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The vertical distribution of mixing ratio of ozone

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सार – प्रस्तुत शोध पत्र में विक्षुब्ध वायुमंडल में ओज़ोन के मिश्रित अनुपात के ऊर्ध्वाधर वितरण के परिकलन के लिये गणित का प्रयोग किया गया है । विभिन्न तुंगताओं पर वायुमंडलीय ओज़ोन के मिश्रित अनुपात के परिकलन के लिये भिन्न वायुमंडलीय स्तर विन्यास लिये गए हैं । क्षोभमंडल तथा ऊपरी समताप मंडल में ओज़ोन के ऊर्ध्वाधर वितरण पर सौर विकिरण की तीव्रता के विचरण पर भी विचार किया गया है ।

ABSTRACT. In the present work a mathematical approach for the calculation of the vertical distribution of mixing ratio of ozone in the turbulent atmosphere is carried out. The different atmospheric stratifications are taken into consideration for the evaluation of the mixing ratio of atmospheric ozone at different altitudes. The variability of solar radiation intensity on the vertical distribution of ozone in the troposphere and upper stratosphere is also considered.

1. Introduction

Metwally (1974) had studied observationally only the vertical distribution of mixing ratio of ozone over Sterling, U. S. A. ($\phi=39^0$ N, $\lambda=171^0$ W) under the effect of cyclone, anticyclone, warm front and cold front. He found that the annual average of mixing ratio is 0.05×10^{-6} at 1000 mb and it increases with height to reach 10.3×10^{-6} at 20 mb.

Goshin (1964) made a nomogram, in which he calculated the vertical distribution of ozone in the atmosphere from $z=0$ to 30 km with considering the turbulence and photochemical reactions but he did not take into consideration the variability of the density of air with height.

Goshin (1965) solved the differential equation of turbulent diffusion of atmospheric impurity taking into consideration the variability of air density with height but he considered the temperature to be constant. This condition is being satisfied in respect of the layer of lower stratosphere because the temperature of this layer is constant.

Gager (1962) found that the heating in the atmosphere is connected with the transfer of atmospheric ozone from Arctic and Subarctic. Teweles and Finger (1958) observed that the heating in the stratosphere in Greenland during January 1958 is accompanied with the increases of total amount of ozone.

2. The method

If ozone is taken as an impurity or if there is a source of ozone then, the differential equation of turbulent diffusion of atmospheric impurities has the form :

$$\frac{\partial r}{\partial t} = -\bar{V}\nabla r + \frac{1}{\rho}\frac{\partial}{\partial x}\left(\rho\, k_x \frac{\partial r}{\partial \alpha}\right) + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\rho\, k_y \frac{\partial r}{\partial y}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\, k_z \frac{\partial r}{\partial z}\right) \quad (1)$$

where r is the mean mixing ratio of ozone in the atmosphere and w is the effect of solar radiation (Goshin 1964) on the mixing ratio of ozone. Let us solve the Eqn. (1) according to the following simple assumptions :

(1) The active transfer of ozone in the troposphere and upper stratosphere equals to zero, *i.e.*, $\bar{V}\nabla r = 0$.

(2) Turbulent transfer of ozone in x and y axes in the mentioned layers equals to zero (*i. e.*, the second and third terms of the r.h.s. are equal to zero).

(3) The density of air varies with height according to the relation :

$$\rho = \rho_0 \left(\frac{T_0 - \alpha z}{T_0}\right)^{\{g/k\alpha\}-1} \quad (2)$$

(a) For the troposphere ρ_0, T_0 are the density and temperature at the earth surface respectively, and α has a positive sign (+6.5°C/km).

(b) For the upper stratosphere ρ_0, T_0 are the density and temperature at the tropopause height, and α has a negative sign (—3.5°C/km).

(4) In these layers $k_z = k$ = constant.

(5) Stationary conditions are considered, *i.e.*, $\partial r/\partial z = 0$

(6) The ozone concentration decreases or increases in the layers under the effect of solar radiation, moreover, the rate of its change is $W = -b\bar{D}r$ (Goshin 1964).

where b is constant and $\bar{D}$ is the average intensity of solar radiation.

For simplicity it is assumed that $\bar{D}$ is independent of height.

