631.164.24 : 519.216 : 582.548.21

MARKOV CHAIN MODELING OF STOCHASTIC PROCESS DEFINED ON DAILY MINIMUM TEMPERATURE AND ITS APPLICATIONS IN WEATHER BASED CROP INSURANCE SCHEME FOR BANANA

1. The paper investigate probability distribution of daily minimum temperature during winter (November to February) based on meteorological data of Jalgaon weather station for the period of agriculture years 1973-74 to 2015-16. We define event A as the event that daily minimum temperature is below 8°C for consecutive three or more days during the period of November to February. To model the occurrence of extreme low temperature events, in particular event A, we define stochastic process, Y_n taking value 1 if daily minimum temperature less than 8°C on nth day during November to February and 0 otherwise. Degenerate sequence of zeros and Markov chains of different orders are fitted to yearly sequences Y_n. The probability of occurrence of the event A is estimated using distributions of number of success runs in the sequence of Markov Bernoulli trials. Distributions of three random variables as the functions of daily minimum temperature are also reported. Our study results are useful on pricing of weather derivatives in Weather Based Crop Insurance Scheme (WBCIS) for banana.

2. Banana is a popular and important commercial fruit crop grown in tropical and sub tropical part of world. Jalgaon district of Maharashtra state of India contributes about 3% in the world's banana production. Also, productivity of banana in Jalgaon district is about 3.5 times of world's productivity. Jalgaon is often known as the Banana Capital of India.

Banana, basically a tropical crop, grows well in a temperature range of 16°C - 38°C with average relative humidity of 65-85% at morning. In India this crop is being cultivated in climate ranging from humid tropical to dry mild subtropical. Soil for banana should have good drainage, adequate fertility and moisture. Deep, rich loamy soil with pH between 6.5-7.5 is most preferred for banana cultivation. There are some adverse climatic conditions for banana crop such as high temperature(above 40°C) with bright sunshine causes sun scorching, very low temperature (below 8°C) causes chilling injury and high wind speed which exceeds 40 km/hr damages the crop. Out of these adverse climatic conditions for banana crop, in this paper we focus our study on estimation of probability of event A based on extreme low temperature (below 8°C). Since 2011-12, Weather Based Crop Insurance Scheme (WBCIS) for banana has been offered to cultivators by Government of Maharashtra as per the guidelines of Agricultural Insurance Company of India. Under this scheme, banana cultivator receives compensation of amount ₹ 25,000 per hectare if the daily minimum temperature is below 8 °C for consecutive 3 or more days during the period of November to February.

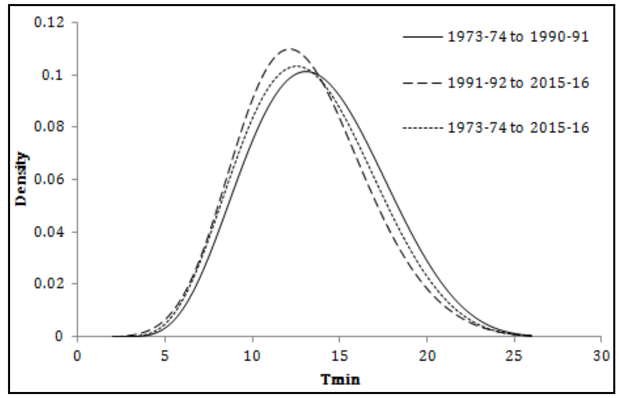

Fig. 1. Change in distribution of T_{min} during November to February

We have collected daily meteorological data for 43 agriculture years (1973-2016) from the Meteorological Observatory situated at Mamurabad near to Jalgaon which is approved by Indian Meteorological Department. These data sets are maintained by Oilseed Research Centre at Jalgaon. This centre works under the Mahatma Phule Krishi Vidyapeeth (MPKV), Rahuri which is Agricultural University in Maharashtra. In this study, we have mainly used data of daily minimum temperature (T_{min}) during winter period (November to February) of each agriculture year from 1973-74 to 2015-16. For the year 2006-07 data were not available for two months November and December. So, we have dropped the year 2006-07 from our study. That is our study is based on the data of 42 agriculture years.

3. *Distribution of T_{min} during winter*: We have obtained month wise descriptive statistics of T_{min} during November to February for distribution of daily minimum temperature during November to February. We have considered two groups for agriculture years as 1973-74 to 1990-91 and 1991-92 to 2015-16. Our results are reported for these two periods and combine period 1973-74 to 2015-16. Table 1 presents month wise descriptive statistics of T_{min} during November to February.

