

# Spatial variation of probability distribution of annual precipitation in India

NITYANAND SINGH

Indian Institute of Tropical Meteorology, Pune

(Received 3 September 1984)

**सार** — भारत में समान रूप से फैले 460 स्टेशनों, जिनमें से प्रत्येक के 1901-1960 तक की दीर्घावधि का समान रूप से रिकार्ड उपलब्ध है, के आंकड़ों का उपयोग करके मानक विचलन और वार्षिक माध्य वर्षा का वर्णन करते हुए एक सरल रेखिक समाश्रयण समीकरण प्राक्कलित किया गया है। इसमें कल्पना की गई है कि भारत के विभिन्न स्थानों में वार्षिक वर्षा सामान्य रूप से बंटी हुई रहती है तथा मानक विचलन एवं माध्य वार्षिक वर्षा में संबंध का उपयोग करके एक आरेख बनाया गया है जिसका नाम वार्षिक माध्य वर्षण आरेख रखा है। यह आरेख देश भर की वार्षिक वर्षा के संभावित बंटन को दर्शाता है। देश के विभिन्न भागों में वर्षा की विशिष्ट मात्रा वाले स्थान को ध्यान में रखते हुए संभाव्यता में विचरण को आधार मानकर वर्षा के क्षेत्रों को रेखांकित किया गया है।

**ABSTRACT.** A simple linear regression describing relationship between standard deviation and the average (mean) annual precipitation in India has been estimated using data of uniformly spread 460 stations, each having homogeneous records for a long period 1901-1960. Assuming that annual precipitation at individual station in India is normally distributed and using relationship between S.D. and the average annual precipitation a diagram has been prepared, designated as average probability diagram (APD), which depicts probability distribution of annual precipitation over the country. Precipitation zones have been delineated in the country based on spatial variation of probability of getting specified amount of precipitation in different parts of the country.

## 1. Introduction

Studies on graduating the frequency distribution of the point precipitation data of various time scales, e.g. annual, seasonal, weekly etc, by suitably selected probability models have been carried out to a considerable extent (see Mooley 1973). Estimating the percentiles/deciles of precipitation from the fitted model and giving it in tabular form stationwise or preparing isohyetal map of the region showing distribution of particular percentile/decile estimate, have been widely practised. Sarker *et al.* (1982) have computed weekly rainfall probabilities assuming it to be gamma distributed and discussed at length its role for agricultural planning in dry farming tract of India. Presently relationships have been obtained showing variation of selected precipitation probabilities (separately) with the spatial variation of the average annual precipitation in India in order to simplify understanding the probability distribution of annual precipitation in the country.

Cocheme and Franquin (1967) prepared a diagram, named 'rainfall probability diagram', for annual rainfall distribution over semiarid areas in Africa (south of Sahara) by making the assumptions : (a) mean annual rainfall to be between 500 and 1000 mm, (b) coefficient of variability of 22.5% at 500 mm and 15% at 1000 mm, and (c) no skewness and shifting of the mean on the high values side of the median. These authors had attempted the problem with a view to generate

information for part of the area having incomplete or no historical records. Diagrams of similar type have been prepared by Gupta and Singh (1981) for annual and summer monsoon (June to September) rainfall of Rajasthan (India). In Rajasthan, the mean annual rainfall varies from 125 to 1000 mm (of which 90% is received during summer monsoon) and coefficient of variation from 80% to 30% (70% to 30% in case of summer monsoon) and results for both the cases were quite satisfactory. These studies are empirical in nature. The motivation behind the present study is to give a theoretical justification of preparing such diagram by taking the example of India. Based on getting specified amount of precipitation with chosen probabilities precipitation zones have been delineated in the country in order to illustrate applications of the study.

