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Some aspects of the large-scale lee-waves due to westerly flow over Peninsular India

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सार — निम्न स्तरीय पछुआ हवा के प्रवाह के क्रमिक विकास का अध्ययन करने के लिए 5 दिनों तक एक त्रिस्तरीय पी. ई. निदर्श समन्वित किया गया है। पछुआ प्रवाह को मॉडल वायुमण्डल की निम्नतम परत पर (जिसकी अधिकतम ऊंचाई एक किलोमीटर तक सीमित रखी गई है) प्रायद्वीपीय भारत, बर्मा तथा तिब्बती आदर्श क्षेत्र से बल मिला।

पहला प्रयोग कर्तनी रहित सम्पूर्ण वायुमण्डल में समान पछुआ प्रवाह की एक साधारण विधि के द्वारा किया गया है। मॉडल परिणामों की रैखिक सतत अवस्था भ्रमिलता समीकरण के विश्लेषणात्मक समाधान से तुलना की गई है कि प्रायद्वीपीय भारत के पूर्व की ओर तथा बर्मा के पर्वतों पर अलग-2 अनुवात-द्रोणी बनती है। दूसरा प्रयोग क्षैतिज तथा ऊर्ध्वाधर कर्तनी वाली आदर्श पवन प्रोफाइल के द्वारा किया गया है। यह प्रयोग निम्नस्तरीय संचरण पर ऊपरी वायुमण्डलीय पूर्वी पवन के प्रभाव का आकलन करने के लिए किया गया था। इस लेख में प्रयोगों के परिणाम प्रस्तुत किए गए हैं। फिर भी, अन्य प्रकार के बल भी हैं जो प्रवाह को प्रभावित करते हैं इनकी अलग से खोज करने की आवश्यकता है।

ABSTRACT A three level P.E. model is integrated for 5 days to study the evolution of the low level westerly flow with the forcing provided by an idealized terrain of Peninsular India, Burma and Tibetan area (maximum height restricted to 1 km) in the lowest layer of the model atmosphere.

The first experiment is conducted with a simple mode of uniform westerly flow throughout atmosphere without any shear. The model results are compared with the analytical solutions of the linear steady state vorticity equation. The results obtained from both the methods showed formation of the lee-troughs on the eastern side of the Peninsular India and Burma mountains respectively. The second experiment is conducted with idealized wind profile having both horizontal and vertical shears. The experiment is conducted to assess the effect of upper tropospheric easterlies on the low level circulation. The results of the experiments are presented. However, there are other types of forcings also influencing the monsoon flow, there is need for investigating them separately.

1. Introduction

The Indian summer monsoon flow is characterised by the lower tropospheric westerlies which dominate in strength and depth near the west coast of India. In the upper troposphere strong easterlies are present. The westerly flow of the lower troposphere generates a northerly component after crossing the barrier of Western Ghats and the trough is observed on the head Bay of Bengal.

Banerji (1930) indicated that the bending of the isobars on the lee side of the Western Ghats is due to effect of westerly flow crossing the barrier of Western Ghats. The problem was also investigated analytically by Gadgil and Sikka (1975) using idealized zonal flow and the idealized topography of Western Ghats. Das (1980) used a three level P.E. model in σ coordinate system and conducted the experiments with uniform westerly flow and with the flow having both horizontal and vertical shears typical to Indian summer monsoon situtation.

In the present study the three level P.E. model in the pressure coordinate system is used to study the problem of this type.

2. Description of the P.E. model

Bavadekar and Khaladkar (1983) tested the performance of the three level P.E. model by integrating it for 5 days. The basic input was westerly flow in geostrophic balance and the obstacle to flow was elliptic type with maximum height of one km at the centre of the domain of integration. The forecast fields for wind and height at all levels, ω-fields, temperature and surface pressure departures from the initial values were presented in the paper. The results show proper simulation of the dynamic effects of orography.

