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# A statistical study of the annual rainfall in the Sutlej catchment and annual run-off at Bhakra dam site

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ABSTRACT. An analysis of the annual rainfall of three selected stations with long term data in Sutlej catchment and the annual run-off at Bhakra dam site has been made for their averages and standard deviations. Each series has been examined for general trend by (*i*) the low pass filter, (*ii*) Mann-Kendall test and (*iii*) comparison of short period averages' with long period average. Further, the periodicities in the data series have also been examined by power spectral analysis.

## 1. Introduction

1.1. The river *Sutlej* has its origin from Mansarovar and Rakas-Tal lake in Tibet. The river enters the Indian territory near Shipki in Himachal Pradesh and takes westsouthwesterly direction on its way to Bhakra where dam is located. The major tributary of the river *Sutlej* is *Spitti* river which drains a large area of the snow covered Himalayas. The total catchment area of *Sutlej* above Bhakra dam site is about 56,300 sq. km of which 20,500 sq. km lies in India. The upper portion of the catchment in the Indian territory is considerably wider compared to the lower portion which is quite narrow upto Bhakra dam site.

### 1.2. *Rainfall*

*Sutlej* catchment receives most of its annual rainfall during southwest monsoon season from June to September. Sometimes the heavy rainfall occurs in the catchment in early October due to the movement of the monsoon depression or the movement of the cyclonic storm. The season is marked by high river flows and floods in river. The winter precipitations are usually confined to the months of December to March and sometimes in November and April also due to the movement of the westerly troughs across the Western Himalayas. Most of the precipitation during winter in the catchment, particularly above Nichar, is in solid state and it generally contributes to the river flow during the following summer. The precipitation, however, in the lower reaches is both in the solid and liquid forms and contributes to the run-off immediately. Little precipitation occurs in the catchment area lying in Tibet as arid conditions prevail in that part of the catchment.

There are 23 raingauge stations in the *Sutlej* catchment, of which long term data are available only in respect of Simla, Kotgarh and Kilba, the data of which have been used in the present study. Some snowgauges are also existing in the upper reaches but reliable data are not available.

### 1.3. *Run-off*

The *Sutlej* river run-off is comprised of two parts : one coming from the melting of the snow and the other resulting from the rainfall in the catchment. The river flow remains more or less constant from year to year except during the month of May when large fluctuation in the run-off occurs due to variation in the air temperature and the extent of snow deposite during the preceding winter in the catchment area.

## 2. Analysis

An attempt has, therefore, been made to analyse statistically the average annual rainfall data series and annual run-off at Bhakra dam site using data of about 60 years from 1910 to 1969. The rainfall data were collected from the *Monthly Weather Report* of the India Meteorological Department for the three stations, *viz.*, Simla, Kotgarh and Kilba selected for this study and continuous records of yearly run-off at Bhakra dam site for the same period obtained from Bhakra Management Board.

TABLE 1

Average and variability etc

| Series | Average | Standard deviation | C. V. (%) | Highest Value | Highest Year | Lowest Value | Lowest Year |
|---|---|---|---|---|---|---|---|
| Rainfall (cm) | 111·53 | 19·91 | 17·86 | 166·37 | 1957 | 56·39 | 1965 |
| Run-off (million acre feet) | 13·30 | 2·25 | 16·91 | 20·28 | 1914 | 9·16 | 1944 |

TABLE 2

Five Year Averages

| Period | Rainfall (cm) Av. | Rainfall (cm) $t_k$ | Rainfall (cm) S.D. | Run-off (maf) Av. | Run-off (maf) $t_k$ | Run-off (maf) S.D. |
|---|---|---|---|---|---|---|
| 1915-1919 | 108·63 | − 0·34 | 13·01 | 13·53 | 0·20 | 1·71 |
| 1920-1924 | 124·00 | 1·46 | 19·94 | 15·09 | 1·81 | 2·12 |
| 1925-1929 | 117·07 | 0·63 | 16·61 | 12·97 | −0·36 | 1·17 |
| 1930-1934 | 98·58 | −1·52 | 21·18 | 12·69 | − 0·65 | 1·89 |
| 1935-1939 | 106·55 | − 0·58 | 13·23 | 12·06 | − 1·29 | 1·36 |
| 1940-1944 | 105·00 | − 0·76 | 14·66 | 11·47 | − 1·92 | 2·49 |
| 1945-1949 | 105·66 | − 0·68 | 17·65 | 31·03 | − 0·30 | 1·12 |
| 1950-1954 | 107·21 | − 0·49 | 9·78 | 13·60 | 0·27 | 1·63 |
| 1955-1959 | 139·21 | 3·51* | 17·07 | 15·60* | 2·38* | 1·91 |
| 1960-1964 | 107·79 | −0·43 | 7·29 | 13·06 | −0·27 | 1·01 |
| 1965-1969 | 99·62 | −1·40 | 21·79 | 11·67 | −1·70 | 1·30 |

