

Budget of absolute angular momentum of monsoon depressions

A. CHOWDHURY

Meteorological Office, Pune

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सार — इस शोध पत्र में परम कोणीय संवेग माध्य और मानसून अवदाबों के साथ उसके परिवहन का विवेचन किया गया है। इस अध्ययन के लिए अवदाब केन्द्र से 1000 कि० मी० इर्दगिर्द सम्पूर्ण क्षोभमंडल में संवेग के संभावित संचयन के सामान्य चित्रण को प्राप्त करने के लिए 1961 से 1974 तक के मानसून का प्रतिनिधित्व करने वाले जुलाई और अगस्त के महीनों में बनने वाले अवदाबों का संयोजन किया गया है। सतही प्रतिलंब का आकलन किया गया है जिसके द्वारा कोणीय वेग स्थानांतरित होता है। कर्षण गुणों और पवन के आनुभविक संबंधों को भी ज्ञात किया गया है।

इस अध्ययन से पता चला है कि अवदाब क्षेत्र में काफी अन्दर चली गई वायु निम्नतर स्तरों में ज्यादा से ज्यादा संवेग खो देती है। लेकिन जब-जब वह ऊपरी क्षोभमंडल में होती है तो सामान्यतः कोणीय संवेग का अविनाश हो जाता है। अवदाब के उच्च वेग क्रोड में संवेग का अधिकतम निष्कारण पाया गया है एवं सतही कर्षण और पवन में संबंधों की प्रकृति द्विघाती पाई गई है।

ABSTRACT. Mean absolute angular momentum and its transport associated with monsoon depressions have been discussed in the paper. For the purpose of this study, depressions which formed in two representative monsoon months of July & August between 1961 & 1974 and moved in the customary west or westnorthwest direction have been pooled to obtain a generalised portrayal of the expected accumulation of the momentum throughout the troposphere, around 1000 km of the depression centre. Estimates have been made of the surface stress by means of which the angular momentum are transferred. Empirical relationship between the drag coefficients and the wind has been deduced.

The study reveals that the air which penetrates deep inside the depression field loses maximum momentum in the lower levels while in the upper troposphere angular momentum is generally conserved. Maximum extraction of momentum is found to occur in the high velocity core of the depression. The nature of relationship between the surface drag and the wind is found to be quadratic. Relatively uniform angle of inflow is observed near the depression centre.

1. Introduction

The study of momentum transfer was first started by Jeffereys in 1926 and was later taken up by a wide group of research workers holding divergent views (*cf.* Starr 1948, Rossby and Starr 1949, Widgar 1949, Palmén 1949). Some of these were of the opinion that momentum transfer is like heat transfer and as such has to be accompanied by eddy motion. On the contrary Riehl and Yeh (1950) and Riehl (1961) concluded that eddy transport is small compared to the transport by the mean motion and hence may be ignored.

Rao (1958) calculated for July & August 1955, meridional transfer of mean momentum association with southwest monsoon. His results indicated that the convergence of meridional transport which would require to balance the frictional torque of the surface west wind, is brought about largely

by strong northward transport across the equator. He also pointed that Indian monsoon areas become a sink for westerly angular momentum on account of surface westerlies. Keshavamurthy (1968) analysed angular momentum balance in the region bounded by equator, 20° N and 50° E and 100° E meridians and concluded that the northward meridional motion in the lower parts of monsoon cell circulation generates enough angular momentum through Ω transport to make up for the frictional torque. Newton (1971) found transport of angular momentum across equator towards the summer hemisphere a maximum in June, July and August and equal to 13×10^{25} gm cm² sec⁻². He also concluded that momentum flows towards the hemisphere in which the rising branch of Hadley circulation (of winter hemisphere) is located. In the present study angular momentum and its transfers for an average