The model design is similar to that given by Okamura (1975). The brief description of the model is given below :

2.1. *Model equations*

u and v momentum equations

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(uu) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial p}(u\omega) - fv + \frac{\partial \phi}{\partial x} = F_x \qquad (1)$$

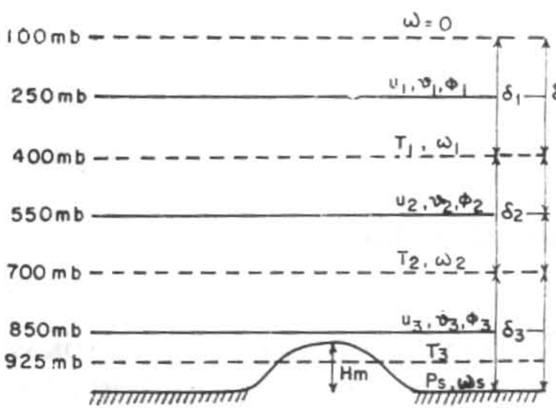


Fig. 1. The vertical structure of the three level P.E. model

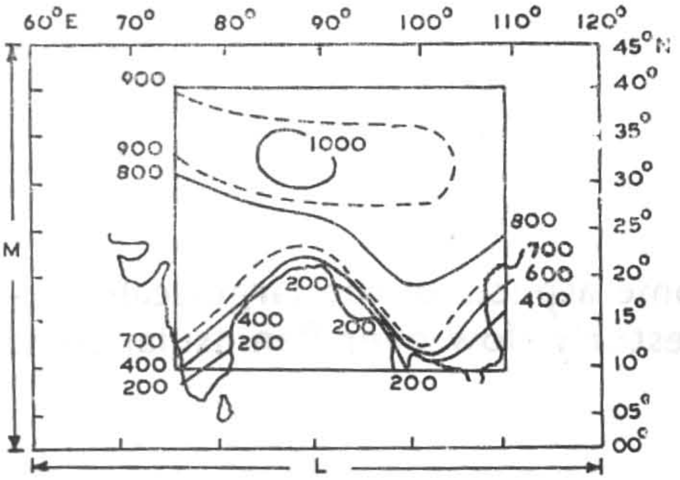


Fig. 2. The domain of integration for the model. The contours for topographic heights are in metres

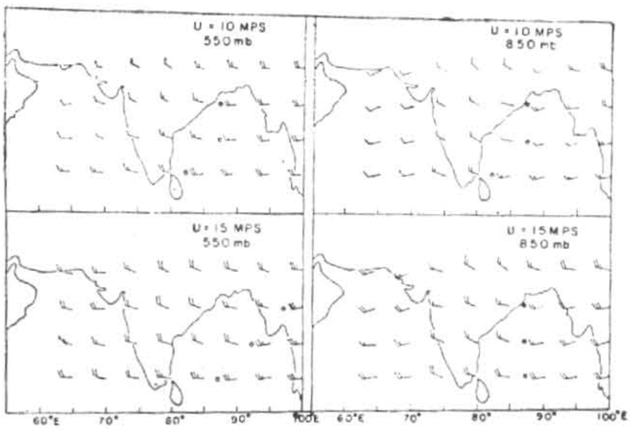


Fig. 3. The forecast wind after 120 hours

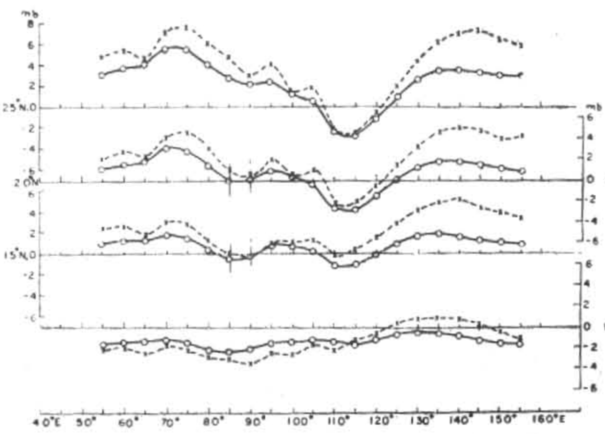


Fig. 4. The surface pressure departures along different latitudes after 120 hours. O—O : $U=10$ mps case, X......X : $U=15$ mps case

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(vu) + \frac{\partial}{\partial y}(vv) + \frac{\partial}{\partial p}(v\omega) + fu + \frac{\partial \phi}{\partial y} = F_y \tag{2}$$