*Significant at 5 per cent level

### 2.1. *Average and Variability*

Table 1 gives the average, standard deviation, coefficient of variation and the extremes for the annual rainfall at Simla, Kotgarh and Kilba and the annual run-off at Bhakra dam site. The average annual rainfall of three stations is 111·5 cm and has a variability of 17·9 per cent only. The highest ever recorded average rainfall of three stations is 166·3 cm in 1957 which is 49 per cent in excess of the normal. The lowest value is 56·3 cm recorded in 1965 which is 28 per cent in deficit. The average annual run-off at Bhakra dam site is 13·3 million acre feet with coefficient of variation (CV) 16·9 per cent. The highest ever recorded run-off is 20·3 million acre feet in 1914 which is 52 per cent in excess. The lowest recorded run-off in 1944 is 9·2 million acre feet which is 31 per cent in deficit.

## 3. Trends in the data

For the study of the trends, the data have been examined as follows :

### 3.1. *Five-year averages*

An examination of the five-year averages has been made so as to see if they differ from the average of the entire period. This helps in examining the changes in the data with respect to time. For this purpose, the data from 1915 to 1969, *i.e.*, for 55 years has been split in 11 short periods of five years each and averages computed and tested against the long period averages by the parameter $t_k$ which is given as (WMO 1966):

$$t_k = T_k \left[ \frac{k(n-2)}{n-k-kT_k^{\,2}} \right]^{1/2}$$

where $T_k = (\bar{x}_k - \bar{x})/S$, $k$ is the number of years in the short period, $n$ is the number of values in the whole series and $S$ is the standard deviation. It can be seen that $t_k$ is distributed as Student's "$t$" with ($n$—2) degree of freedom. Table 2 gives the statistical average of the five-year periods together with the $t_k$ values.

### 3.2. *Rainfall — Five-year means*

The five-year period 1955-1959 has a significantly high average as this period contains the years of 1955 and 1957 when heaviest rain was recorded in the catchment. The five-year period 1930-1934 has the lowest mean. This gives an impression of the existence of a cycle of 50 years or so in the annual data series as the lowest and highest periods are separated by an interval of 25 years.

3.3. *Run-off — Five-year means*

The five-year period 1955-1959 is having a highest value and is significantly higher than the average. This period also contains the year of highest run-off of 19·3 million acre feet recorded in 1955 in the span from 1915 to 1969. The period 1940-1944 has the lowest mean, but it is just within the statistical significance ($t_k=1 \cdot 918$). This suggests the existence of a cycle of 30 years in the annual data series as the lowest and highest five-year periods are separated by an interval of 15 years.

#### 4. Testing for homogeneity of data and variances

Following the study of David, Hartley and Pearson (1954) the homogeneity of data is tested. For this purpose the ratio of range ($w$) to the standard deviation ($S$) of the sample is calculated in order to find out whether $w/S$ exceeds the 5 per cent and 1 per cent values. The rainfall and run-off data were, therefore, subjected to this test and it was found that the data series considered in the present study were homogeneous.

Bartlett's test, described in Pearson and Hartley (1958), has been utilized to test the homogeneity of variances. For this purpose the ratio $S^2_{max}/S^2_{min}$ has been calculated and compared with the standard tabulated values for appropriate values of $k$ (number of sub-groups) and $n$ (number of terms in each sub-group). This test confirms the homogeneity of variances in both the data series.

#### 5. Persistency in the data series

It is also interesting to know whether any year's rainfall (or run-off) is having a direct relationship with the rainfall(or run-off) that occurred in the previous years. To study this aspect serial correlation coefficients have been computed for lags upto 10 years and presented for two data series in Table 3. No definite relationship could be established from these data series.