We observed that values of all statistics are decreased for all months and combine four months from period 1973-1991 to 1991-2016. This shows increase in coldness during winter at Jalgaon. Probability distributions fitted to T_{min} using Easy-fit software for all

TABLE 1

Descriptive statistics of T_{min} (°C) for the months November to February

Period	Statistic	Nov	Dec	Jan	Feb	Nov to Feb
1973-74 to 1990-91	Mean	15.59	12.90	12.36	14.12	13.72
	SD	4.17	3.31	3.27	3.27	3.74
	Min	5.60	4.00	3.00	5.40	3.00
	Max	25.60	22.10	21.00	22.00	25.60
	Q1	12.28	10.40	10.10	11.50	11.00
	Q2	15.45	12.90	12.30	14.10	13.40
	Q3	19.00	15.40	15.00	16.50	16.30
1991-92 to 2015-16	Mean	15.05	11.74	11.37	13.50	12.88
	SD	3.76	3.19	3.09	3.10	3.61
	Min	5.00	4.00	2.00	4.50	2.00
	Max	24.80	21.00	20.20	21.70	24.80
	Q1	12.10	9.30	9.00	11.30	10.20
	Q2	15.05	11.50	11.30	13.50	12.50
	Q3	17.93	14.00	13.50	15.70	15.40
1973-74 to 2015-16	Mean	15.28	12.24	11.80	13.77	13.24
	SD	3.95	3.29	3.20	3.19	3.69
	Min	5.00	4.00	2.00	4.50	2.00
	Max	25.60	22.10	21.00	22.00	25.60
	Q1	12.10	9.80	9.50	11.48	10.50
	Q2	15.20	12.00	11.60	13.60	13.00
	Q3	18.50	14.60	14.00	16.20	15.78

TABLE 2

Probability distributions fitted for T_{min}

Month	Period	Distribution	Parameters
Nov	1973-74 to 1990-91	Error	$k = 4.6201$ $\sigma = 4.1724$ $\mu = 15.586$
	1991-92 to 2015-16	Johnson SB	$\gamma = 0.17654$ $\delta = 1.293$ $\lambda = 21.989$ $\xi = 4.7173$
	1973-74 to 2015-16	Error	$k = 3.6587$ $\sigma = 3.9470$ $\mu = 15.28$
Dec	1973-74 to 1990-91	Error	$k = 2.6511$ $\sigma = 3.3071$ $\mu = 12.899$
	1991-92 to 2015-16	Johnson SB	$\gamma = 0.9473$ $\delta = 1.4895$ $\lambda = 22.422$ $\xi = 3.6789$
	1973-74 to 2015-16	Johnson SB	$\gamma = 0.56325$ $\delta = 1.4935$ $\lambda = 22.148$ $\xi = 3.0395$
Jan	1973-74 to 1990-91	Gen. Extreme Value	$k = -0.24806$ $\sigma = 3.2188$ $\mu = 11.155$
	1991-92 to 2015-16	Gen. Extreme Value	$k = -0.24191$ $\sigma = 3.0243$ $\mu = 10.225$
	1973-74 to 2015-16	Gen. Extreme Value	$k = -0.23842$ $\sigma = 3.1311$ $\mu = 10.602$
Feb	1973-74 to 1990-91	Error	$k = 3.1705$ $\sigma = 3.2747$ $\mu = 14.124$
	1991-92 to 2015-16	Gen. Extreme Value	$k = -0.27936$ $\sigma = 3.0991$ $\mu = 12.397$
	1973-74 to 2015-16	Error	$k = 2.6366$ $\sigma = 3.1888$ $\mu = 13.768$
Nov-Feb	1973-74 to 1990-91	Johnson SB	$\gamma = 0.66606$ $\delta = 1.8119$ $\lambda = 29.757$ $\xi = 1.3699$
	1991-92 to 2015-16	Gen. Extreme Value	$k = -0.17989$ $\sigma = 3.4051$ $\mu = 11.465$
	1973-74 to 2015-16	Johnson SB	$\gamma = 0.83663$ $\delta = 1.8864$ $\lambda = 30.812$ $\xi = 0.99715$