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \tag{3}$$

Hydrostatic approximation

$$\frac{\partial \phi}{\partial p} = -\frac{RT}{p} \tag{4}$$

Thermal equation

$$\frac{\partial}{\partial t}(c_p T) + \frac{\partial}{\partial x}(c_p Tu) + \frac{\partial}{\partial y}(c_p Tv) + \frac{\partial}{\partial p}(c_p T\omega) + \frac{\partial \phi}{\partial p}\,\omega = D_t \tag{5}$$

Pressure tendency equation

$$\frac{\partial P_s}{\partial t} + u_s \frac{\partial P_s}{\partial x} + v_s \frac{\partial P_s}{\partial y} - \omega_s = 0 \tag{6}$$

The dependent variables are u, v, ω, ϕ, T and P_s

where u and v are horizontal wind components, ω is the vertical p-velocity, ϕ is the geopotential height. T is the temperature and P_s is the surface pressure. The coriolis parameters f is expressed in β plane as

$$f = F_o + \beta y \tag{7}$$

where,

$$F_0 = 1.0 \times 10^{-4}\ \text{sec}^{-1} \quad \text{and}$$

$$\beta = 1.619 \times 10^{-11}\ \text{m}^{-1}\ \text{sec}^{-1}$$

The above parameters correspond to latitude of 45°N. F_0 and β so chosen, give the coriolis parameter positive

TABLE 1

The maximum temperature departures ($\triangle T = T_{120} - T_0$) and their locations. (negative : cooling)

Case	Levels	$\triangle T$ (°C)	Lat.	Long.	$\triangle T$ (°C)	Lat.	Long.
U=10 mps	400	−1.8	35°N	75°E	1.6	25°N	115°E
	700	−2.9	40°N	95°E	1.9	25°N	115°E
U=15 mps	400	−2.2	30°N	75°E	1.5	30°N	125°E
	700	−3.3	30°N	75°E	1.5	35°N	140°E

value upto 15°S. This is purposely done to keep the domain of terrain approximately at the central position and thus keeping it away from the north and south boundary walls. The model domain is thus, from 15°S to 65° N and 50° E to 160° E. F_x, F_y and D_t are the frictional and diffusion terms of the momentum and thermal equations.

$$F_x = -\mu u + v \nabla^2 u \tag{8}$$

$$F_y = -\mu v + v \nabla^2 v \tag{9}$$

where μ is the coefficient of surface friction and v is the horizontal diffusion coefficient which is given by

$$v = | \nabla \zeta | d^3 \tag{10}$$

as proposed by Leith (1969). ζ is the relative vorticity and 'd' is the grid length.

2.2. *Vertical structure*

The vertical structure of the model is given in Fig. 1. The wind components u, v and the geopotential height ϕ are specified at 250, 550 and 850 mb levels. The temperature T is specified at 400, 700 and 925 mb levels. The temperature T_3 at 925 mb is kept constant throughout the period of integration. The vertical velocity ω is computed at 400 and 700 mb levels and P_s and ω_s are the pressure and vertical velocity respectively at the surface with the terrain height H.

2.3. *Boundary conditions*

ω=0 at 100 mb level and ω_s is computed by surface pressure tendency Eqn. (6). North and south walls are assumed to be rigid and slipping boundary conditions are assumed at these walls. The cyclic boundary conditions are assumed in the east-west directions.

2.4. *Finite difference schemes*

Mass and energy conserving finite difference schemes as derived by Okamura (1975) are used for the space differential terms in the primitive equations and Matsuno's (1966) scheme is used for the time integration. The grid-length $d = 5 \times 10^5$ m and the time-step for integration is 12 minutes.

2.5. *Initial data for uniform westerly flow*

The initial wind 'U' is assumed westerly without any shear in the model atmosphere. The height field is given by

$$z_l = \bar{z}_l - \frac{U_l}{g} \int_0^y f\, dy$$

where l varies from 1 to 3

$\bar{z}_1$=10326 m, $\bar{z}_2$=4848 m, $\bar{z}_3$=1452 m

The centre of the domain is at 25° N and 105° E

The sea level pressure is given by

$$P_{sea} = 1013 - \frac{1013 - 850}{\bar{z}_3} \frac{U_3}{g} \int_0^y f\, dy \tag{11}$$

Orography of the region is shown in Fig. 2. The maximum height of the orography is assumed to be one km in the present study.

