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Characteristics of boundary layer parameters

M. K. GUHA and M. P. SHEWALE
Meteorological Office, Pune
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सार — लघुगुणकीय पवन प्रोफाइल एवं सतह प्रतिबल τ_0 का उपयोग करके ऊंचाई, समय और वास्तविक वेग के संदर्भ में ग्रहीय परिसीमा स्तर के अन्दर घर्षण वेग u_* की निर्भता को ज्ञात किया गया है । किसी स्तर के पवन चाल को निम्नतम स्तर पवनचाल के रूप में और स्तरी ऊँचाई को घातांकी फलन के रूप में व्यक्त किया गया है तथा निदर्श के आधार पर प्राप्त परिणामों का किसी क्षण विशेष पर वास्तविक पवन से मिलान किया गया है । परिणामों में काफी समानता पाई गई । यहां यह भी दर्शाया गया है कि संवेग कर्षण गुणांक C_D पूर्णतः अचर नहीं है, बल्कि बदलता रहता है । निदर्श के आधार पर अधिक ऊँचाई पर पवनचाल विश्लेषिक विधि से जारी रखी जा सकती है । पुन : इस विधि से प्राप्त C_D का मान C_D के वास्तविक मानों की परास के पर्याप्त निकट पाया गया ।

ABSTRACT. The dependence of frictional velocity u_* within the planetary boundary layer (PBL) with reference to height, time and actual velocity has been obtained by using the logarithmic wind profile and surface stress τ_0. The wind speed at any level has been expressed in forms of the lowest level wind speed and the level height in the form of exponential function and the results obtained according to the model have been compared with the actual wind at any instant. The results are in good agreement. It has been also shown here that the momentum drag coefficient C_D is not absolutely constant but possesses a height variation. The wind speed can be continued analytically to a greater height according to the model and the value of C_D obtained by this continued value is well in agreement with the existing range of C_D.

1. Introduction

Different types of numerical values for frictional velocity and drag coefficient for momentum, heat and moisture processes have been proposed in the past. The derivation of frictional velocity profile and drag coefficients have involved different considerations. Pannel and LeMone (1974) have related the virtual potential temperature, mixing ratio, wind components and turbulence statistics to vertical transport of kinetic energy. Zilitenkevich and Deardorff (1974) related the vertical wind profile to suitable dimensionless parameters. Blackader and Panofsky (1969) developed a relationship between surface Rossby number and buoyancy functions developed by Clarke (1970). Fielder and Panofsky (1972) evolved a logarithmic wind profile relationship within planetary boundary layer. Such logarithmic relation is valid only under neutral condition and implies constant stress with reference to height and a linear increase of mixing length.

Difficulties encountered for obtaining the characteristics of boundary layer parameters are :

(*i*) determination of drag coefficient C_D as it is a function of height and

(*ii*) nature of distribution of wind profiles in a condition other than neutral. Under neutral stratification, the logarithmic profile can be assumed with a fair accuracy.

In this discussion an attempt has been made to obtain the wind profiles as a function of lowest level wind velocity and height. The wind velocity obtained according to the model has been also compared with the upto-date data and found in good agreement. The drag coefficients have been also computed by using wind speeds at different levels and the nature of variation with reference to height has also been shown. It has been frequently discussed that the frictional velocity u_* cannot be independent of the actual wind velocity and here it has been reported that the frictional velocity decreases nearly exponentially with the increase of wind speed and practically becomes steady at a wind speed $V = 11$ km ph. The frictional velocity u_* has been determined by using wind profiles and stress profiles for comparison.