**TABLE 3**

**Serial Correlations**

| Lags | Rainfall | Run-off |
|---|---|---|
| 1 | 0·136 | 0·047 |
| 2 | 0·138 | 0·196 |
| 3 | 0·175 | 0·351* |
| 3 | —0·122 | —0·071 |
| 5 | —0·119 | 0·386* |
| 6 | —0·250* | 0·007 |
| 7 | —0·203* | 0·015 |
| 8 | —0·227* | 0·195 |
| 9 | 0·028 | —0·096 |
| 10 | 0·048 | 0·066 |

*Significant at 5 per cent level

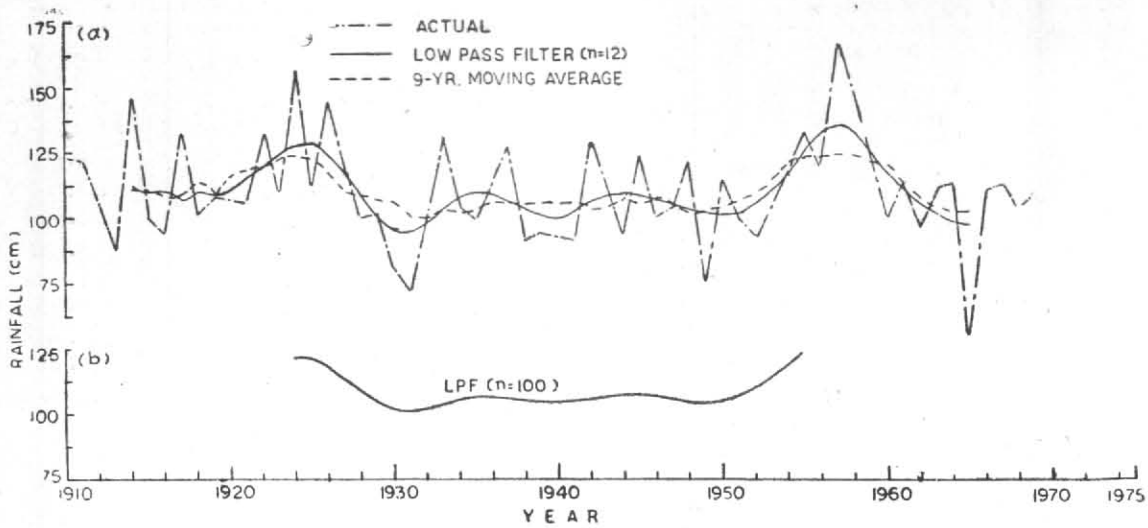
#### 6. Trend analysis by low pass filter and Mann-Kendall Test

6.1. A plot of the annual rainfall (or run-off) data shows that the year to year fluctuations are so large that it is impossible to detect any regular short or long period fluctuations from it. In order to suppress the yearly fluctuations 5, 9 and 11 years ordinary moving averages have been worked out. The resultant series still showed fluctuations, although on a considerably reduced scale. The low pass filter technique (WMO 1966) has, therefore, been utilised to damp very short period fluctuations of 2 to 3 years. This technique has also been employed by Koteswaram and Alvi (1970) for the study of secular trends and variations in rainfall in Indian regions. In the present study also two low pass filters have been utilised, one with nine effective binomial weights, ($n=12$) and the other with 31 effective weights ($n=100$) where $n$ is the total number of binomial coefficients utilised for calculating the concerned effective weights. Mann-Kendall test has also been applied for the significance of the trend indicated by the low pressure filter ($n=100$).