Temperatures at different levels are obtained by using hydrostatic approximation, assuming no mountains. After getting the temperature, the hydrostatic approximation is used again to compute surface pressure at different grid points with the terrain height H. The procedure described above gives the initial values for the model.

3. Results of integration

Two cases of westerly flows, U=10 mps and U=15 mps, are investigated by integrating the model for 5 days each so as to get steady state solutions. The forecast winds at 850 and 550 mb levels for the region of Peninsular India and the Bay of Bengal are shown in Fig. 3. For both the levels the wind generated northerly components after crossing the barrier of Western Ghats. The average magnitudes of the northerly wind components at 850 mb level are v=−2.0 mps for U=10 mps case and v=−1.2 mps for U=15 mps case. The points marked on the latitude sections give the approximate location eastward of which the wind is having southerly wind component. For heigher wind speed case (U=15 mps) at 550 mb level, the turning of the wind towards north over the Bay of Bengal is not observed. The surface pressure departures for 10° N to 25° N are shown in Fig. 4. The leeward trough of Western Ghats and Burma mountains are clearly seen. The trough position on the Bay of Bengal is more towards east for U=15 mps case. This is observed for 15° N and 20° N.

3.1. *Forecast fields of temperature*

The forecast fields for temperature are obtained for 400 and 700 mb levels for both U=10 and U=15 mps cases. In general the cooling is observed on the windward side and is due to forced ascent of the westerly flow over western part of the terrain. The warming is observed on the leeward side and is due to downward vertical velocity developed on the leeside of the terrain. Table 1 gives the locations where the maximum values of temperature departures are obtained.

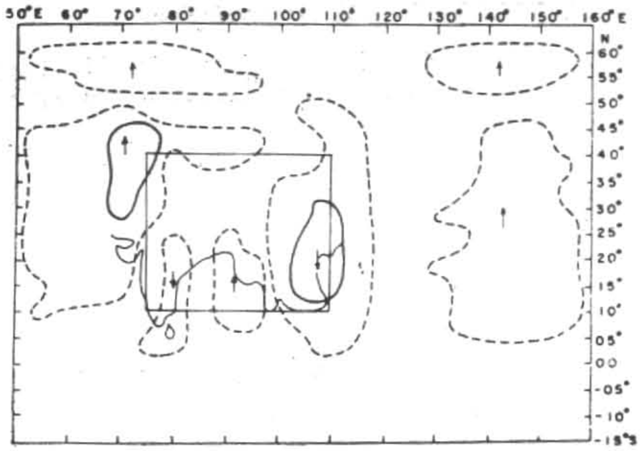
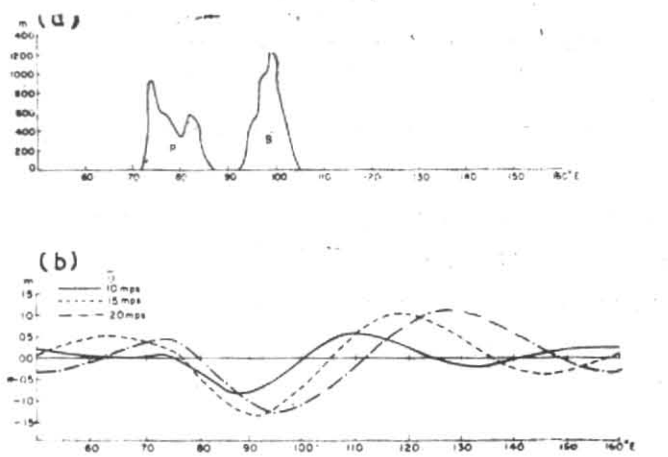
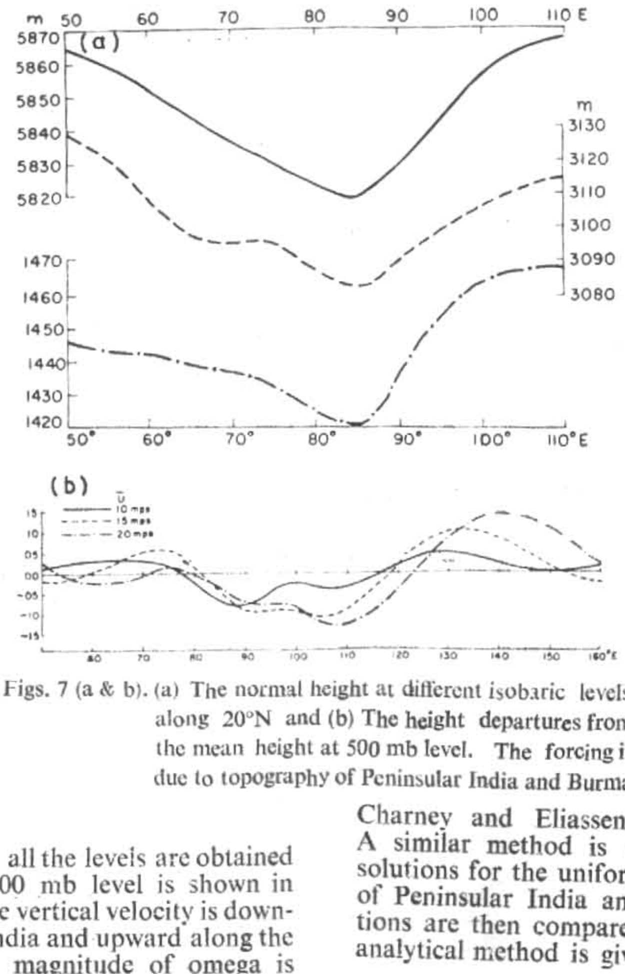