## 2. Data used and methodology

### 2.1. Relationship between the average annual precipitation and its standard deviation

Rao *et al.* (1971) had computed Fisher's 'g' statistic (both  $g_1$  for skewness and  $g_2$  for kurtosis) of annual precipitation of 460 stations well distributed in India, each having homogeneous records for a long period (1901-60), and concluded that it is normally distributed over major parts of the country. Then, considering that annual precipitation over various Indian

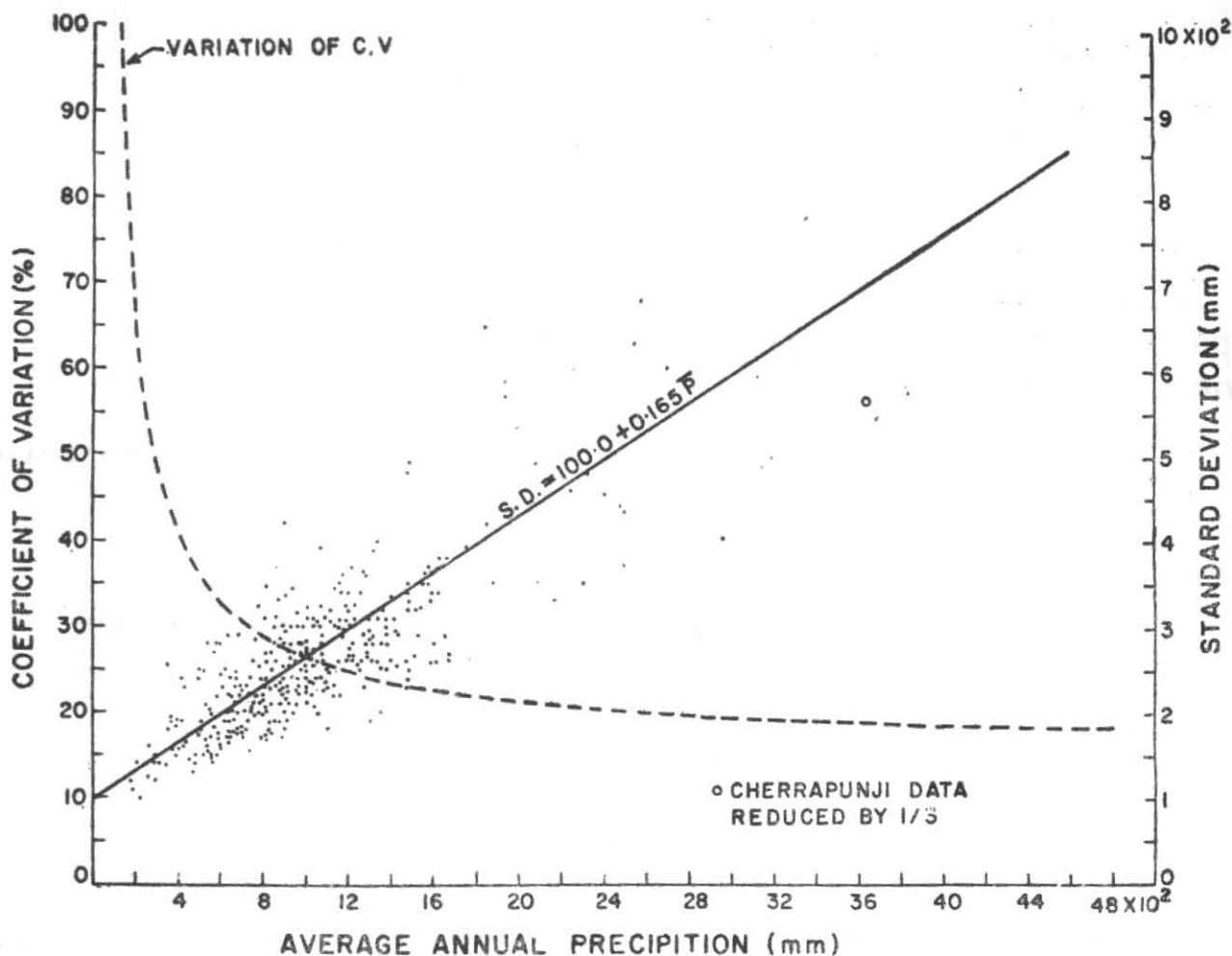


Fig. 1. Relationship between standard deviation and the mean annual precipitation of India (1901-60)