2. Method

Fundamental equations are the equations of momentum, continuity, energy, state and thermal expansion, *viz.*, in tenser notation.

$$\frac{DU_j}{Dt} + \epsilon_{jkl} f_k U_l = \frac{\partial}{\partial x_k}\left(-\overline{u_k u_j}\right) - $$

$$-\frac{\partial p}{\partial x_j} - g_j \beta\theta \qquad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho U_i\right) = 0 \qquad (2)$$

$$\frac{D\theta}{Dt} + \frac{\partial}{\partial x_k}\left(\overline{u_k \theta}\right) = 0 \qquad (3)$$

$$p = \rho R T \qquad (4)$$

$$\rho = \rho_0 (1 - \beta T) \qquad (5)$$

where p is the mean kinematic pressure, $g_j = (0, 0 - g)$— the gravity vector, $f_j = (0, f_y, f_z)$ the coriolis parameter and

$$\beta = -\frac{\left(\frac{\partial \rho}{\partial T}\right)}{\rho}$$

the coefficient of thermal expansion, v is the coefficient of eddy viscosity.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_k \frac{\partial}{\partial x_k}$$

Our interest is to obtain the vertical distribution of wind profiles within the PBL under the following assumption of PBL flow, *i.e.*,

(*i*) turbulency of PBL-flow exists, (*ii*) turbulence and mean flows are stationary, (*iii*) turbulance and mean flows are horizontally homogeneous, *i.e.*, $\vec{\nabla}_h = 0$ except $\vec{\nabla}_h p$, for geostropic wind existence and $\vec{\nabla}_h T$, with this the momentum equations can be written in the form :

$$fv = -\alpha \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial z^2} \qquad (6)$$

$$fu = -\alpha \frac{\partial p}{\partial y} + v \frac{\partial^2 v}{\partial z^2} \qquad (7)$$

$$\text{Therefore,} \quad f\frac{\partial w}{\partial p} = v \frac{\partial^2 \zeta}{\partial z^2} \qquad (8)$$

showing the relationship between the vertical component of vorticity and vertical velocity in *p*-coordinate. Assuming further no turning of the wind we can write :

$$fv = v \frac{\partial^2 u}{\partial z^2} \qquad (9)$$

$$-fu = v \frac{\partial^2 v}{\partial z^2} \qquad (10)$$

TABLE 1

Actual wind (km p h)	Computed wind (km ph)	Level (m)
23.1	23.616	1.8
20.1	20.418	1.8
20.8	21.156	2.1
20.4	21.149	2.1
21.2	21.91	2.1
15.8	15.99	1.8
21.6	23.61	1.8

One of the approximate solutions is given by :

$$u = u_0 \, e^{K_z} \qquad (11)$$

which is distinct from Ekman's solution. The value of the constant K has been determined from a set f observation obtained at Pune observatory and is givenby :

$$K = 0.115$$

Table 1 gives the comparison of wind obtained according to the above model and actual wind. The solution of Eqn. (9) is furnished below :

$$\text{We have the equations,} \quad fv = v \frac{\partial^2 u}{\partial z^2} \qquad (12)$$

$$-fu = v \frac{\partial^2 v}{\partial z^2} \qquad (13)$$

Differentiating Eqn. (12) twice w.r.t. z, and inserting $\partial^2 v/\partial z^2$ from Eqn. (13) we get :

$$\frac{d^4 u}{dz^4} + \frac{f^2}{v^2} u = 0 \qquad (14)$$

Consequently, roots for the complementary function of the differential equations are :

$$\pm\sqrt{(if/v)}, \qquad \pm\sqrt{-(if/v)}$$

Therefore, the solution of the type furnished in the Eqn. (11).

However, the Eqn. (9) can be directly solved as follows:

$$u\frac{d^2 u}{dz^2} = -v\frac{d^2 v}{dz^2} = g \ (Say)$$

where g is an absolute constant.