Fig. 5. The vertical p - velocity at 700 mb level for U=10 mps case (Forecast : 120 hours) Unit : 10^{-3} mb sec^{-1}. The contour spacing is at 0.3 unit interval



Figs. 6 (a & b). (a) Topography of Peninsular India and Burma along 20°N and (b) The height departures from the mean height at 500 mb level. The forcing is due to topography of Peninsular India



Figs. 7 (a & b). (a) The normal height at different isobaric levels along 20°N and (b) The height departures from the mean height at 500 mb level. The forcing is due to topography of Peninsular India and Burma

3.2. *Omega fields*

The vertical velocities ω for all the levels are obtained and the typical pattern for 700 mb level is shown in Fig. 5, for $U=10$ mps case. The vertical velocity is downward along the east coast of India and upward along the west coast of Burma. The magnitude of omega is highest at the surface and decreases aloft. The magnitudes are also higher, in general, for $U=15$ mps case compared to that of $U=10$ mps case.

4. Results with analytical method

Analytical treatment of the linear steady state vorticity equation with uniform zonal flow is given by Charney and Eliassen (1949) and Okamura (1976). A similar method is adopted to get the analytical solutions for the uniform westerly flow over the terrain of Peninsular India and Burma. The analytical solutions are then compared with the model results. The analytical method is given below :

The linear steady state vorticity equation to homegeneous atmosphere is given by :

$$\bar{U}\frac{\partial \zeta}{\partial x}+\beta\,v+f_0\,D+\mu\,\zeta=0 \qquad (12)$$

where $\bar{U}$ is the constant zonal wind, μ is the coefficient of surface friction and D is the horizontal divergence term. The other symbols have their usual meanings.

TABLE 2

The numerical values of different constants

Symbol	Description	Value
L	Length of the domain	11×10^3 km
M	Width of the domain	8×10^3 km
z_0	Height of the homogeneous atmosphere	8.0 km
μ	Coefficient of surface friction	5×10^{-6} sec^{-1}
β	Beeta parameter at 20°N	2.15×10^{-11}m^{-1} sec^{-1}
g	Acceleration due to gravity	9.8 m sec^{-2}
f_o	The coriolis parameter	0.4988×10^{-4} sec^{-1}

The divergence is given by :

$$D = \frac{\overline{U}}{z_o} \frac{\partial H}{\partial x} \tag{13}$$

where z_0 is the mean height of the incompressible homogeneous atmosphere and H is the height of terrain above mean sea level.