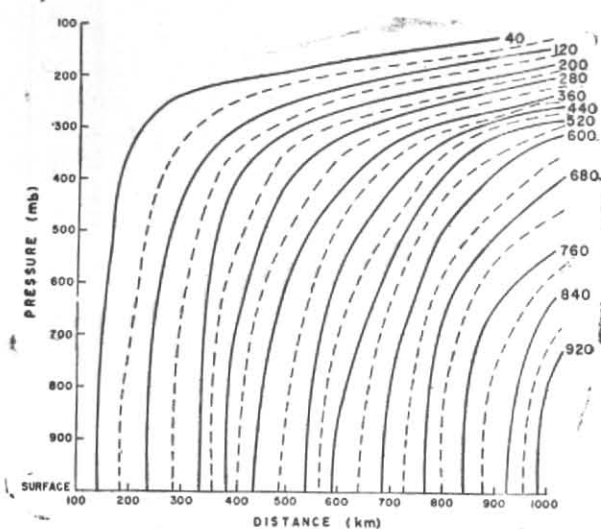


Fig. 1. Vertical cross section of angular momentum (units : 100 km²/hr)

"depression" has been examined. Only the transport due to mean motion has been calculated. In this connection it may be mentioned that, for Atlantic hurricanes, Riehl (1961) has shown that the eddy transport of angular momentum is small compared to the transport of the mean motion.

2. Data

In this study, data for 27 depressions which formed and moved in W/WNW directions during July and August months between 1961 & 1974 have been utilised. A distance upto 1000 km from the depression centre has been considered in this analysis. Thus all available winds (pibal & radio) for standard millibaric level in lower, middle and upper troposphere have been included. Each wind was resolved into radial and tangential components according to the methodology adopted earlier by Jordon (1952). Composite technique suggested by George (1975) has been adopted to obtain mean values of the wind components at various radial distances from the surface position of the depression centre.

3. Absolute angular momentum pattern

The tangential velocities were used to construct a vertical cross-section of the absolute angular momentum, $M = v_{\theta} r + \frac{1}{2} f r^2$.

The absolute angular momentum (Fig. 1) obviously decreased as the parcel spiralled inwards towards the depression centre. This is more true in the case of inflow layer and can directly be attributed to the frictional drag exerted on the surface by the moving air, above it. Upto about 600 mb the isopleths are nearly vertical suggesting that upto this level stirring was quite effective in the inflow layer. At the upper outflow layers, the absolute angular momentum of the parcel was more likely to be conserved and as such the isopleths at these levels, should become horizontal.

4. Computation of the transfers

The budget of the absolute angular momentum M was computed using mean values of v_r and v_{θ} , the radial and tangential components of the winds respectively for the appropriate radius and thickness. These calculations are basically similar to the computations of Riehl & Malkus (1961), Miller (1962) and Hawkins and Rubsam (1968). The absolute angular momentum per unit mass of air expressed in the tangential coordinate system, is :

$$M = v_{\theta} r + \frac{1}{2} f r^2 \quad (1)$$

where f is the coriolis force

The tangential equation of motion can be transformed into a momentum equation, giving :

$$\frac{dM}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial \theta} + \frac{r}{\rho} \left(\frac{1}{r} \frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial r \tau_{\theta r}}{r \partial r} + \frac{\partial \tau_{\theta z}}{\partial z} \right) \quad (2)$$

where τ_{ij} refers to shearing stresses and ρ is the air density.

Eqn. (2) states that the changes in absolute angular momentum of a parcel of air is the result of production by pressure forces and dissipation due to friction. Now,

$$\frac{dM}{dt} = \frac{\partial M}{\partial t} + v_{\theta} \frac{\partial M}{r \partial \theta} + v_r \frac{\partial M}{\partial r} + w \frac{\partial M}{\partial z} \quad (3)$$

where w is the vertical velocity.