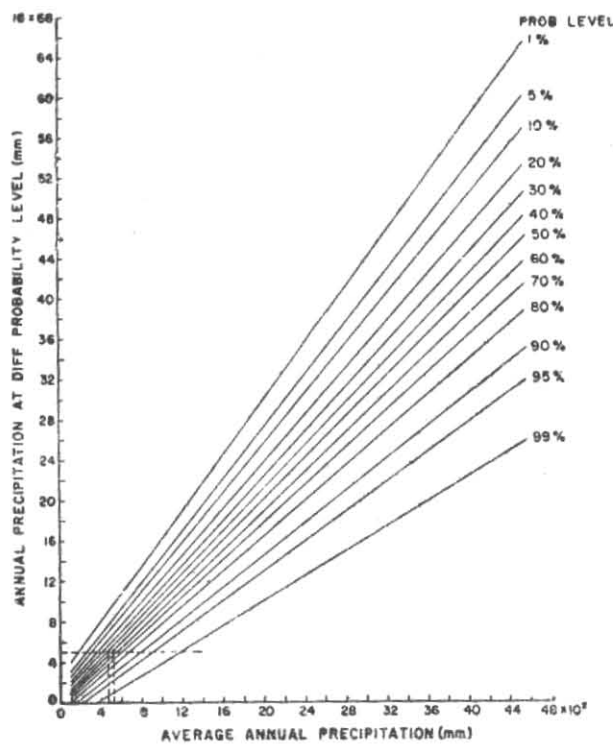


Fig. 2. Average probability diagram of the annual precipitation of India

stations is normally distributed, a linear regression of standard deviation (SD) on the average annual precipitation ( $\bar{P}$ ) has been estimated using data of these 460 stations from the above publication which is as follows,

$$SD = 100.0 + 0.165 \bar{P} \quad (1)$$

where SD and  $\bar{P}$  are in mm. The correlation coefficient of 0.851 between SD and  $\bar{P}$  is significant at 0.1% level. Antarkar (1965) estimated similar regression equations for east and west Rajasthan separately, and found it as  $SD = 121.2 + 0.211 \bar{P}$  ( $r = 0.61$ ) and  $SD = 63.8 + 0.31 \bar{P}$  ( $r = 0.85$ ) respectively. The study is of limited utility for area-specific problems. The scatter diagram between SD and  $\bar{P}$  for India is shown in Fig. 1, the value of  $\bar{P}$  varies from 99 mm (Leh) to 10881 mm (Cherrapunji). The broken line in Fig. 1 shows variation of coefficient of variation (CV) with  $\bar{P}$  whose equation can be had by dividing both sides of Eqn. (1) by  $\bar{P}$  and multiplying by 100 (since CV is expressed in percent), i.e.,

$$CV = 100 \times \frac{SD}{\bar{P}} = 16.5 + 100\bar{P} \quad (2)$$

which represents the equation of a rectangular hyperbola.

2.2. Relationship between the average and the probability estimates of annual precipitation

In the case of normal distribution the relationship among the probability estimates ( $x_p$ ), the mean ( $\bar{x}$ ) and standard deviation ( $\sigma$ ) is given by

$$x_p = \bar{x} + z_p \cdot \sigma \quad (3)$$

where  $z_p$  is the probability point of unit normal variate. SD and  $\bar{P}$  in Eqn. (1) are based on long-term data and can be taken as population standard deviation ( $\sigma$ ) in mm and the mean ( $\bar{x}$ ) in mm respectively. Substituting  $\sigma$  from Eqn. (1) into (3) we get,

$$x_p = \bar{x} + z_p \cdot (100.0 + 0.165 \bar{x}) \quad (4)$$

For constant  $z_p$ , Eqn. (4) represents equation of a straight line.  $z_p$  for 13 selected probability levels, e.g., 1, 5, 10, 20 ..... 90, 95 and 99% has been taken from the normal table.

