{a^2}\frac{\partial}{\partial \phi} \tag{14}$$

where u and v are components of $\mathbf{V}_H$ along λ-axis and ϕ-axis. In the present study, constant grid spacing of $\triangle\lambda = \triangle\phi = 1.875°$ has been used. Centred space finite difference scheme is used for calculation of divergence and vorticity. The Laplacian is expressed in 5-point finite difference form.

In a regular latitude longitude spherical grid

($\triangle\lambda = \triangle\phi$ = constant), if we define :

$$DX = a\cos\phi \triangle\lambda \tag{15}$$

$$DY = a \triangle \phi \tag{16}$$

then DX will be a function of latitude ϕ and will be equal to DY only at the equator.

Using Eqns. (12) to (16); the Eqns. (1) and (2) can be represented in the following finite difference forms :

$$\frac{X(I+1,J)+X(I-1,J)-2X(I,J)}{DX(J)\times DX(J)}+\frac{X(I,J+1)+X(I,J-1)-2X(I,J)}{DY\times DY}-\frac{[X(I,J+1)-X(I,J-1)]}{2.0\times DY\times a}\tan\phi(J)$$
$$=\frac{u(I+1,J)-u(I-1,J)}{2.0\times DX(J)}+\frac{v(I,J+1)-v(I,J-1)}{2.0\times DY}-\frac{v(I,J)}{a}\tan\phi(J) \quad (17)$$

and

$$\frac{\psi(I+1,J)+\psi(I-1,J)-2\psi(I,J)}{DX(J)\times DX(J)}+\frac{\psi(I,J+1)+\psi(I,J-1)-2\psi(I,J)}{DY\times DY}-\frac{[\psi(I,J+1)-\psi(I,J-1)]}{2\cdot 0\times DY\times a}\tan\phi(J)$$
$$=\frac{v(I+1,J)-v(I-1,J)}{2\cdot 0\times DX(J)}-\frac{[u(I,J+1)-u(I,J-1)]}{2\cdot 0\times DY}+\frac{u(I,J)}{a}\tan\phi(J) \quad (18)$$

As discussed in section 3, in order to solve these Poisson equations at the boundary points, normal derivative boundary conditions (3) and (4) are used to eliminate the unknowns outside the domain. In the numerical approximation of the boundary conditions (8)-(11) for the four boundaries, the gradients are always taken outwards from the boundary.

In order to obtain ψ at the boundary points, the modified version of the partial differential Poisson Eqn. (18) with the help of normal derivative boundary conditions, (8) and (9) for the four boundaries are as follows:

for west boundary $(I=1)$:

$$\psi(1,J)=G(J)\frac{[\psi(2,J)}{[DX(J)]^2}+\frac{\psi(1,J+1)+\psi(1,J-1)}{(DY)^2}-\frac{\{\psi(1,J+1)-\psi(1,J-1)\}}{2.0\times a\times DY}\tan\phi(J)]+$$
$$+G(J)\left[\frac{u(1,J+1)-u(1,J-1)}{2.0\times DY}-\left\{\frac{v(1,J)+v(2,J)}{2.0\times DX(J)}\right\}+\frac{X(1,J)-X(1,J-1)}{DY\times DX(J)}-\frac{u(1,J)}{a}\tan\phi(J)\right] \quad (19)$$

for east boundary $(J=M)$:

$$\psi(M,J)=G(J)\left[\frac{[\psi(M-1,J)}{[DX(J)]^2}+\frac{\psi(M,J+1)+\psi(M,J-1)}{(DY)^2}-\frac{\{\psi(M,J+1)-\psi(M,J-1)\}}{2.0\times DY\times a}\tan\phi(J)\right]+$$
$$+G(J)\left[\frac{u(M,J+1)-u(M,J-1)}{2.0\times DY}+\frac{v(M,J)+v(M-1,J)}{2.0\times DX(J)}-\frac{\{X(M,J)-X(M,J-1)\}}{DY\times DX(J)}-\frac{u(M,J)}{a}\tan\phi(J)\right] \quad (20)$$

for south boundary $(J=1)$:

$$\psi(I,1)=F1\left[\frac{\psi(I,2)}{(DY)^2}+\frac{\psi(I+1,1)+\psi(I-1,1)}{DX(1)\times DX(1)}-\frac{\{\psi(I,2)-\psi(I,1)\}}{DY\times a}\tan\phi(1)]\right]-$$
$$-F1\left[\frac{v(I+1,1)-v(I-1,1)}{2.0\times DX(1)}-\frac{\{u(I,1)+u(I,2)\}}{2.0\times DY}+\frac{X(I,1)-X(I-1,1)}{DX(1)\times DY}+\frac{u(I,1)}{a}\tan\phi(1)\right] \quad (21)$$

and for north boundary $(J=N)$:

$$\psi(I,N)=FN\left[\frac{\psi(I,N-1)}{(DY)^2}+\frac{\psi(I+1,N)+\psi(I-1,N)}{[DX(N)]^2}-\left\{\frac{\psi(I,N)-\psi(I,N-1)}{DY\times a}\right\}\tan\phi(N)\right]-$$
$$-FN\left[\frac{v(I+1,N)-v(I-1,N)}{2.0\times DX(N)}+\frac{u(I,N)+u(I,N-1)}{2.0\times DY}-\left\{\frac{X(I,N)-X(I-1,N)}{DX(N)\times DY}\right\}+\frac{u(I,N)}{a}\tan\phi(N)\right] \quad (22)$$

Similarly for obtaining X, the modified versions of the Poisson equation at the boundaries in the finite difference form can be expressed with the help of relations (17), (10) and (11) as :

for west boundary $(I=1)$:

$$X(1,J)=G(J)\left[\frac{X(2,J)}{[DX(J)]^2}+\frac{X(1,J+1)+X(1,J-1)}{DY\times DY}-\left\{\frac{X(1,J+1)-X(1,J-1)}{2.0\times DY\times a}\right\}\tan\phi(J)\right]-$$
$$-G(J)\left[\frac{u(1,J)+u(2,J)}{2.0\times DX(J)}+\frac{v(1,J+1)-v(1,J-1)}{2.0\times DY}+\frac{\psi(1,J)-\psi(1,J-1)}{DX(J)\times DY}-v\frac{(1,J)}{a}\tan\phi(J)\right] \quad (23)$$

for east boundary $(I=M)$:

$$X(M,J)=G(J)\left[\frac{X(M-1,J)}{[DX(J)]^2}+\frac{X(M,J+1)+X(M,J-1)}{DY\times DY}-\left\{\frac{X(M,J+1)-X(M,J-1)}{2.0\times a\times DY}\right\}\tan\phi(J)\right]+$$
$$+G(J)\left[\frac{u(M,J)+u(M-1,J)}{2.0\times DX(J)}-\left\{\frac{v(M,J+1)-v(M,J-1)}{2.0\times DY}\right\}+\frac{\psi(M,J)-\psi(M,J-1)}{DX(J)\times DY}+\frac{v(M,J)}{a}\tan\phi(J)\right] \quad (24)$$

for south boundary $(J=1)$:

$$X(I,1)=F1\left[\frac{X(I,2)}{(DY)^2}+\frac{X(I+1,1)+X(I-1,1)}{[DX(1)]^2}-\frac{\{X(I,2)-X(I,1)\}}{DY\times a}\tan\phi(I)\right]-$$
$$-F1\left[\frac{u(I+1,1)-u(I-1,1)}{2.0\times DX(1)}+\frac{v(I,1)+v(I,2)}{2.0\times DY}-\left\{\frac{\psi(I,1)-\psi(I-1,1)}{DX(1)\times DY}\right\}-\frac{v(I,1)}{a}\tan\phi(1)\right] \quad (25)$$

and for north boundary $(J=N)$:

$$X(I,N)=FN\left[\frac{X(I,N-1)}{DY\times DY}+\frac{X(I+1,N)+X(I-1,N)}{[DX(N)]^2}-\frac{\{X(I,N)-X(I,N-1)\}}{DY\times a}\tan\phi(N)\right]-$$
$$-FN\left[\frac{u(I+1,N)-u(I-1,N)}{2.0\times DX(J)}-\left\{\frac{v(I,N)+v(I,N-1)}{2.0\times DY}\right\}+\frac{\psi(I,N)-\psi(I-1,N)}{DX(N)\times DY}-\frac{v}{a}(I,N)\tan\phi(N)\right] \quad (26)$$

where,
$$F1=\frac{DX(1)\times DX(1)\times DY\times DY}{2.0\times DY\times DY+DX(1)\times DX(1)}$$

$$FN=\frac{DX(N)\times DX(N)\times DY\times DY}{2.0\times DY\times DY+DX(N)\times DX(N)}$$

$$G(J)=\frac{DX(J)\times DX(J)\times DY\times DY}{DY\times DY+DX(J)\times DX(J)\times 2.0}$$

