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# Stochastic dynamic model for long range prediction of monsoon rainfall in Peninsular India

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**सार** – इस शोध पत्र में प्रसंगांक गतिज प्रदर्श विकसित करने का प्रयास किया गया है । इसे भारत के बड़े उप–प्रभाग जैसे प्रायद्वीप में जून से सितम्बर तक की मानसूनी वर्षा के पूर्वानुमान के लिए उपयोग में लाया जा सकता है । इस प्रकार के मॉडल को बनाने के लिए वायुमंडल को एक रेखित गतिज प्रणाली माना गया है । यह विभिन्न आगमों को निर्गमों में परिवर्तित करता है : जैसे वर्षा । इस अध्ययन में 75° पू० देशान्तर की ओर 500 मि० बार पर अप्रैल की उप–क्षोभमंडलीय पर्वत श्रेणी स्थिति के माध्य को वायु मण्डल के आगम के लिए उपयोग में लाया गया है । पिछले 38 वर्षों (1939–1976) के आगम एवं निर्गम आंकड़ों को प्रदर्श के बनाने के लिए उपयोग मे लाया गया है । इसमें वायुमंडल की गतिकी का उपयोग किया गया है और वर्षा के पूर्वानुमान के लिए अरिमा प्रक्रिया का भी उपयोग किया गया है ।

प्रति चयन और जांच (1977 से 1980 तक) अवधियों में मॉडल का कार्य काफी अच्छा रहा है । यहां तक कि कोटि अनावृष्टि और ज्यादा वर्षा के वर्षों में पूर्वानुमानित और वास्तविक मानों में काफी निकटता देखी गई है । दीर्घ परिसीमा पूर्वानुमानों के लिए भारत मौसम विज्ञान विभाग में आजकल सात श्रेणियां उपयोग में लाई जा रहीं हैं । जांच अवधि में प्रदर्श पूर्वानुमान का कौशल अंक उच्चतम रहा जो एक पूर्वानुमान सूत्र के हो सकते हैं । इसमें यह सुझाया गया है कि इस विकसित प्रसंगांक गतिज प्रदर्श को प्रायद्वीप में ऋतु के आने के एक महीने पहले मानसूनी वर्षा की ठीक-ठीक दीर्घ सीमा पूर्वानुमान जारी करने के लिए उपयोग में लाया जा सकता है इस से मानसून के अधिक प्रबल हो जाने से संभावित विनासकारी प्रभाव से बचने के लिए योजना बनाने में पर्याप्त समय मिल जायेगा ।

ABSTRACT. An attempt has been made to develop a Stochastic dynamic model that could be used for forecasting monsoon rainfall, June to September, in the larger sub-division of India, *viz.*, Peninsula. For building such a model the atmosphere has been considered as a linear dynamic system that converts various inputs into the output, say the rainfall. In this study the 500 mb mean April sub-tropical ridge position along Long. 75° E has been used as input to the atmosphere. The input and output data for the recent 38 years (1939-1976) have been utilised for developing the model which utilises the dynamics of the atmosphere and also that of the ARIMA process to forecast the rainfall.

The performance of the model has been found good during the sample and the test (1977 to 1980) periods. Even in rank drought and excess rainfall years the closeness of the predicted and realised values stands out well. In terms of seven categories currently being used by the India Meteorological Department for describing its long range forecasts, the skill score of the model forecast for the test period has been found equal to one which is the highest that a forecast formula can have. This suggests that the Stochastic Dynamic Model developed here can, therefore, be used for issuing more accura[illegible]

TABLE 1

Final estimates of $\delta$'s and $\omega$'s

| $i$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $\delta_i$ | — | —0.8986 | —0.5319 | —0.3459 | —0.2007 |
| $\omega_i$ | +0.0554 | —0.0520 | —0.0385 | —0.0197 | —0.0056 |