Assuming geostrophic approximation and introducing $\chi = 2\pi x/L$, the Eqn. (12) can be put in the following form :

$$\frac{\partial^3 z}{\partial \chi^3} + s^2 \frac{\partial z}{\partial \chi} + \sigma \left(\frac{\partial^2 z}{\partial \chi^2} - m^2 z \right) = - \eta^2 \frac{\partial H}{\partial \chi} \tag{14}$$

where,

$$s^2 = \left(\frac{L}{2\pi} \right)^2 \left(\frac{\beta}{\overline{U}} - m^2 \right) \tag{15}$$

$$\sigma = \left(\frac{L}{2\pi} \right) \frac{\mu}{\overline{U}} \tag{16}$$

$$m^2 = \left(\frac{L}{2\pi} \right)^2 m_1^2 \tag{17}$$

and $$\eta^2 = \left(\frac{L}{2\pi} \right)^2 \frac{f_0^{\,2}}{g z_o} \tag{18}$$

where L represents the length of the domain and g is the acceleration due to gravity.

The assumption for height field having sine dependency with respect to y is made and is accounted for by putting $\frac{\partial^2 z}{\partial y^2} = - m_1^2 z$ (Charney and Eliassen 1949).

where,

$$m_1 = \frac{\pi}{M} \tag{19}$$

and M represents the width of the channel.

TABLE 3

Experiments with topography of Peninsular India

Zonal wind (mps)	Position of the trough	Estimated wave length (km)	Wave length L_x*
10	89°E	5000	4380
15	92°E	5900	5470
20	95°E	6700	6448

L_x*is obtained by Rossby-Haurwitz formula.

The solution of the Eqn. (14), is given by

$$z(\chi) = \eta^2 \int_0^{2\pi} H(\chi') \phi (\chi - \chi') \, d\chi' \tag{20}$$

$z(\chi)$ represents the height deviation from the mean height at 500 mb level. $H(\chi')$ represents the numerical values of terrain height along the latitude circle and $\phi(\chi)$ is the Green's function which is given by

$$\phi(\chi) = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\left(n^2 - s^2\right) \cos n\chi - \sigma \left(n + \frac{m^2}{n}\right) \sin n\chi}{\left(n^2 - s^2\right)^2 + \sigma^2 \left(n + \frac{m^2}{n}\right)^2} \tag{21}$$

The numerical values for different constants are given in Table 2. The Green's function as given by the expression (21) is evaluated for the given speed of westerly wind and taking suitable number of 'n' values for the summation. The values of elevation of the earth at one degree interval are obtained from the table provided by Berkofsky and Bertoni (1960). The topography of Peninsular India and Burma mountains along the section of 20° N is shown in Fig. 6(a).

4.1. *Solution of height field due to topographic forcing due to Peninsular India*

The solutions for different wind speeds as given in Table 3 due to the topographic forcing caused by Peninsular India are shown in Fig. 6(b). The positions of the lee troughs are also given in Table 3. The trough is more intense (large-amplitude) and the position is more eastwards as the speed increases. The wave-length of the lee-wave is estimated by using the expression of Rossby-Haurwitz formula for the channel flow. The expression for the phase-speed of the Rossby waves is given by

$$c = \overline{U} - \frac{\beta + \lambda^2 \overline{U}}{k^2 + m_1^2 + \lambda^2} \tag{22}$$

where,

$$k = \frac{2\pi}{L_x} \text{ and } \lambda^2 = \frac{f_o^2}{g z_0}$$

m_1 is given by (19)