To obtain ψ field over the entire domain we have to simultaneously solve a set of 5 Poisson partial differential Eqns. (18)-(22) by an iterative scheme. And similarly for obtaining X field the Eqns. (17) and (23)-(26) are to be solved simultaneously.

The accelerated Liebmann relaxation technique is used with an over relaxation coefficient equal to 0.7 as suggested by Miyakoda (1962) and Shukla and Saha (1974).

In using such an iterative scheme, it is necessary to provide a reasonable intitial approximation to the solution. As the convergence capability of the iterative method is sensitive to the initial approximation to the solution, it is important to start the iterative solution with a reasonably accurate initial guess for ψ and X fields. There are a number of ways to obtain the initial approximation for ψ and X. In the present method the actual wind fields are used to obtain initial value for ψ and X from the relations (3) and (4). In the first step, the X field is determined for the interior points by solving Eqn. (17) with $X=0$ as the initial guess over the entire domain. While initial values of ψ in all steps and X in the subsequent steps are obtained using Eqns. (3) and (4) to step along from point to point throughout the entire domain starting with zero values at the first point. In the subsequent steps the value at the starting point is also modified by linear interpolation from the values at the surrounding 6 points.

5. Results

In order to evaluate the efficiency of the various methods, both qualitative and quantitative intercomparisons were carried out. For the purpose of qualitative evaluation, isotach and streamline analyses were carried out over the domain for the actual wind field and the reconstructed wind fields from the ψ and X fields obtained by the various methods. Stream function isopleth analysis was also done in order to compare the computed stream functions with streamline analysis of the actual wind. And as a measure of the quantitative intercomparison, the following parameters were computed. :

(a) Root mean square vector error (r.m.s.v.e.) between the actual wind and the reconstructed non-divergent wind field (r_ψ),

$$r_\psi=\sqrt{\frac{1}{M}\Sigma\left[(u_a-u_\psi)^2+(v_a-v_\psi)^2\right]} \quad (27)$$

(b) Root mean square vector error between the actual wind field and the reconstructed total wind field $r_{\psi X}$,

$$r_{\psi X}=\sqrt{\frac{1}{M}\Sigma\left[u_a-u_{\psi X})^2+(v_a-v_{\psi X})^2\right]} \quad (28)$$

where u_a and v_a are the observed horizontal wind components, u_ψ, v_ψ are the non-divergent horizontal wind components obtained from the ψ field and $u_{\psi X}$, $v_{\psi X}$ are the total horizontal wind components calculated by the relations (3) and (4) from ψ and X fields. M=Total number of grid points considered for verification.

The basic data for computation for ψ and X fields consists of observed wind components u_a and v_a at 1.875° regular latitidue-longitude grid points obtained from the FGGE level-IIIb data archieves of European Centre for Medium Range Weather Forecast, U.K. The area of computation extends from 45°E to 105°E and 3.75°N to 30°N. As 50 per cent of this area is covered by data sparse oceanic region, it is difficult to have a reliable wind fields at regular grid points to undertake a comparative study. This problem is overcome in the present study as FGGE provided a unique data base for the first time, for this region.

5.1. *Quantitative intercomparison*

In the present study data for 500 mb of 0000 GMT on 1 December 1978 has been used for intercomparison of the methods I-V, described in the earlier sections.