TABLE 2

Values of white noise $a_t$ for recent 21 years (1960-1980)

| $j$ | $a_j$ | $j$ | $a_j$ | $j$ | $a_j$ |
|---|---|---|---|---|---|
| 1960 | —0.2721 | 1967 | 0.1333 | 1974 | —0.0151 |
| 61 | 0.1555 | 68 | 0.1025 | 75 | 0.0116 |
| 62 | —0.1500 | 69 | —0.0893 | 76 | —0.0752 |
| 63 | —0.0166 | 70 | 0.1787 | 77 | 0.0642 |
| 64 | —0.2308 | 71 | —0.2190 | 78 | —0.0430 |
| 65 | —0.1197 | 72 | —0.0765 | 79 | —0.1396 |
| 66 | —0.0504 | 73 | 0.1028 | 80 | —0.0208 |

been developed by regarding the output as linear aggregate of a series of super-imposed impulse response functions scaled by the input. By following this usual identification procedure, the dynamic transfer function can be expressed as follows :

$$y_t = \omega_0 x_t + \sum_{i=1}^{4}(-1)\,\omega_i x_{t-i} + \sum_{i=1}^{4}\delta_i y_{t-i} \qquad (9)$$

where,

$$y_t = \log (R_t/R_{t-1}),$$

$$x_t = X_t - X_{t-1} \qquad (10)$$

$R_t$ is monsoon rainfall in centimetres over Peninsula and $X_t$ is the position of the 500 mb April sub-tropical ridge in degrees latitude along Long. 75° E. The values of $\delta$'s and $\omega$'s are given in Table 1.

Eqn. (9) gives the exact mathematical expression for the dynamic transfer function of the forecast model for Peninsula. For representing the complete Stochastic dynamic model, however, the mathematical expression for the noise transfer function is also required. The procedure for identifying the noise transfer function has been described in detail by the author (Thapliyal 1981 a) elsewhere and it may be sufficient to mention that the noise transfer function has been developed by using the dynamics of the ARIMA process which postulates that a time series can be usefully represented as an output from a dynamic system to which the input is a white noise. By following this usual identification procedure, the noise transfer function can be expressed as follows :

$$a_t = n_t - .52a_{t-1} - .15a_{t-4} - .28a_{t-13} \qquad (11)$$

Here

$$n_t = \nabla N_t \qquad (12)$$

$$N_t = (y_t)_A - (y_t)_F \qquad (13)$$

where $(y_t)_A$ is the actual rainfall defined in Eqn. (10) and $(y_t)_F$ is the forecast rainfall obtained from Eqn. (9) and $a$'s represent independent shocks.

For recent 21 years (1960 to 1980), the values of $a$'s have been obtained from Eqn. (11) and are given in Table 2.

On using Eqns. (9) and (11) in Eqn. (8) the dynamic (dynamic transfer function noise) model for Peninsula can be expressed as follows :

$$y_t = \left[\omega_0 x_t + \sum_{i=1}^{4}(-)\,\omega_i x_{t-i} + \sum_{i=1}^{4}\delta_i y_{t-i}\right] + \left[\hat{a}_t - .52a_{t-1} - .15a_{t-4} - .28a_{t-13}\right] \qquad (14)$$

where $\hat{a}_t$ represents the residuals.

In Eqn. (14), the expressions in the first and the second brackets represent, the dynamic and the noise transfer functions, respectively. The former accounts for the dynamic relatonship between the April ridge position over India and the monsoon rainfall in Peninsula while the latter takes into account the effects of all those inputs which have not been considered while developing the dynamic transfer function of the model.