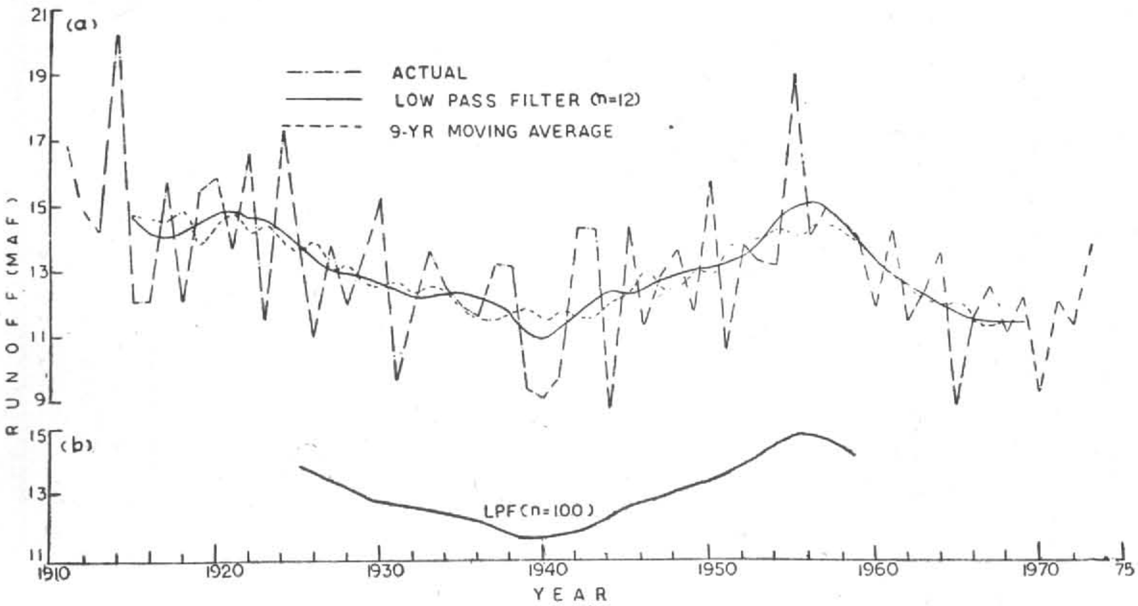
Figs. 1 (a, b) and 2 (a, b) show the annual rainfall and run-off series together with (*i*) 9-year ordinary moving average, (*ii*) low pass filter $n=12$ and (*iii*) low pass filter $n=100$.

6.2. The nine-year weighted moving average series for the rainfall (Fig. 1 a) indicates a long term trend of 32 years or so, with short oscillations of the order of 10 years duration. An examination of Fig. 1 (b) reveals that the rainfall series treated with the low pass filter ($n=100$) has a wavy pattern with a minima in 1931. The difference between the minima and the end of the series is about 22·0 cm. This gives an increase of 0·86 per cent per year which is insignificant in accordance with Mann-Kendall test. This conforms to the earlier study of Rao and Raghavendra (1972) based on individual stations including Simla, Kotgarh and Kilba in Western Himalayas. They have shown that there is no statistically significant change in the pattern of rainfall of those stations during the past century.

6.3. It can be seen from Fig. 2 (a) that the low pass filter ($n=12$) for the run-off series has long term linear trend of 34 years or so. Further Fig. 2(b) shows that the run-off series treated with the low pass filter ($n=100$) has a minima in 1940. Thereafter, a continuous increase




Figs. 1. (a&b). Graphs showing low pass filter of yearly average rainfall (cm) in *Sutlej* catchment




Figs. 2. (a&b). Graphs showing low pass filter of yearly run-off at *Bhakra* Dam site

with maxima in 1956 is observed. This increase from 1940 to 1956 is about 4·0 million acre feet which shows an increase of 2·24 per cent per year. The Mann-Kendall test indicates a significant trend.

The results of low pass filter and Mann-Kendall test mentioned above are presented in Table 4.

**7. Search of periodicities by power spectral estimates**

7.1. Power spectrum analysis also known as generalized harmonic analysis, is derived rom the principles first enunciated by Wiener (1930, 1949). It is based on the premise that time series are not necessarily composed of a finite number of oscillations, each with a discrete wave length (as one tacitly assumes when one applies classical harmonic analysis), but rather that they consist of virtually infinite number of small oscillations spanning a continuous distribution of wave lengths. The spectrum, therefore, yields a measure of the distribution of variance in a time series over a continuous domain of all possible wavelengths — each arbitrarily close to the next ranging from an infinite wave length

TABLE 4

**Average increase or decrease in percentage and test of randomness against trend**

| S. No. | Series | Average (cm) | Period From | Period To | Increse per year (%) | Rank statistics $\tau$ | Rank statistics $(\tau)_t$ | Remarks regarding trend |
|---|---|---|---|---|---|---|---|---|
| 1 | Rainfall | 111·5 | 1910 | 1969 | 0·86 | +0·0339 | +0·1736 | Insignificant |
| 2 | Run-off | 13·3 | 1911 | 1973 | 2·24 | −0·1777 | −0·1692 | Significant |

(linear trend) to the shortest wave length that can be resolved by any scheme of harmonic analysis (equal to twice the interval between successive observations in the series).

The actual determination of spectrum analysis is a harmonic analysis of the autocorrelation resulting in a smoothing of the calculated variances. The procedure for calculating the spectrum analysis is as follows.