The verification parameters r_ψ and $r_{\psi X}$ computed for the methods I-V are presented in Table 1. From

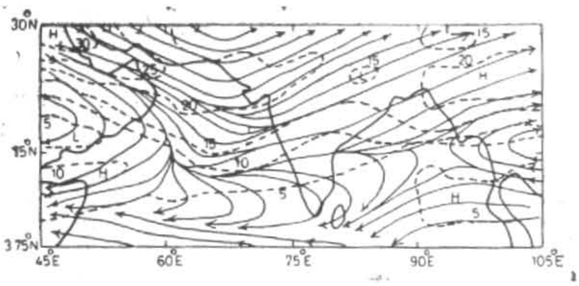

Fig. 1. The stream lines and isotachs of the original wind field at 500 mb for 0000 GMT, 1 December 1978 (isotachs in m/sec)

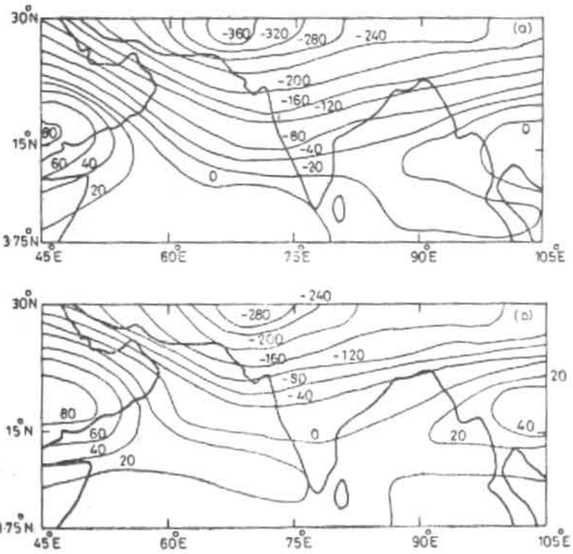

Fig. 2. The stream function fields at 500 mb for 0000 GMT 1 December 1978, computed by (a) method IV and (b), method V (units $10^5 m^2/sec$)

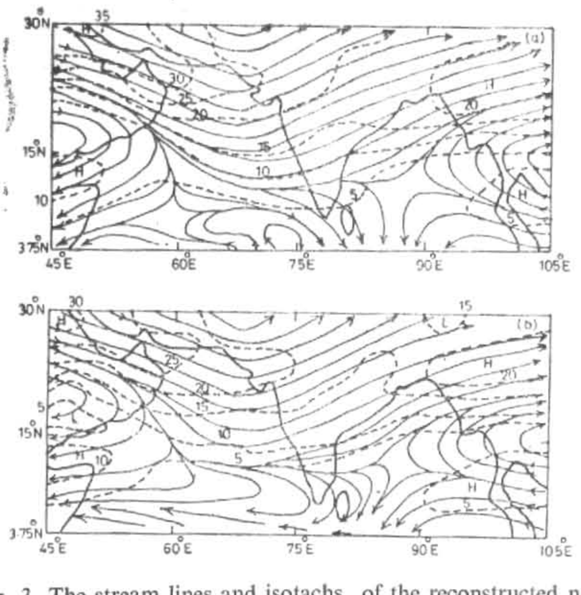

Fig. 3. The stream lines and isotachs of the reconstructed non-divergent wind, obtained from the stream function fields, computed by (a) method IV and (b) method V (isotachs in m/sec)

TABLE 1

Intercomparison among the results of computed reconstructed wind field from ψ and X by different methods

Verification parameter	Method					
	I	II	III	IV		V
				$N=1$	$N=3$	$N=1$
r_{ψ}	11.33	2.83	2.30	2.30	3.49	2.02
$r_{\psi X}$	8.54	2.64	1.59	1.59	1.38	1.17

TABLE 2

Intercomparison between the results for the method by Shukla and Saha (1974) (method IV) and the method suggested in the present study (method V)

Date (May '79)	Method		r_{ψ}	$r_{\psi X}$
16	IV	$N=1$	1.371	.899
	V	$N=1$	1.230	.695
17	IV	$N=1$	1.645	.857
		$N=4$	4.465	.839
	V	$N=1$	1.558	.666
18	IV	$N=1$	1.332	.802
	V	$N=1$	1.279	.640
19	IV	$N=1$	1.470	.696
		$N=2$	1.688	.691
	V	$N=1$	1.433	.629
20	IV	$N=1$	1.461	.862
		$N=5$	4.226	.684
	V	$N=1$	1.302	.593
25	IV	$N=1$	1.630	.945
		$N=2$	1.998	.923
	V	$N=1$	1.604	.895
		$N=3$	1.954	.869
26	IV	$N=1$	1.359	.884
		$N=2$	1.881	.844
	V	$N=1$	1.227	.667
27	IV	$N=1$	1.592	.963
		$N=5$	3.952	.838
	V	$N=1$	1.491	.786
		$N=5$	1.808	.761
28	IV	$N=1$	1.333	.781
		$N=2$	2.673	.752
	V	$N=1$	1.296	.707
		$N=5$	1.969	.680

these results the following interesting remarks can be made :

(*i*) In case of all the 5 methods the value of $r_{\psi X}$ is lower than that of r_{ψ}. This is due to the fact that the inclusion of irrotational component into the non-divergent wind field always gives a better representation of the original observed wind than the non-divergent wind field alone.