TABLE 3

Expected probability of occurrence of event 'A' based on model of Y_n

Year	Model for Y_n (Deg Seq or Markov chain of order 0/1/2)	Distribution of Degenerate Sequence Y_n or Estimated Transition Probabilities of Markov chain Y_n of order 0/1/2				P(A)
1973-74	2	$P_0 = 0.0532$	$P_1 = 0.5882$	$P_2 = 0.4444$	$P_3 = 0.4848$	0.9052
1974-75	2	$P_0 = 0.0526$	$P_1 = 0.4211$	$P_2 = 0.4800$	$P_3 = 0.4828$	0.8366
1975-76	2	$P_0 = 0.0745$	$P_1 = 0.4783$	$P_2 = 0.4194$	$P_3 = 0.4516$	0.9087
1976-77	1	$P_{00} = 0.9732$	$P_{01} = 0.0268$	$P_{10} = 0.4286$	$P_{11} = 0.5714$	0.6364
1977-78	2	$P_0 = 0.0288$	$P_1 = 0.6000$	$P_2 = 0.4375$	$P_3 = 0.4737$	0.7336
1978-79	0	$P_{00} = 0.9917$	$P_{01} = 0.0083$	$P_{10} = 0.9917$	$P_{11} = 0.0083$	0.0000
1979-80	Degenerate	$P(Y_n = 0) = 1$				0
1980-81	0	$P_{00} = 0.9741$	$P_{01} = 0.0259$	$P_{10} = 1.0000$	$P_{11} = 0.0000$	0.0000
1981-82	1	$P_{00} = 0.9815$	$P_{01} = 0.0185$	$P_{10} = 0.1818$	$P_{11} = 0.8182$	0.7675
1982-83	1	$P_{00} = 0.9828$	$P_{01} = 0.0172$	$P_{10} = 0.5000$	$P_{11} = 0.5000$	0.3941
1983-84	1	$P_{00} = 0.9727$	$P_{01} = 0.0273$	$P_{10} = 0.3333$	$P_{11} = 0.6667$	0.7556
1984-85	2	$P_0 = 0.0183$	$P_1 = 0.4286$	$P_2 = 0.4444$	$P_3 = 0.5000$	0.4927
1985-88	Degenerate	$P(Y_n = 0) = 1$				0
1988-89	1	$P_{00} = 0.9739$	$P_{01} = 0.0261$	$P_{10} = 0.7500$	$P_{11} = 0.2500$	0.1706
1989-90	0	$P_{00} = 0.9833$	$P_{01} = 0.0167$	$P_{10} = 0.9833$	$P_{11} = 0.0167$	0.0000
1990-91	1	$P_{00} = 0.9913$	$P_{01} = 0.0087$	$P_{10} = 0.2000$	$P_{11} = 0.8000$	0.4809
1991-92	1	$P_{00} = 0.9730$	$P_{01} = 0.0270$	$P_{10} = 0.3750$	$P_{11} = 0.6250$	0.7060
1992-93	1	$P_{00} = 0.9737$	$P_{01} = 0.0263$	$P_{10} = 0.6000$	$P_{11} = 0.4000$	0.3838
1993-94	2	$P_0 = 0.0381$	$P_1 = 0.3846$	$P_2 = 0.4000$	$P_3 = 0.5000$	0.6884
1994-95	0	$P_{00} = 0.9256$	$P_{01} = 0.0744$	$P_{10} = 0.9256$	$P_{11} = 0.0744$	0.0929
1995-96	1	$P_{00} = 0.9828$	$P_{01} = 0.0172$	$P_{10} = 0.6667$	$P_{11} = 0.3333$	0.1991
1996-97	0	$P_{00} = 0.9$	$P_{01} = 0.1$	$P_{10} = 0.9$	$P_{11} = 0.1$	0.1011
1997-98	2	$P_0 = 0.0174$	$P_1 = 0.4000$	$P_2 = 0.4000$	$P_3 = 0.5000$	0.4381
1998-99	2	$P_0 = 0.0667$	$P_1 = 0.5000$	$P_2 = 0.4286$	$P_3 = 0.5556$	0.9398
1999-00	1	$P_{00} = 0.9444$	$P_{01} = 0.0556$	$P_{10} = 0.5455$	$P_{11} = 0.4545$	0.7239
2000-01	1	$P_{00} = 0.9636$	$P_{01} = 0.0364$	$P_{10} = 0.4444$	$P_{11} = 0.5556$	0.7240
2001-02	1	$P_{00} = 0.9636$	$P_{01} = 0.0364$	$P_{10} = 0.4444$	$P_{11} = 0.5556$	0.7240
2002-03	0	$P_{00} = 0.9917$	$P_{01} = 0.0083$	$P_{10} = 0.9917$	$P_{11} = 0.0083$	0
2003-04	0	$P_{00} = 0.9917$	$P_{01} = 0.0083$	$P_{10} = 0.9917$	$P_{11} = 0.0083$	0
2004-05	0	$P_{00} = 0.9583$	$P_{01} = 0.0417$	$P_{10} = 0.9583$	$P_{11} = 0.0417$	0.0082
2005-06	0	$P_{00} = 0.9583$	$P_{01} = 0.0417$	$P_{10} = 0.9583$	$P_{11} = 0.0417$	0.0082
2007-08	2	$P_0 = 0.0408$	$P_1 = 0.5714$	$P_2 = 0.4545$	$P_3 = 0.4815$	0.8366
2008-09	1	$P_{00} = 0.9911$	$P_{01} = 0.0089$	$P_{10} = 0.1429$	$P_{11} = 0.8571$	0.5388
2009-10	2	$P_0 = 0.0291$	$P_1 = 0.6000$	$P_2 = 0.4375$	$P_3 = 0.5000$	0.7533
2010-11	1	$P_{00} = 0.9556$	$P_{01} = 0.0444$	$P_{10} = 0.1333$	$P_{11} = 0.8667$	0.9805
2011-12	1	$P_{00} = 0.9646$	$P_{01} = 0.0354$	$P_{10} = 0.6667$	$P_{11} = 0.3333$	0.3608
2012-13	1	$P_{00} = 0.9643$	$P_{01} = 0.0357$	$P_{10} = 0.5714$	$P_{11} = 0.4286$	0.5268
2013-14	1	$P_{00} = 0.9821$	$P_{01} = 0.0179$	$P_{10} = 0.2857$	$P_{11} = 0.7143$	0.6559
2014-15	2	$P_0 = 0.0190$	$P_1 = 0.5714$	$P_2 = 0.4545$	$P_3 = 0.5625$	0.6338
2015-16	1	$P_{00} = 0.9802$	$P_{01} = 0.0198$	$P_{10} = 0.4000$	$P_{11} = 0.6000$	0.5638
Estimated P(A) based on Models fitted for period 1973-74 to 1990-91(Average of P(A))						0.3944
Estimated P(A) based on Models fitted for period 1991-92 to 2015-16 (Average of P(A))						0.4635
Estimated P(A) based on Models fitted for period 1973-74 to 2015-16 (Average of P(A))						0.4346