If there are $N$ terms of a series $x_i$, the serial covariances $C_\tau$ for all lags $\tau = 0$ to $\tau = m$ (where $m < N$) are computed according to the standard formula:

$$C_\tau = \frac{1}{(N-\tau)} \sum_{i=1}^{N-\tau} (x_i - \bar{x})(x_{i+\tau} - \bar{x})$$

where $\bar{x}$ is the mean of all $x_i$ in the series.

Raw spectral estimates $\hat{S}_k$ are then obtained directly from these $C_\tau$ values by the equations:

$$\hat{S}_0 = \frac{1}{2m}(C_0 + C_m) + \frac{1}{m}\sum_{\tau=1}^{m-1} C_\tau$$

$$\hat{S}_k = \frac{C_0}{m} + m \sum_{\tau=1}^{m-1} C_\tau \cos\left(\frac{\pi k \tau}{m}\right) + \frac{1}{m} C_m (-1)^k, \quad k=1,\ldots\ldots, m-1$$

$$\hat{S}_m = \frac{1}{2m}\left\{ C_0 + (-1)^m C_m \right\} + \frac{1}{m}\sum_{\tau=1}^{m-1} (-1)^\tau C_\tau$$

A better estimate of the smoothed spectrum function is obtained by weighting the coefficients

$$S_0 = \tfrac{1}{2}(\hat{S}_0 + \hat{S}_1)$$

$$S_k = \tfrac{1}{4}(\hat{S}_{k-1} + 2\hat{S}_k + \hat{S}_{k+1}),\ k = 1, 2, \ldots, m-1$$

$$S_m = \tfrac{1}{2}(\hat{S}_{m-1} + \hat{S}_m)$$

7.2. The techniques discussed above has been used in this paper to bring out the significant periodicities present in the series at different levels. Koteswaram and Alvi (1970); Raghavendra (1972, 1973) etc have adopted same technique for their studies.

7.3. Figs. 3 (a, b) contain the spectral estimates for two lag times of 15 and 20 years. The spectrum is that of white noise. The null continuum as well as the significant level of 95 per cent has been shown for each diagram. It can be seen that of the two lag limits considered the spectrum of only one exceeds the 95 per cent confidence limit in each of them. Their periodicities are listed below:

| *Lag limit* | *Periodicity (Year)* |
|---|---|
| 20 | 13·3-20·0 |
| 15 | 10·0-15·0 |

The common period in these two lag limits considered lies between 13·3 and 15·0 years. It would appear that a periodicity in this range is significantly indicated in the data.

### 7.4. *Run-off analysis*

Figs. 4(a, b) contain the spectral estimates of run-off data series for two lag limits of 15 and 20 years, together with the null continuum and the 95 per cent confidence limit. The spectrum is that of a white noise. It can be seen from the figure that in both the lag limits considered, two periods are statistically significant.

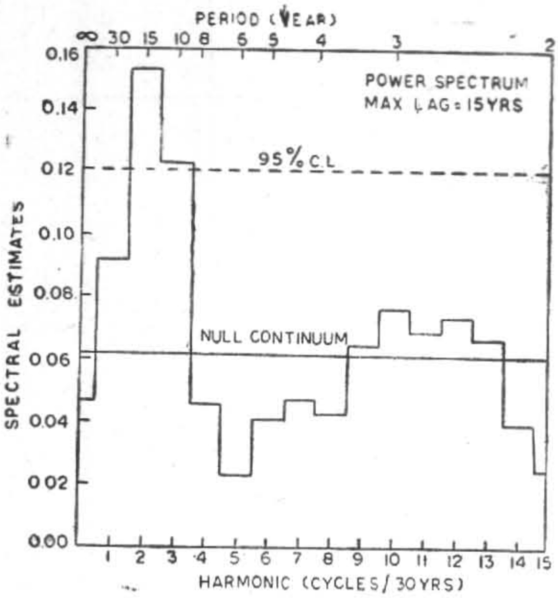



Fig. 3 (a). Power spectrum of average rainfall (Simla, Kotgarh and Kilba) series in *Sutlej* catchment