By studying the behaviour of the auto and partial correlation functions of the residual $\hat{a}_t$ along with that of the cross correlation function between input, $x_t$ and the residual, $\hat{a}_t$ it has been found that the dynamic and the noise transfer functions of the model are adequate and need no further refinements. In addition, Portmanteau lack of fit test (Box and Jenkins 1970) has also not revealed any adequacy of the model. From Eqn. (14) we thus obtain the long range forecast dynamic model for Peninsula as follows :

$$R_t = (R_{t-1})\left\{(R_{t-1}/R_{t-2})^{\delta_1} \cdot (R_{t-2}/R_{t-3})^{\delta_2} \cdot (R_{t-3}/R_{t-4})^{\delta_3} \cdot (R_{t-4}/R_{t-5})^{\delta_4}\right\} \cdot \exp\left\{\omega_0(X_t - X_{t-1}) + \sum_{i=1}^{4}(-1).(X_{t-i} - X_{t-i-1}) - (.52a_{t-1} + .15a_{t-4} + .28a_{t-13})\right\} \qquad (15)$$

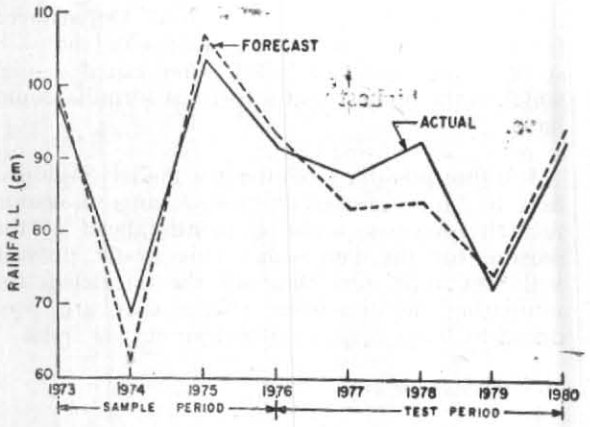


Fig. 4. Performance of model during recent 8 years (1973—1980)

where $R_t$ is the amount of monsoon rainfall in centimetres for the year $t$ over Peninsula, $X_t$ is the position of the 500 mb April sub-tropical ridge in degrees latitude for the year $t$ along 75° E meridian. The values of $\delta$'s and $\omega$'s are given in Table 1 and that of $a$'s are calculated from Eqn. (11). However for ready reference its values for recent 21 years (1960-1980) are given in Table 2. It is, therefore, possible from the model, given in Eqn. (15) to obtain an optimal forecast in early May for the monsoon rainfall in Peninsula. From the dynamic model, the amount of monsoon rainfall in Peninsula could thus be forecast a month earlier than the start of the season.

**5. Performance and superiority of the model**

In the previous section the Stochastic dynamic model has been developed for forecasting the monsoon rainfall in Peninsula. To assess the performance of the model, the forecast rainfall amounts for recent 8 years (1973-1980) have been calculated from the model and are shown in Fig. 4. In the same figure the actual rainfall amounts have also been presented so that the accuracy of the forecasts can be examined. It is seen from the figure that the forecast rainfall amounts are quite close to those realized. Even in the rank drought years, like 1974 and 1979, the closeness of the prediction and realized values stands out remarkably well. In addition, during extreme years such as 1975 and 1974, the model has correctly indicated the excess and deficient monsoon rainfall in Peninsula. The Root Mean Square Error (RMSE) for the recent 8 years (1973-1980) has been found equal to 4.4 cm. Considering the fact that the significant fluctuations in the Indian monsoon rainfall have occurred (Thapliyal 1981 b) during these years, the amount of the RMSE for the forecast is significantly small. In view of this it can be concluded that the performance of the model forecast during the recent period is good.

By utilizing the new model, experimental forecasts have been prepared for recent 4 years (1977-1980). The data of these years were not utilized to formulate the model and hence the accuracy of the forecasts during this test period would indicate the degree of confidence which could be placed in the model forecasts. In Fig. 4 the forecast and the actual rainfall amounts for the test period have also been presented. It is seen from the figure that the amounts of rainfall predicted by the model are quite close to those actually realized in the Peninsula. This indicates that the performance of the model is good during the test period (1977-1980) also and more confidence can, therefore, be placed in the forecast obtained from the dynamic model.