Using equation of continuity this may be written as :

$$\rho \frac{dM}{dt} = \frac{\partial \rho M}{\partial t} + \frac{\partial \rho v_{\theta} M}{r \partial \theta} + \frac{1}{r} \frac{\partial}{\partial r} \rho v_r r M + \frac{\partial}{\partial z} \rho w M \quad (4)$$

Substituting (4) in (2), after neglecting radial and vertical fluxes of momentum and integrating the resulting equation over volume α , gives :

$$\frac{\partial}{\partial t} \int_{\alpha} \rho M d\alpha = \frac{1}{g} \int_{p_1}^{p_2} \int_0^{2\pi} v_r (v_{\theta} r + \frac{1}{2} f r^2) r d\theta dp - \int_{\alpha} \frac{\partial p}{\partial \theta} d\alpha + \int_{\alpha} r \left(\frac{\partial \tau_{\theta\theta}}{r \partial \theta} + \frac{\partial r \tau_{\theta r}}{r \partial r} + \frac{\partial \tau_{\theta z}}{\partial z} \right) d\alpha \quad (5)$$

where α is the volume of cylinder with radius r and top and bottom at pressure levels, p_1 and p_2 respectively.

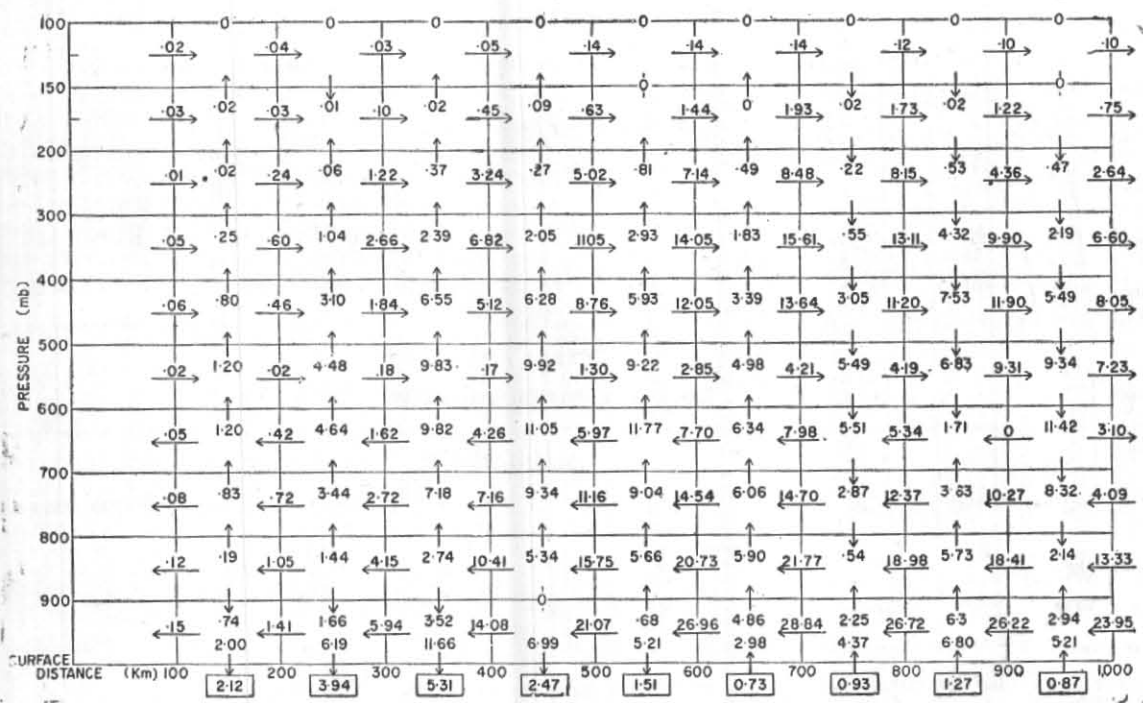


Fig. 2. Flux of angular momentum (units of 10^{23} gm cm^2 sec^{-2}) for the "mean depression". In the lower boxes, are the momentum lost to the underlying surface (\downarrow) or gained from it (\uparrow) computed to the top of the atmosphere (assumed 100 mb). The lower most small enclosed boxes contain the momentum loss or gain per unit area in units of 10^8 gm sec^{-2}

For an axially symmetric system like depression, it may be assumed that the production term $(\partial p / \partial \theta) da$ vanishes.