All the different lines for  $z_p$  values for probability levels 1 to 99% are drawn in the same diagram, since, the diagram presents the relationship between precipitation at different levels and the average precipitation it is designated, therefore, as average probability diagram (APD) of annual precipitation of India (Fig. 2). The equations of the lines in APD will be called as average probability relationships (APRs) and are given below :

$$x'_1 = 232.6 + 1.3838 \bar{x}$$

$$x'_5 = 164.5 + 1.2714 \bar{x}$$

$$x'_{10} = 128.2 + 1.2117 \bar{x}$$

$$x'_{20} = 84.2 + 1.1389 \bar{x}$$

$$x'_{30} = 52.4 + 1.0865 \bar{x}$$

$$x'_{40} = 25.3 + 1.0417 \bar{x}$$

$$x'_{50} = 0 + 1.0000 \bar{x}$$

$$x'_{60} = -25.3 + 0.9583 \bar{x}$$

$$x'_{70} = -52.4 + 0.9135 \bar{x}$$

$$x'_{80} = -84.2 + 0.8611 \bar{x}$$

$$x'_{90} = -128.2 + 0.7883 \bar{x}$$

$$x'_{95} = -164.5 + 0.7286 \bar{x}$$

$$x'_{99} = -232.6 + 0.6162 \bar{x} \quad (5)$$

where  $x'_p$  is the amount of precipitation (in mm) exceeded with probability  $p$  at the station for which the average annual precipitation is  $\bar{x}$  (in mm). For convenience Fig. 2 is prepared for the annual precipitation only upto 4800 mm but it can be extended for any desired amount of average precipitation.

2.3. Delineating precipitation zones

In order to illustrate the applicability of the study precipitation zones have been delineated in the country depending upon spatial variation of probability of occurrence of specified amount of precipitation. Suppose, for instance, 500 mm precipitation is required annually to run an enterprise, and we wish to know the regions of the country where 500 mm precipitation is expected with probabilities 10, 20... and 90 per cent and less than 10% and more than 90%—the probabilities will indicate percent of years where precipitation exceeds the specified amount in the region. For determining average annual precipitation of the places where 500 mm precipitation is expected with different probabilities, a line at 500 mm parallel to abscissa is drawn in Fig. 2. From intersections of this line with those showing probability levels vertical projections are drawn on the abscissa and the average precipitation can be evaluated. This will give average precipitation of the places corresponding to specified precipitation amount with 1,5,10,20, .90,95 and 99 per cent probabilities.

However, to delineate a zone where 500 mm precipitation is expected with 10% probability, the average annual precipitation of the places expecting this amount with 5% and 15% are to be determined. For this purpose average probability relationships for 5% and 15% probabilities are used and average precipitations calculated as follows: Put  $x'_5 = 500.0$  in the expression  $x'_5 = 164.5 + 1.2714 \bar{x}$ ,  $\bar{x}$  become equal to 263.9. Likewise, by putting  $x'_{15} = 500.0$  in the expression  $x'_{15} = 103.6 + 1.171 \bar{x}$ ,  $\bar{x} = 338.5$ .