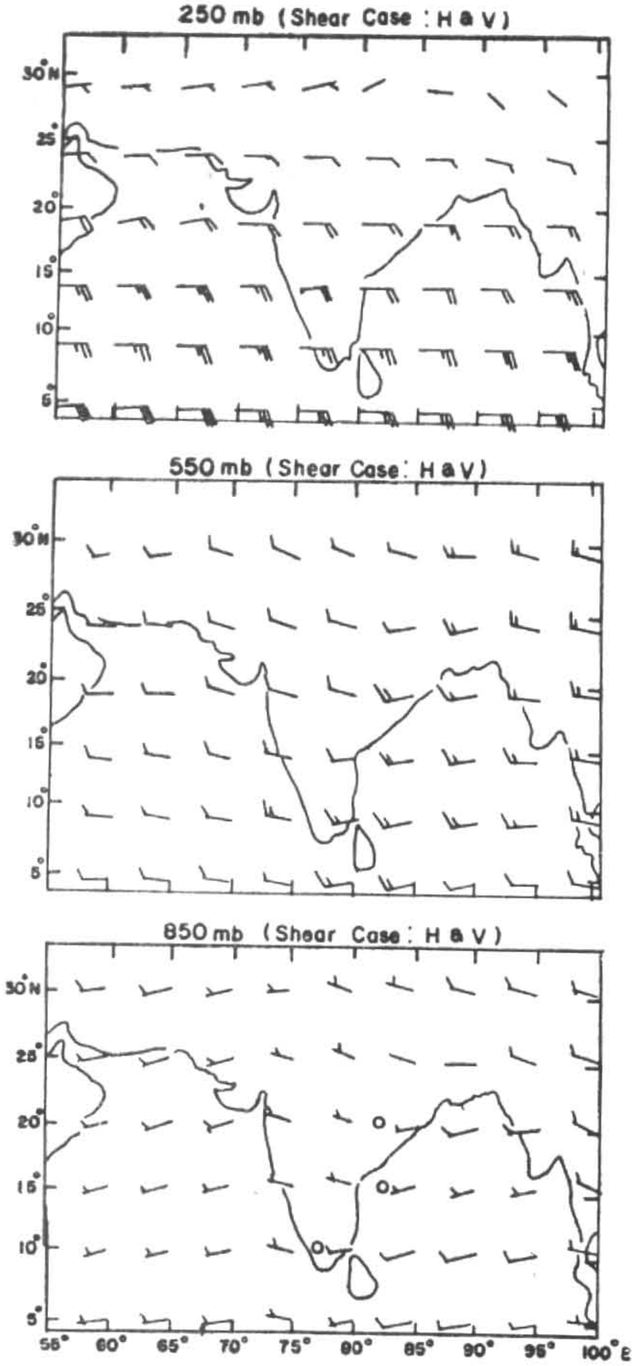


Fig. 8. The forecast wind (shear case) after 120 hours

TABLE 4

Normal values of height at different longitudes along 20°N for the month of July

Levels	50	55	60	65	70	75	80	85	90	95	100	105	110°E
850	1445	1443	1442	1439	1437	1432	1424	1421	1436	1454	1463	1467	1469
700	3129	3121	3109	3098	3096	3096	3087	3083	3091	3099	3107	3112	3115
500	5864	5858	5852	5843	5836	5830	5820	5819	5831	5840	5856	5863	5867

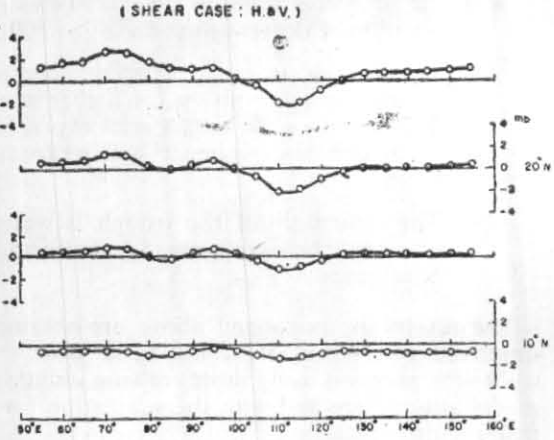


Fig. 9. The surface pressure departures (shear case) along different latitudes after 120 hours

TABLE 5

Experiments with topography of Peninsular India and Burma

Zonal wind (mps)	Position of the first trough	Position of the second trough
10	88°E	107°E
15	92°E	105°E
20	*	108°E

*The position of the trough minimum cannot be determined.

TABLE 6

The initial zonal wind (in mps) for shear case

Levels (mb)	Lat. °(N) 65°	60°	55°	50°	45°	40°	35°	30	25°	20°	15°	10°	05°	00°	−05°	−10°	−15°
250 mb	12.0	13.6	13.8	12.7	10.3	6.9	2.9	−1.5	−5.6	−9.3	−12.0	−13.6	−13.8	−12.7	−10.3	−6.9	−2.9
550 mb	10.2	9.7	9.4	9.1	8.8	8.4	8.1	7.8	7·5	7.2	6.9	6.6	6.3	5.9	5.6	5.3	5.0
850 mb	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0

For steady state $c=0$. The equation for L_x is then given by

$$L_x = 2\pi \sqrt{\overline{U}\Big/\left(\beta - \frac{\pi^2\overline{U}}{M^2}\right)} \tag{23}$$

The computed wave-length from (23) can be compared with the estimated wave length. The agreement for the two estimates is better for the higher wind speed.