The first two terms in the third integral in the right hand side of Eqn. (5) are the lateral exchange of angular momentum by small scale stress and are, in general, considered small in comparison with the third term, i.e., vertical transport term. Assuming further a steady state Eqn. (5) becomes :

$$\int_a r \left(\frac{\partial \tau_{\theta z}}{\partial z} \right) d\alpha = - \int_{p_1}^{p_2} \int_0^{2\pi} v_r (v_{\theta} r + \frac{1}{2} f r^2) r d\theta \frac{dp}{g} \quad (6)$$

In the present study, horizontal fluxes were calculated at 100 km interval radially and 100 mb interval vertically upto 200 mb and thereafter at 50 mb interval upto 100 mb.

The momentum extracted by the underlying surface or given by it to the surface immediately over it, was calculated as below. The values were derived using net horizontal fluxes into and out of the annular volumes from surface to 100 mb level. The net difference of momentum (or residual), i.e., the import less the export per annulus represent momentum lost to the surface through drag at the interface or momentum gained by it.

Expected accumulation of angular momentum is shown in Fig. 2. As may be seen, angular momentum is transported into the depression field in the lower troposphere. This momentum flow is most prominent in the areas of strong monsoon activity, viz., 200-400 km from the centre. The accumulated momentum is transported downwards and transferred to the earth's surface by friction at the boundary layer upto 600 km and this is made up by the transports across the walls. This amount tends to diminish at smaller radii due mainly to decreasing area. However, this was not uniformly true and the values at 500 and 600 km radii are lower than those at 300 and 400 km radii, where higher radial velocities, apparently more than compensates the decrease in area and drag. In the outer region of the field, i.e., beyond 600 km there is a net gain in the angular momentum. In this zone to produce necessary balance in the rings, angular momentum is transported through friction to the winds above. It means that air penetrating in the zone picks up momentum from the earth's surface. Vertical fluxes have also been computed so as to produce a balance in each box.

In Fig. 2 values given in the lower enclosed rectangles are the momentum losses per centimetre square (in units of 10^8 gm sec^{-2}) within the annular rings. This shows that the momentum extraction in the high speed core of the depression is maximum, viz., within the 300-400 km annulus.

TABLE 1
Drag coefficients and tangential surface stress computed from the angular momentum balance

	Radius (km)									
	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
$c_d \times 10^{-3}$	2.5	1.8	1.8	1.9	1.6	1.0	0.7	0.6	0.4	0.3
$\tau_{\theta\theta}$ (dyne/cm ²)	4.67	4.59	5.01	5.21	3.72	2.48	1.40	0.80	0.41	0.21

TABLE 2
Inflow angle and total surface stress

	Distance (km)									
	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
δ (degrees)	18	19	20	20	20	19	17	12	11	8
τ_0 (dyne/cm ²)	4.92	4.86	5.33	5.53	3.66	2.62	1.46	0.82	0.42	0.21

5. Angular momentum and surface stress

Estimates have been made in the study of the surface stress, by means of which angular momentum are transferred. From the absolute angular momentum and the radial and tangential velocities, expression of the stress can be derived as below:

$$\frac{dM}{dt} = \frac{d}{dt} \left(v_{\theta} r + \frac{1}{2} f r^2 \right) = r(dv_{\theta}/dt) + v_{\theta} v_r + r f v_r \quad (7)$$

$$\begin{aligned} \frac{d v_{\theta}}{dt} &= \frac{\partial v_{\theta}}{\partial t} + v_{\theta} \frac{\partial v_{\theta}}{r \partial \theta} + v_r \frac{\partial v_{\theta}}{\partial r} + \\ &+ w \frac{\partial v_{\theta}}{\partial z} = -f v_r - \frac{v_r v_{\theta}}{r} - \\ &- \frac{1}{\rho} \frac{\partial p}{r \partial \theta} + \frac{1}{\rho} \frac{\partial \tau_{\theta z}}{\partial z} \end{aligned} \quad (8)$$