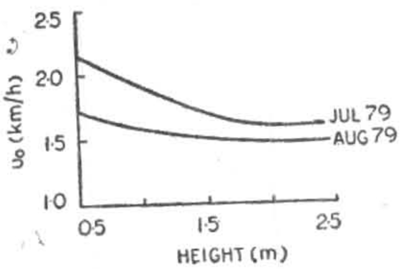

Fig. 2. Frictional velocity and height

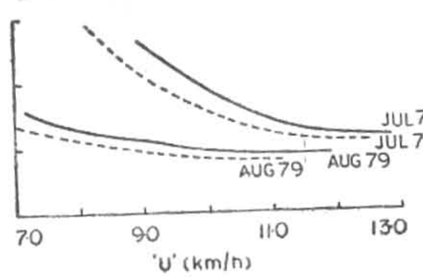

Fig. 2. Frictional velocity and actual velocity

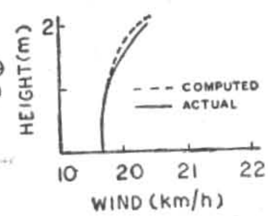

Fig. 3. Wind profile

TABLE 2

Ht. (m)	u (km p h)	u_* (km p h)	$C_D \times 10^{-3}$
0.5	6.3	1.441	5.234
	8.8	2.013	5.231
	7.3	1.670	5.234
1.8	9.3	1.296	1.979
	12.3	1.713	1.659
	10.0	1.393	1.941
2.1	9.7	1.286	1.758
	12.7	1.685	1.760
	10.6	1.406	1.760

Multiplying both sides by $2\,\frac{du}{dz}$, we have

$$\frac{d}{dz}\left[\left(\frac{du}{dz}\right)^2\right] = 2\,g\,\frac{d}{dz}\,(\ln u) \tag{15}$$

$$\text{or,}\quad \int_{u_0}^{u}\frac{du}{\sqrt{\ln u}} = \sqrt{2g}\int_{z_0}^{z} dz \tag{16}$$

$$\text{or,}\quad \left[\ln u\right]_{u_0}^{u} = \sqrt{2g}\,(z - z_0) \tag{17}$$

as the solution of Eqn. (9) (*cf.* Snedon *et al.* 1965)

The relation between the surface stress τ_0 and the frictional velocity is defined by, $\tau_0 = \rho C_D\, u^2 = \rho\, u_*^2$

Therefore, $C_D = [u_* / u]^2$ (18)

Again , if we assume the log-wind profile, then we have :

$$u = \frac{u_*}{k}\,\ln\left(\frac{z + z_0}{z_0}\right)$$

$$C_D = \left[\,k \Big/ \ln\left(\frac{z + z_0}{z_0}\right)\right] \tag{19}$$

where k is Von-Karman constant and is equal to 0.41 while z_0 is the roughness length which has been taken here 5 cm according to the usual norm of classification of roughness. Table 2 gives the experimental values for the drag coefficients in relation to different heights, frictional velocity and actual velocity for the average wind distribution from July to August.

Fig. 1 is the variation of frictional velocity with height of measurement. The actual velocities are measured at different levels and thence u_* determined by using Eqn. (18).

It is to be noted that the height variation of the frictional velocities for different months can be described by the exponential decaying function. The trend shows that the frictional velocity decreases with the height of actual wind measurement. It is perfectly clear from Eqn. (18) that the frictional velocity is a function of actual wind speed and Fig. 2 shows the variation of frictional velocity with the actual wind speed and it shows the variation of frictional velocity. It shows the frictional velocity decreases with the increase of actual wind speed. The continuous line represents the variation according to the model analysis while the dotted line is the coumputed one according to Eqn. (18). Fig. 3 is the wind profile according to the model and computed for 7 July of 1981 for Pune, where the experiment is being carried on. It is only for verification of the model results with the actual observations.

3. Conclusion

The evaluation of drag coefficient C_D, for the determination of frictional velocity and subsequently describing the wind profile within the PBL is an important problem of atmospheric boundary layer meteorology. In this analysis we have attempted to describe the highest variation of C_D, the momentum drag coefficient and thence obtained the frictional velocity at various levels by using a mathematical relation. The computed wind according to the model has been compared with the actual wind for verification only. It is to be noted that (*i*) frictional velocity decreases with height and approaches a definite value & (*ii*) the frictional velocity decreases with the increase of actual wind speed and has a tendency of approaching a constant value.

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