TABLE 4

Observed values of X_1, X_2, X_3

Agriculture Year	X_1	X_2	X_3	Agriculture Year	X_1	X_2	X_3	Agriculture Year	X_1	X_2	X_3
1973-1974	16	3	4	1987-1988	0	0	0	2001-2002	9	1	5
1974-1975	14	2	3	1988-1989	4	0	2	2002-2003	1	0	1
1975-1976	14	1	3	1989-1990	2	0	1	2003-2004	1	0	1
1976-1977	7	1	5	1990-1991	5	1	5	2004-2005	5	0	2
1977-1978	9	1	4	1991-1992	8	1	6	2005-2006	5	0	2
1978-1979	1	0	1	1992-1993	5	0	2	2007-2008	13	2	4
1979-1980	0	0	0	1993-1994	7	1	3	2008-2009	7	1	7
1980-1981	3	0	1	1994-1995	9	0	2	2009-2010	10	1	5
1981-1982	11	2	6	1995-1996	3	0	2	2010-2011	30	3	10
1982-1983	4	1	3	1996-1997	12	0	2	2011-2012	6	0	2
1983-1984	9	2	5	1997-1998	3	0	2	2012-2013	7	1	4
1984-1985	5	1	3	1998-1999	20	4	5	2013-2014	7	1	6
1985-1986	0	0	0	1999-2000	11	1	4	2014-2015	9	1	6
1986-1987	0	0	0	2000-2001	9	2	4	2015-2016	5	1	4

TABLE 5

Probability distributions fitted to X_1, X_2 , X_3

Period	Distribution fitted for random variables (P:Poisson, Geo: Geometric, NB: Negative Binomial)		
	X_1	X_2	X_3
1973-74 to 1990-91	$Geo(p = 0.14754)$	$P(m = 0.83333)$	$NB(n = 5 \;\; p = 0.67208)$
1991-92 to 2015-16	$P(m = 8.4167)$	$P(m = 0.875)$	$P(m = 3.7917)$
1973-74 to 2015-16	$NB(n = 2, p = 0.21541)$	$P(m = 0.85714)$	$P(m = 3.2619)$

separately for four months and combine for four months are reported in Table 2 along with their estimated parameters.