Assuming a steady state relative to the moving system and that tangential gradients of p and v_{θ} are negligible, combination of (7) and (8) gives:

$$\frac{dM}{dt} = -r g \frac{\partial \tau_{\theta z}}{\partial p} \quad (9)$$

In the polar cylindrical coordinates, angular momentum can be written as:

$$\frac{dM}{dt} = \frac{\partial M}{\partial t} + v_{\theta} \frac{\partial M}{r \partial \theta} + v_r \frac{\partial M}{\partial r} + w \frac{\partial M}{\partial p} \quad (10)$$

Neglecting local variations in M and its tangential gradients and employing equation of continuity:

$$\begin{aligned} \frac{\partial w}{\partial p} + \frac{1}{r} \frac{\partial}{\partial r} (r v_r) &= 0, \\ \text{i.e., } \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial w}{\partial p} &= 0 \end{aligned} \quad (11)$$

Eqn. (10) now becomes:

$$\begin{aligned} \frac{dM}{dt} &= v_r \frac{\partial M}{\partial r} + w \frac{\partial M}{\partial p} + M \left(\frac{\partial v_r}{\partial r} + \right. \\ &\left. + \frac{v_r}{r} + \frac{\partial w}{\partial p} \right) = 0 \end{aligned} \quad (12)$$

From (9) and (12) it follows that:

$$-\frac{\partial \tau_{\theta z}}{\partial p} = \frac{1}{r^2 g} \left[\frac{\partial}{\partial r} (r v_r M) + \frac{\partial}{\partial r} (w r M) \right] \quad (13)$$

Integrating the above expression from surface (p_s) to the top of atmosphere, taken as 100 mb it follows:

$$\begin{aligned} - \int_{p_s}^{100} \frac{\partial \tau_{\theta z}}{\partial p} dp &= \frac{1}{g} \int_{p_s}^{100} \frac{1}{r^2} \left[\frac{\partial}{\partial r} (r v_r M) + \right. \\ &\left. + \frac{\partial}{\partial r} (w r M) \right] dp \end{aligned} \quad (14)$$

The surface stress in the θ - z plane $\tau_{\theta 0}$ becomes:

$$\tau_{\theta 0} = \frac{1}{g} \int_{p_s}^{100} \frac{1}{r^2} \frac{\partial}{\partial r} (r v_r M) dp \quad (15)$$

The numerical integration of the surface stress have been presented in Table 1.

The tangential shearing stresses are low, i.e., less than 1 dyne/cm² in the outer circles where the wind speed are weak (less than 10 mps). In the region where the winds were of maximum strength (of about 15 mps), i.e., between 200 & 400 km, the surface stresses are of the order of 5 dyne/cm².

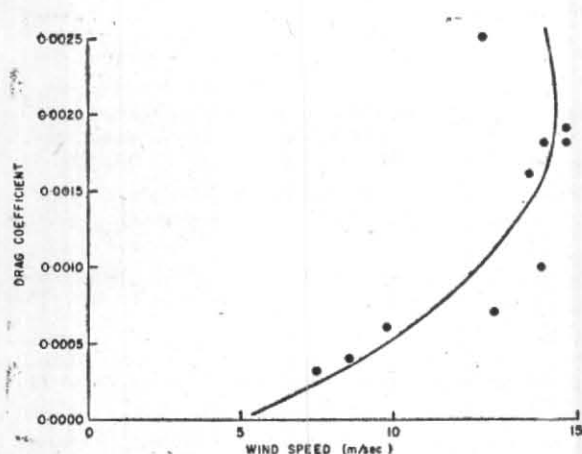


Fig. 3. Drag coefficients against the wind speed (m/sec)

In the two innermost rings, the stress ranged between 4 & 5 dynes.