Therefore Eqn. (1) becomes :

$$\left[\frac{\partial^2 r}{\partial z^2} + \frac{C_1}{C_2 - \alpha z}\frac{\partial r}{\partial z}\right] = C_3 r \qquad (3)$$

where,

$$C_1 = \alpha - g/k, \quad C_2 = T_0 \text{ and } C_3 = b\bar{D}/k$$

Putting

$$C_2 - \alpha z = e^x$$

Eqn. (3) becomes :

$$\frac{\partial^2 r}{\partial x^2} - C\frac{\partial r}{\partial x} = \epsilon e^{2x} r \qquad (4)$$

where $C = 1 + \frac{C_1}{\alpha}$ and $\epsilon = \frac{C_3}{\alpha^2} << 1$

Considering the solution of the unperturbed equation :

$$\frac{\partial^2 r_0}{\partial x^2} - C\frac{\partial r_0}{\partial x} = 0 \quad \text{as}$$

$$r_0 = A_0 + B_0 e^{cx} \qquad (5)$$

To introduce the perturbation term let

$$r = r_0 + \epsilon r_1 + \epsilon^2 r_2 + \ldots\ldots \qquad (6)$$

Substituting (6), Eqn. (4) yields according to different orders in ϵ to

$$\frac{\partial^2 r_0}{\partial x^2} - C\frac{\partial r_0}{\partial x} = 0 \qquad (7)$$

$$\frac{\partial^2 r_1}{\partial x^2} - C\frac{\partial r_1}{\partial x} = r_0 e^{2x} \qquad (8)$$

$$\frac{\partial^2 r_2}{\partial x^2} - C\frac{\partial r_2}{\partial x} = r_1 e^{2x} \qquad (9)$$

The solution of Eqn. (7) is given by Eqn. (5) with A_0, B_0 determined from the boundary conditions. Substituting Eqn. (5) into Eqn. (8) we get :

$$\frac{\partial^2 r_1}{\partial x^2} - C\frac{\partial r_1}{\partial x} = A_0 e^{2x} + B_0 e^{(2+C)x}$$

whose solution is :

$$r_1 = A_1 + B_1 e^{Cx} + \frac{A_0 e^{2x}}{4-2C} + \frac{B_0 e^{(C+2)x}}{4+2C} \qquad (10)$$

A_1 and B_1 are determined from the initial conditions.

Substituting Eqn. (10) into Eqn. (9) we get :

$$\frac{\partial^2 r_2}{\partial x^2} - C\frac{\partial r_2}{\partial x} = \left[A_1 + B_1 e^{Cx} + \frac{A_0 e^{2x}}{4-2C} + \right.$$
$$\left. + \frac{B_0}{4+2C} e^{(2+C)x}\right] e^{2x}$$

whose solution is :

$$r_2 = A_2 + B_2 e^{Cx} + \frac{A_1}{4-2C} e^{2x} + \frac{B_1}{4+2C} e^{(C+2)x}$$
$$+ \frac{A_0 e^{4x}}{(4-2C)(16-4C)} + \frac{B_0 e^{(C+4)x}}{(4+2C)(16+4C)} \qquad (11)$$

4. Results and calculations

$$e^x = C_2 - \alpha z = e^{\ln(C_2 - \alpha z)}$$

$$e^{Cx} = e^{\ln(C_2 - \alpha z)^C} = (C_2 - \alpha z)^C$$

(A) *In the troposphere*

$$r_0 = A_0 + B_0 (C_2 - \alpha z)^C \qquad (12)$$

$$C = 1 + \frac{C_1}{\alpha} = 1 + \frac{\alpha - g/R}{\alpha} = -0.28$$

$r_0 = 0$ at $z = 0$

$r_0 = 0.5 \times 10^{-6}$ at $z = 10$ km (Goshin 1965)

Substituting in Eqn. (12) we get :

$A_0 = -7.08 \times 10^{-6}$ and $B_0 = 34.97 \times 10^{-6}$

We can also determine A_1 and B_1 when :

$$r_1 = 0 \quad \text{at} \quad z = 0$$

and $\frac{dr_1}{dz} = 0$ at $z = 0$

$\therefore A_1 = 0.184$ and $B_1 = 0.68$

since ϵ is small we can neglect $\epsilon^2 r_2$

and if $C_2 = T_0 = 300°$K.