(*ii*) Intercomparison of the methods I-IV confirms the earlier results of Shukla and Saha (1974). However, from the results of methods IV and V, it may be noted that both r_{ψ} and $r_{\psi X}$ are the least for the method V.

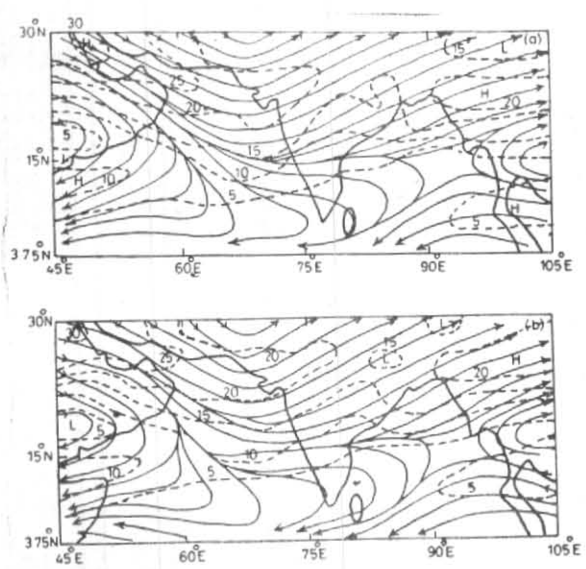

Fig. 4. The stream lines and isotachs of the reconstructed total wind obtained from the stream function and velocity potential fields, computed by (a) method IV and (b) method V (isotachs in m/sec)

(*iii*) In order to obtain the optimum number of steps (N) at which stage $r_{\psi\chi}$ is minimum, at each step r_ψ is calculated and compared with the values at the previous steps and this process is continued for a large number of steps (N=30). It is found that the minimum $r_{\psi\chi}$ occurred at N=3 in method IV and at N=1 in method V. From later experiments with a large number of cases, it was found, in contrast to the results of Shukla and Saha (1974), that the minimum $r_{\psi\chi}$ did not occur for a fixed value of N.

(*iv*) For both the methods IV and V, it is observed that r_ψ is minimum for N=1 and then it started diverging with the increase in number of steps and thus r_ψ and $r_{\psi\chi}$ were not always minimum at the same stage.

In order to confirm the above findings methods IV and V were tried for a number of cases with FGGE level IIIb data for 500 and 850 mb level during the period of 16 to 31 May 1979.

The results for all these cases confirm the above findings based on Table 1 for a single observation. The results for only nine cases (16-20 May 1979, 25-28 May 1979) are illustrated in Table 2, for further discussion. Table 2 reveals the following features :

(*i*) In all cases method V gives the least values of r_ψ and $r_{\psi\chi}$.

(*ii*) The minimum value of $r_{\psi\chi}$ by methods IV and V are not always obtained at the same stage. Further as stated above, optimum values of N for methods IV and V are not fixed values but vary between 1 & 5.

(*iii*) In all cases, values of both r_ψ and $r_{\psi\chi}$ by method V for N=1 are smaller than the corresponding values obtained by method IV.

(*iv*) The method V improves the reconstructed total wind $V_{\psi\chi}$ with negligible deterioration of V_ψ field, which is quite important in computation of balanced geopotential height from wind fields in the tropics whereas improvement of $r_{\psi\chi}$ by method IV leads to remarkable increase in r_ψ value.

5.2. *Qualitative analysis*

From quantitative intercomparison, it became clear that methods IV and V give considerably lower values of r_ψ and $r_{\psi\chi}$ compared to the corresponding values by other methods (Table 1). Therefore, we have restricted the qualitative evaluation of ψ and X fields only to methods IV and V.