As mentioned above, the forecast model developed here utilizes the dynamics of the system and also of the ARIMA process and hence the accuracy of the model forecast must be higher than those obtained from the conventional long range forecast methods because the techniques that are currently being used for preparing the operational forecasts do not use the dynamics of the system. In a recent study, this aspect has been examined by the author (Thapliyal 1981 a) in detail. He has reported higher forecast errors for the regression equation than for the dynamic model considered here. In fact, during recent 8 years (1973-1980) the RMSE for the regression has been found nearly twice to the model and it can, therefore, be inferred that the performance of the dynamic model is better than that of the regression technique.

Presently the India Meteorological Department prepares long range monsoon forecast by using the regression equation given below :

$$R = -58.1595 + 2.42 I_T + 1.127 A_R - 0.296 S_p$$

where, $R$ is rainfall in inches, $I_T$ is Indian temperature (mean minimum March temperature of Jaisalmer, Jaipur and Calcutta), $A_R$ is April ridge position and $S_p$ is South American pressure of April and May. The Department describes its long range forecasts in seven categories such as large defect (below 50 per cent of the normal), moderate defect (50-74), slight defect (75-89), normal (90-110), slight excess (111-125), moderate excess (126-150) and large excess (above 150). The figures given in the brackets represent the percentage of the normal rainfall for the forecast area. During the test period (1977-1980) the multiple regression equation used by the Department gave correct forecasts on 75 per cent occasions. When expressing the model forecasts in terms of the above categories, it has been found that the model gave correct forecasts on all occasions during the test period (1977-80). This indicates that the model has skill score for the test period equal to one which is the highest that a forecast formula could have. This suggests that the Stochastic dynamic method can provide more accurate forecasts as compared to multiple regression technique which itself is considered to be superior to many of the other conventional long range

weather forecast methods. It can, therefore, be concluded that the Stochastic dynamic models provide better predictions and can be termed as superior to other long range forecast methods.

**6. Concluding remarks**

It is possible from the Stochastic dynamic model, developed here, to forecast in early May the monsoon rainfall in Peninsula. The model utilizes the dynamic relationship associating the input and the output and also the noises that account for the effects of numerous other inputs of the atmosphere. For formulating the model, the atmosphere has been considered as a linear dynamic system that converts various inputs into the rainfall. The input used in this study is not a direct input but is an indicator of a large number of inputs of the atmosphere. The interaction among the prevailing winds, heat sources and sinks results in the formation of the anomalies which in turn produce some typical synoptic patterns like ridges and troughs at different levels of the atmosphere. Since the 500 mb level separates two different circulation regimes of the upper and lower troposphere, particularly in the tropics, it may be reasonable to assume that the locations of the trough and the ridge are governed by a large number of inputs of the atmosphere. Besides, the location of the ridge in the month of April has shown a positive correlation with the monsoon rainfall in India. It can, therefore, be assumed that more northerly location of the ridge indicates a good degree of organization of monsoon circulation ahead or *vice-versa*. All these considerations suggest that the input selected here does not represent a single factor but a resultant of a large number of direct inputs of the atmosphere. Thus, it is obvious that the model developed here may provide more accurate forecasts than those obtained from a single input forecast model.

The Stochastic dynamic model developed here has provided more accurate forecasts than those obtained from the regression and the multiple regression techniques which are considered to be superior to many of the other conventional long range forecast methods, currently being used in the world. During extremely deficient and excess rainfall years, the model has provided correct forecasts for the monsoon rainfall in Peninsula. Even in rank drought years, the closeness of the prediction and realized values stands out remarkably well. In terms of the seven categories currently being used by the India Meteorological Department for describing its long range forecasts, the skill score of the model has been found equal to one which is the highest that a forecast formula could have.

It is thus possible from the new model developed here to issue more accurate long range monsoon rainfall forecasts, about a month ahead of the season, for the Peninsula. This would provide sufficient time for planning the strategies for mitigating the disastrous effects that are produced by large vagaries of monsoon over India.

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