Change in the distributions fitted to T_{min} for combine four months is demonstrated in Fig. 1. In Fig. 1, we have plotted density functions of T_{min} for combine four months using fitted distributions: Johnson SB for the periods 1973-74 to 1990-91 and 1973-74 to 2015-16 and General Extreme Value distribution for the period 1991-92 to 2015-16.

4. *Markov Chain Modeling*: We define stochastic process, $Y_n$taking value 1 if daily minimum temperature less than 8°C on nth day during November to February and 0 otherwise. We have formed $Y_n$sequence taking values 0 and 1 for each year during 1973-74 to 2015-16. Markov chain modeling for year-wise is done by using MATLAB program based on Minimum Akaike Information Criterion Estimation (MAICE). Our study shows that out of 42 years, for nine years zero order Markov chain that are sequence of independent random variables, for 18 years one order Markov chain, for 11 years two order Markov chain are fitted and for four years Y_n is degenerate sequence at 0.

Results of model fitted are reported in Table 3. For four years 1979-80 and 1985-88, we observed that Y_n is degenerate sequence with $P(Y_n = 0) = 1$. Hence for these four years, $P(A) = 0$ where, A denote the event that daily minimum temperature is below 8 °C for consecutive three or more days during the period of November to February. Markov chains of order 0, 1 & 2 are fitted to sequence of Bernoulli Trials, Y_n for 9, 18 and 11 years respectively.

We define random variable $G_{n,k}$ as the number of success runs of length greater than or equal to k in n Bernoulli Trials. We have obtained $P(A)$ as $P(A) = 1 - P(G_{n,k} = 0)$ by using exact probability distribution as studied in Shinde and Kotwal(2006) and Kotwal and Shinde(2006). MATLAB programs are used to evaluate exact probability distribution of $G_{n,k}$. We have used n=120 (number of days during November to February) and k=3 (due to consecutive three or more days in reference to event A). Transition probabilities, P_{00}, P_{01}, P_{10} and P_{11} for Markov chain with order zero and one are estimated based on observed Y_n sequence for each year. For Markov chain of order two, we have converted Markov chain in order one and corresponding transition probabilities (Kotwal and Shinde(2006)) are estimated based on observed Y_n sequence. In this case we need estimates of only four probabilities P_0, P_1, P_2 and P_4one from each row of transition probability matrix of order 4×4.

We have obtained model based $P(A)$ for each year and presented in Table 3. Average of these values $P(A)$ over all years gives model based estimates of $P(A)$ under the period of study. Model based estimated $P(A)$ evaluated for three periods 1973-74 to 1990-91, 1991-92 to 2015-16 and 1973-74 to 2015-16. These values of estimated $P(A)$ can be used for pricing of weather derivatives in Weather Based Crop Insurance Scheme (WBCIS) for banana. Generally estimated probabilities of risk events based on previous25 years weather data are considered for fixing premium. Hence we prefer estimated $P(A)$ based on the period 1991-92 to 2015-16. For this period model based estimated value of $P(A) = 0.4635$.

5. *Probability Distributions of some random variables of interest*: We consider following three random variables as functions of minimum temperature, T_{min} during November to February.

X_1: Number of days for which daily minimum temperature less than 8°C during November to February,

X_2: Number of occurrences of daily minimum temperature less than 8°C for at least three consecutive days during November to February and

X_3: Maximum length of consecutive number of day's daily minimum temperature less than 8°C during November to February.

In Table 4, we have reported year wise observed values of X_1, X_2 and X_3 during years 1973-74 to 2015-16. Distribution fitting for random variables X_1, X_2 and X_3 is done by using Easy-fit software. Standard probability distributions (Geometric, Poisson and Negative Binomial) are fitted to these random variables. Fitted distributions with estimated parameters are reported in Table 5.

6. We have estimated probability of extreme low temperature event A using Markov chain modeling. This probability can be used to estimate premium to be charged to cultivators of banana from Jalgaon district. Estimated premium can be calculated as multiplication of $P(A)$and sum assured to cover losses due to occurrence of event A. Here, event A is closely related to random variable X_2. One may also estimate $P(A)$ as $P(X_2 = 0)$. The random variable X_2 is highly correlated with X_1 and X_3 with correlation coefficient 0.813 and 0.706 respectively, hence extreme low temperature event can be defined on X_1such as Event B: X_1 greater than 8, Event C: X_1 greater than 5. Events which are based on joint probability distribution of X_1, X_2 and X_3 can also be defined and used in estimation of premium for WBCI for banana which covers risk events related to extreme low temperature. Method and results of this paper can show the parallel path to study estimation of premium for WBCI for other crops.

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