For the purpose of qualitative intercomparison, the streamlines and isotachs of observed wind and reconstructed wind fields and stream functions obtained by methods IV and V are illustrated in Figs. 1-4. From these figures, the following general remarks can be made :

(*i*) In general the stream function fields obtained by both the methods (Fig. 2) are in good agreement with the actual wind field streamlines (Fig. 1). However, the ridge observed south of 15°N is better represented by method V (Fig. 2b). The stream functions by method IV (Fig. 2a) depicts a trough of low extending from 30° N, 67°E to 3.75°N, 105°E (southeast corner of the domain) against the observed trough (Fig. 1) which extends from 30°N, 67°E to 15°N, 63°E only. This trough is well represented in Fig. 2 (b).

(*ii*) There are remarkable differences between streamlines and isotachs of reconstructed wind fields V_ψ from the ψ fields alone by methods IV and V (Fig. 3) at the boundaries whereas they are almost similar in the interior of the domain. The isotachs and streamlines of V_ψ field obtained by method V (Fig. 3b) are in good agreement with the observed wind analysis (Fig. 1) over the entire region. V_ψ obtained by method IV (Fig. 3a) shows westerly flow close to the southern boundary against easterly and northeasterly flow as observed in actual. The magnitude of V_ψ by method IV near the northern boundary is higher compared to the actual wind speed. Also inside the interior region the trough orientation is better represented by method V than by method IV. Besides these major differences, a number of dissimilarities in the small scale processes are evident in V_ψ field by method IV, when compared to the actual wind field which are absent with V_ψ from method V.

(*iii*) The streamline and isotachs of the reconstructed total wind $V_{\psi X}$ from ψ and X fields by methods IV and V (Fig. 4) are almost similar to each other and are in excellent agreement with the observed wind (Fig. 1). The southern end of the trough in $V_{\psi X}$ from method IV (Fig. 4a) is slightly oriented to the east instead to the west as in Fig. 1. Further, though almost all the small scale processes are well represented by $V_{\psi X}$ obtained from method V some of the processes are not properly brought out by method IV.

Thus the qualitative evaluation confirms the quantitative intercomparison findings that though the total wind fields $V\psi_X$ obtained by methods IV and V are very close to the observed wind field the stream function field and hence the reconstructed $V\psi$ field by method IV differs significantly from the actual wind whereas the $V\psi$ obtained by method V shows a good agreement with the observed wind analysis.

6. Conclusions

On the basis of the above results the following general conclusions may be drawn :

(*i*) There is no unique method of specifying the realistic boundary conditions though it appears that the result of computation of ψ and X fields depends heavily upon the type of boundary conditions.

(*ii*) In comparison to a fixed stream function, velocity potential boundary conditions (Dirichlets type) the normal derivative condition is more realistic and less restraining. Further the results obtained in the present study with normal derivative boundary conditions have provided significant improvement of ψ and X fields close to the boundary as they are adjusted to the interior grid point values in the process of iteration.

(*iii*) The quantitative intercomparison among various methods based upon root mean square vector error between the observed and reconstructed wind fields confirms the results of Shukla and Saha (1974) that the method IV is superior to earlier methods (I-III). However, both qualitative and quantitative evaluations with a large number of cases reveal that the method suggested in the present study (Method V) yields better results compared to the method IV.

(*iv*) It is found that in the process of minimizing $r\psi_X$ by a number of interative steps the value of $r\psi$ increases. However the deterioration of $V\psi$ is much less in method V than in method IV.

(*v*) From qualitative intercomparison there is a clear evidence that the quantitative superiority of method V to method IV is mainly due to significant improvement of ψ and X fields at the boundaries.

(*vi*) The present method (method V) with $N=1$ gives quite reasonable results in all the cases which are better than the results of method IV with even $N=5$. Therefore, for practical purposes in order to minimize the computational time, the method V (with $N=1$) may be used without any successive steps of iterations.

From a large number of case studies it appears to the authors that the method proposed in the present study is the most appropriate for use in numerical as well as diagnostic studies in tropics where wind observations are given more weightage. In all these cases, balanced height fields were estimated from the wind field. It is found that the root mean sequare error (r.m.s.e.) between the observed and computed geopotential heights were within the inherent observational error limit. The entire process of computation of ψ, X and balanced geopotential fields does not take more than 2 minutes CPU time on an IBM-360/44 system. These are some of the favourable points of the method V over other methods for computation of ψ and X fields.

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