We can calculate the distribution of mixing ratio of ozone in the troposphere at different values

TABLE 1

The mixing ratio ($\times 10^{-6}$) of ozone in troposphere

r	r_0	$r = r_0 + r_1$			
		$\frac{b\bar{D}}{k}=$ 0.001	.002	.003	.004
1	0.04	0.00	0.00	0.00	0.00
2	0 09	0.05	0.01	0.00	0.00
3	0.13	0.09	0.05	0.01	0.00
4	0.18	0.14	0.10	0.06	0.02
5	0.23	0.19	0.15	0.11	0.07
6	0.28	0.24	0.20	0.16	0.12
7	0.33	0.29	0.25	0.21	0.17
8	0.39	0.35	0.31	0.27	0.23
9	0.45	0.41	0.37	0.33	0.29
10	0.50	0.50	0.50	0.50	0.50

TABLE 2

The mixing ratio ($\times 10^{-6}$) of ozone in the upper stratosphere

r	r_0	$r = r_0 + r_4$			
		$\frac{b\bar{D}}{k}=$ .001	.002	.003	.004
30	12.70	12.70	12.70	12.70	12.70
35	11.73	12.43	13.13	13.83	14.35
40	10.19	10.52	10.85	11.18	11.51
45	7.78	7.80	8.02	8.14	8.26
50	4.13	4.14	4.15	4.16	4.17
54	0.00	0.00	0.00	0.00	0.00

of $b\bar{D}/k = .001, .002, .003, .004$, these values are taken from (Goshin 1965). The results are given in Table 1.

Eqn. (3) and its solution give a new knowledge on the vertical distribution of ozone and its relation with thermal structure of the atmosphere and it appears that the mixing ratio of ozone (r) reaches its maximum value in the winter $b\bar{D}/k$ is small (Table 1), and it reaches its minimum value in the summer ($b\bar{D}/k$ is large).

The mixing ratio of ozone increases with height in the tropospheric layer. It begins from zero, at $z=0$ to 0.5×10^{-6} at $z=10$ km with gradient 0.05×10^{-6} per km.

From Table 1, we can also see that the mixing ratio of ozone becomes zero at $z=1$ km. when the value $b\bar{D}/k=0.001$. It also becomes zero at 2 km, for $b\bar{D}/k=.003$ or .004, but at 3 km it becomes zero when $b\bar{D}/k=.004$.

TABLE 3

The actual mixing ratio ($\times 10^{-6}$) of ozone at different latitudes and heights

Ht. (km)	Latitude (season)		
	9.4° N (spring 1962)	30.4° N (spring 1963)	40.5° N (winter 1963)
4	0.02	0.01	0.09
5	0.06	0.05	0.14
6	0.08	0.09	0.19
7	0.12	0.14	0.23
8	0.17	0.19	0.28
30	14	13	12
35	12	13	10
40	11	10	8

According to the previous discussions we can conclude that mixing ratio of ozone at lower troposphere decreases when the intensity of solar radiation increases.

(B) *In the upper stratosphere*

If $\alpha = -3.5$°C/km and $C_2=170°$ K, the temperature of tropopause, the value of C in the stratosphere equals 6.94, using Eqn. (3) as had been followed in the troposphere we can get :

$$A_0 = 1.3898\times10^{-6},$$

$$B_0 = -3.9745\times10^{-22}$$

and

$$A_1 = 4.57\times10^{-2},$$

$$B_1 = 2.35\times10^{-18}$$

Using these values in the solution of Eqn. (3) we can calculate the values of the mixing ratio of ozone in the upper stratosphere (Table 2).

From Table 2 it appears that the mixing ratio of ozone in the upper stratosphere reaches its maximum value in summer (when $b\bar{D}/k$ is large) but the mixing ratio of ozone reaches its minimum value in the winter when the value is $b\bar{D}/k$ small. These results coincide with the results of photochemistry.

The mixing ratio of ozone r decreases with height, it begins from 12.70×10^{-6} at $z=30$ km to zero at $z=54$ km.

To test the theoretical results, we use Table 3. The actual observations of mixing ratio of ozone in spring at Panama Canal (Lat. 9.4° N) coincide with the results in Tables 1 and 2 when $b\bar{D}/k = 0.004$. The actual observations coincide with the theoretical results in Florida (Lat. 30.4° N) when $b\bar{D}/k=0.002$, also the actual observations agree with the theoretical results in Colorado

(Lat. 40.6° N) when using the solution of unperturbed equation.

Thus when $b\bar{D}/k$ is large, our equation is suitable for low latitudes, but when $b\bar{D}/k$ is small (less than 0.003) it is suitable for latitude less than 40°. The solution of unperturbed equation is suitable for middle latitudes.

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