The normal values for height (*Upper air Atlas of India and neighbourhood*, IMD-1972) for the month of July for 50°E to 110°E along 20°N are plotted in Fig. 7(a) for different isobaric surfaces (*see* Table 4 also). The trough position obtained by the analytical solution for $U=$ 10 mps case is at 89° E which compares satisfactorily with the normal position of the trough at 85° E.

4.2. *Effect of Burma mountains on the height solution*

The analytical solutions for different wind speeds are also obtained due to topographic forcings caused by Peninsular India and Burma mountains. The solutions are shown in Fig. 7(b). Increase in wind speed, in this case also, shifts the position of the first trough (the trough leeward side of Peninsular India) towards east. The position of the two troughs are given in Table 5. For $U=10$ mps case, the trough due to Peninsular India is well developed but for $U=20$ mps case, the trough of Burma is more intense. The orographic response for high wind speed is thus unfavourable for Bay of Bengal trough, as its position is more eastward from the observed normal trough position and also the cyclonic turning of the wind eastward of the trough position is also reduced.

The results obtained from the three level P.E. model and from the analytical solutions are properly simulated as the positions of the leeward troughs, shift in the position of the trough towards east for higher wind speed, estimated wave lengths, generation of the northerly wind components on the leeside of Western Ghats etc are more or less in agreement.

TABLE 7

Position of the simulated Bay of Bengal trough at different levels

Levels (mb)	Shear case		U=10 mps		U=15 mps	
	15°N	20°N	15°N	20°N	15°N	20°N
Surface	85°E	85°E	85°E	85°E	90°E	90°E
850	82.5°E	82.5°E	87.5°E	87.5°E	87.5°E	87.5°E
550	77.5°E	82.5°E	87.5°E	87.5°E	92.5°E	97.5°E

5. The orographic response with the initial model atmosphere having horizontal and vertical shears

The results obtained with uniform westerly flows showed some interesting features at the lower troposphere. In these experiments uniform westerly wind throughout the atmosphere was assumed. The atmospheric flows over this region, however, are not simple and both the horizontal and vertical shears exist during summer monsoon season. Apart from orographic forcing, the complex thermal forcing is also present. In the experiment conducted below, the effect of thermal forcing is not considered and the orographic response only is tested by studying the evolution of the flows which have got both the horizontal and vertical shears at the beginning.

The initial winds for the experiment are given in Table 6. The wind structure is more or less similar to that given by Das (1980). The westerly wind is assumed to be uniform in the lowest layer but in the two layers above the lowest layer the wind is not uniform. With the initial wind as given in Table 6, the model is integrated for 5 days. The forecast fields for wind and surface pressure departures are shown in Fig. 8 and Fig. 9 respectively. The easterly flow at 250 mb level is not disturbed due to topographic forcing. The westerly flows at 850 mb and 550 mb levels show southward turning on the lee side of the Western Ghats. The points marked on the latitude sections of 10° ,15° and 20° N show the position of the trough axis. In Table 7 the vertical tilt of the trough at the head Bay of Bengal is given for shear case and for uniform westerly wind cases. The vertical tilt of the trough with wind shear westward whereas it is eastward, for the model with uniform wind flow.

6. Conclusions

(*i*) The response of orography of Western Ghats on the generation of the large-scale lee trough when the westerly flow of the lower troposphere crosses the barrier, is brought out by the above experiments. The trough is located near normal trough position at the surface for $U=10$ mps case and for the shear case.

(*ii*) The analytical solution for $U=10$ mps gives the position of the trough at 88° E for 500 mb level

(*iii*) The trough is more eastward compared to normal trough position for higher wind speed. This is shown by both model experiments and the analytical experiments with uniform westerly flows.

(*iv*) The vertical tilt of the trough is westward for shear case but is eastward for uniform westerly wind cases.

The results as mentioned above are obtained under simplified conditions of atmospheric flow. Further work is in progress using more realistic initial values for model atmosphere and with the simulation for the thermal forcing effects.

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