A comparison of the stresses with those obtained for mean flow in tropical cyclones by Palmen & Riehl (1957), revealed that on an average beyond 1.5° latitude circle, the values agree, well. As is well known, wind speed in the inner core of tropical cyclones are much stronger than those in depressions. The higher values within 1.5° latitude circle, therefore, accounts for larger stress values in the tropical cyclones than observed at corresponding distances in the monsoon depression.

By making use of the empirical relationship :

$$\tau_{\theta 0} = \rho c_d v_{\theta} V \approx \rho c_d v_{\theta}^2 \quad (16)$$

where V is the total wind, the dimension-less drag coefficients have been determined for different annular rings and the corresponding tangential wind speed (Table 1) and plotted against the wind speed and shown in Fig. 3. These coefficients ranged from 0.3×10^{-3} to 2.5×10^{-3} . Palmen and Riehl (1957) computed drag coefficients from the mean hurricane data of Hughes (1952) and Jordon (1952) by making use of angular momentum budget. Their values ranged from 1.1×10^{-3} to 2.1×10^{-3} . It is, therefore, interesting to note that, in spite of various assumptions made in the present study, the values of drag coefficient fairly agrees with that observed in case of mean hurricanes. These values also agree with those in trade wind belt where according to Sheppard & Omar (1952), 1.6×10^{-3} can be expected.

The distribution of the coefficients also suggest a non-linear relationship with the wind. A second degree least square fit to the data yields :

$$c_d = -0.003030 + 0.000539 V - 0.000015 V^2 \quad (17)$$

This expression gives somewhat unrealistic values of c_d for lower velocities (5 m sec^{-1} or less).

The values of c_d obtained in the study for wind speeds 10, 15 and 20 mps are 0.86×10^{-3} , 1.68×10^{-3} and 1.75×10^{-3} respectively.

5.1. Inflow angle and total surface stress

The ratio of radial (v_r) and tangential (v_{θ}) components of the wind, i.e. v_r/v_{θ} , enables to determine the indraft angle or the angle of inflow, δ . From the angle of inflow, the total surface stress τ_0 can be computed from the equation.

$$\tau_0 = \tau_{\theta 0} \sec \delta \quad (18)$$

The calculated values of δ and τ_0 are given in Table 2. It is remarkable to note that relatively uniform inflow occurs upto about 700 km of the centre. In this field of general steady indraft, a relative maxima is observed in 300 to 500 km belt. The outer periphery of the depression field, has, however, rather low angle of inflow. The values of inflow angles obtained in this study, by and large, agrees with those computed for tropical systems by Hughes (1952) and Palmen and Riehl (1957).

Since $\sec \delta$ is greater than 1 in the present case, τ_0 , the total surface stress, would naturally be greater than $\tau_{\theta 0}$, the tangential stress. A comparison between Table 1 and Table 2, however, reveal that τ_0 is only marginally more than $\tau_{\theta 0}$. In fact, the increase observed did not exceed 7 per cent and is mostly confined to about 600 km from the depression centre.

Atmospheric surface stresses drag coefficient and inflow angle are highly dependent on the turbulent nature of fluid. These values, in the present study are based on mean flow and are, as such, it may be emphasised likely to be different from those in the free atmosphere. As may be seen, the estimates of error in computing surface stresses drag coefficient and inflow angle according to procedures mentioned above are, not large and are well within tolerable limits.

6. Summary

Studies on the structure of absolute angular momentum and its maintenance in the monsoon depression field have been rather inadequate. The vertical and horizontal distribution of the momentum fluxes have been computed in this paper. Estimates of the surface stress by which the angular momentum are transported, have been made and exchange coefficient evaluated. The following salient features emerge out of the analysis :

- (i) Upto about 600 km from the depression centre, momentum is lost to the underlying surface while in the outer regions angular momentum is transported through friction to air aloft.
- (ii) Maximum extraction of momentum occurs in the annular zone of 300-400 km where the wind speed are the strongest.

- (iii) Values of the surface drag coefficients obtained bear a quadratic relationship with the wind.
- (iv) Relatively uniform angle of inflow in the range of $17-20^\circ$ is observed upto 700 km of depression centre.

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