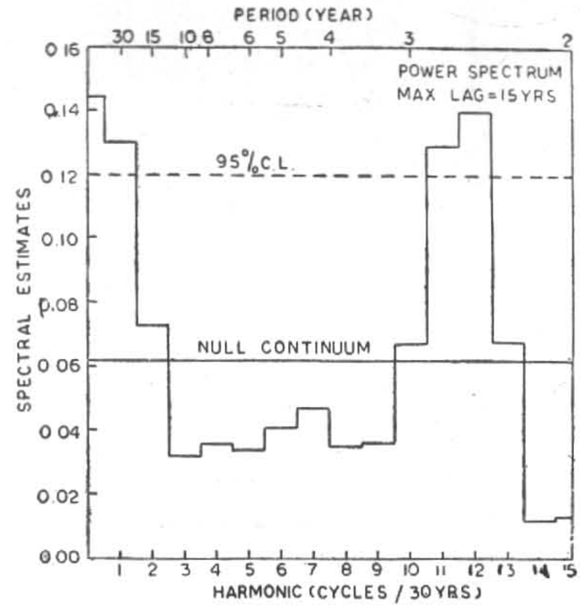



Fig. 4 (a). Power spectrum of *Bhakra* run-off annual series

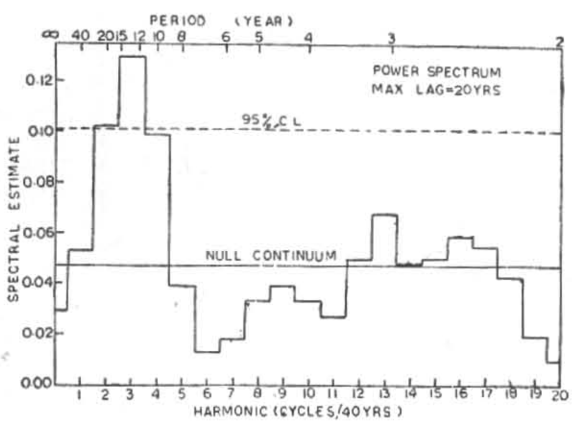



Fig. 3 (b). Power spectrum of average rainfall (Simla, Kotgarh and Kilba) series

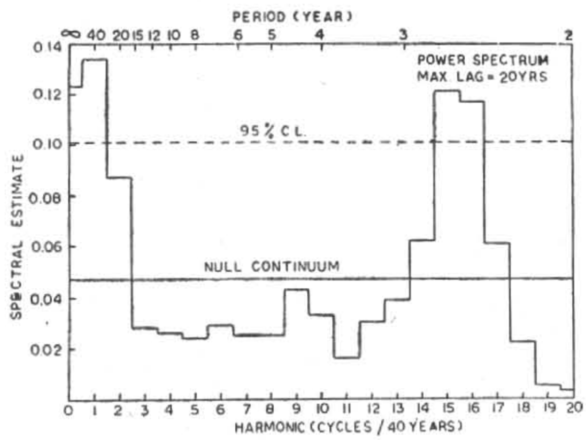



Fig. 4 (b). Power spectrum of Bhakra run-off annual series

(*i*) *Lag limit* (*15 years*) — The first harmonic is significant even at 95 per cent level and hence there is no doubt that there is a long period trend in the series ⩾30 years, which has also been shown by low pass filter. The 11th and the 12th spectral values representing a frequency of 2·7 and 2·5 years respectively are also significant at 95 per cent level.

(*ii*) *Lag limit* (*20 years*) — The significant periods are one in the range ⩾40 years and the other in the range of 2·7 and 2·5 years.

The common period in the two lag limits considered is ⩾40 years and between 2·5 and 2·7 years. The long term period is one seen in the low pass filter and the other is the quasi biennial oscillation (QBO).

## 8. Conclusions

8.1. The average annual rainfall of 3 stations, *viz.*, Simla, Kotgarh and Kilba in *Sutlej* catchment is 111·5 cm with a coefficient of variation 17·9 per cent. Average annual run-off at Bhakra dam site is 13.3 million acre feet with a coefficient of variation 16·9 per cent.

8·2. The five-year period 1930-1934, for rainfall data series, has the lowest mean and the period 1955-1959 has the highest mean which indicates a cycle of 50 years. In the case of annual run-off series the period 1940-1944 has the lowest mean while 1955-1959 has the highest mean, thus showing a cycle of 30 years in the data.

8·3. An examination of the long term trend by low pass filter ($n=12$) in the rainfall and run-off series suggest a linear trend of 32 years and 34 years respectively. The Mann-Kendall test of randomness against trend confirms the significance of trend in run-off series while the trend for rainfall series is significant.

8·4. The power spectral estimates of the data indicate the existence of a periodicity of 13·3 to 15·0 years for rainfall and a long term trend with periodicity$\geqslant$40 years for the annual run-off. In addition the power spectrum of run-off indicated the existence of the quasi-biennial oscillation.

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