Though the probability range is 5 to 15 per cent, the area between isohyets of 263.9 mm and 338.5 mm may be taken as precipitation zone where 500 mm precipitation is expected with 10% probability. Similarly

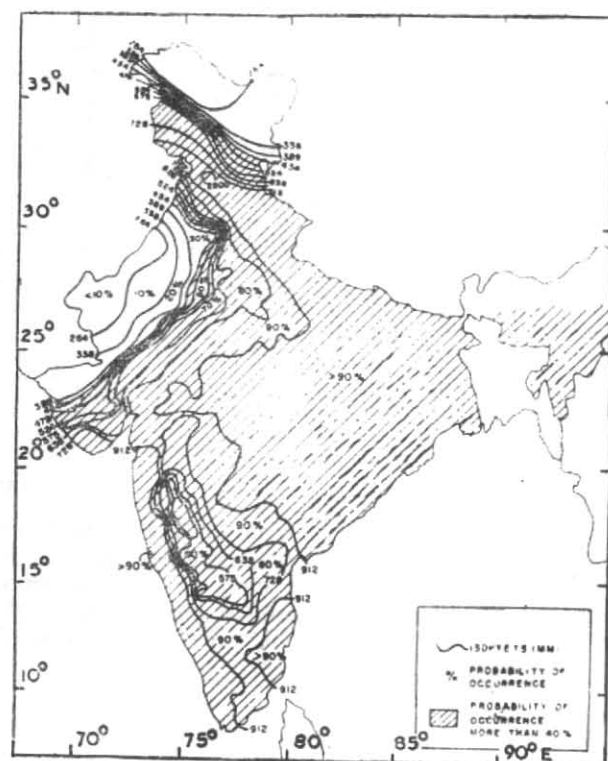


Fig. 3. Probability distribution of occurrence of 500 mm of annual precipitation over India

zones for 20, 30 . . . and 90 per cent probabilities have been demarcated after computing average precipitation of the places expecting 500 mm precipitation with 15, 25, 35 . . . and 95 per cent probabilities (col. 2, Table 1.)

The tolerance of  $\pm 5\%$  in precipitation probabilities would, however, seem high. This can be reduced to  $\pm 2.5\%$  or even less by demarcating zones with 5, 10, 15, 20 . . . etc per cent probabilities and less.

Following similar procedure, average precipitation has been determined for delineating zones for 1000, 1500 and 2000 mm precipitation separately (Table 1). The isohyets of these average precipitation amounts are now drawn on the map of India. For illustration the delineated zones for precipitation amount of 500 is only presented in Fig. 3; hatching in the figure indicates occurrence of the said precipitation amount with 40 per cent or more probability.

#### Results and conclusion

(i) Correlation between standard deviation and the average annual precipitation in India is high.

(ii) The decrease in variability with the precipitation amount is rectangular hyperbolic in nature.

(iii) The idea of precipitation zonation will facilitate in assessing the feasibility of an enterprise which requires certain specified amount of precipitation, in any area of its interest.

Similar study for short period, e.g., seasonal, monthly, weekly etc, precipitation would be of great help for

TABLE 1

The magnitude of annual isohyets (in mm) that can be used for delineating precipitation zones

Probability of occurrence (%)	Precipitation amount (mm)			
	500	1000	1500	2000
5	264	564	928	1293
15	338	726	1140	1554
25	389	821	1266	1710
35	434	898	1366	1834
45	478	967	1456	1946
55	524	1033	1544	2054
65	575	1102	1634	2166
75	638	1179	1734	2290
85	728	1274	1860	2446
95	912	1436	2072	2707

practical purposes. Moreover, normal distribution model does not give good fit to short period precipitation data. To derive average probability relationships for short period precipitation using important probability models, e.g., log-normal, gamma, kappa (2), kappa (3) etc, applicable to precipitation series may be of considerable aid in generating detailed precipitation statistics for places having incomplete or no historical records. So in all we have to interpolate average precipitation of any time scale for the station (with no record) from the average isohyetal map and other precipitation probabilities will be evaluated from average probability relationship of precipitation for that particular time scale.

#### Acknowledgements

The author acknowledges gratefully to Dr. Bh. V. Ramana Murthy, Director, Indian Institute of Tropical Meteorology, Pune for necessary facility to pursue the study. He is also grateful to anonymous referee for his valuable suggestions for modifying the paper. Thanks are also due to Dr. K. Rupa Kumar for his help in computer programming. The manuscript is neatly typed by Miss S. R. Kamble.

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