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Role of convection in cyclone

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सार — अधिकतम पवन के घेरे में चक्रवात के अन्दरूनी क्षेत्र में स्थायी लक्षण होते हैं जो कि लघुमान तरंग, समान क्षोभों को नहीं बनने देते। वायुमंडलीय संवहन को लघुमान तरंग समान क्षोभों के साथ सम्बद्ध दिखाया गया है । चक्रवात केन्द्र की ओर अधिकतम पवन के घेरे के अंदर जाते समय लघुमान संवहन क्षोभ सर्पिल लक्षणों की शुद्धगतिक स्थिरता के कारण लुप्त हो जाते हैं और क्षोभ की गतिज ऊर्जा के एक भाग को चक्रवात को तीव्र बनाने में सहयोग देते हुये दिखाया गया है ।

ABSTRACT. The inner area of a cyclone inside the ring of maximum wind is shown to have a stable feature of not permitting small scale wave-like perturbations. Atmospheric convection is shown to be associated with small scale wave-like perturbations. The small scale convective perturbations while proceeding inside the ring of maximum wind towards cyclone centre will vanish due to kinematic stability of spiral feature and a portion of kinetic energy of perturbation is shown to contribute to cyclone intensification.

1. Introduction

1.1. It is a fact of observation that tropical cyclones called typhoons in Pacific and hurricanes in Atlantic and simply cyclones in Indian area, are associated with convective clouds. Charney and Eliassen (1964) proposed convective instability to hurricane growth. Moisture convergence due to cyclone in lower layers of atmosphere adjacent to earth's surface makes convective cloud formation possible and the heat energy released by the condensation of water vapour contributes to cyclone growth and this type of interaction leads to a large-scale self amplification. This cooperative intensification process is called Convective Instability of Second Kind (CISK).

1.2. Cyclones are relatively large scale phenomena with a large life period as compared to short-lived convective clouds like cumulus and cumulonimbus occurring in smaller areas. Convection is viewed also as perturbation or turbulence. Charney and Eliassen (1964) state that "If one applies perturbation techniques to the study of small amplitude perturbations of a conditionally unstable saturated atmosphere, one finds, not surprisingly that the smallest scale modes grow at the greatest rate and ultimately predominate". Ooyama (1982) viewing convection as turbulence states that "according to the theory of three dimensional turbulence, energy cascades into smaller and smaller scales and no deterministic prediction is possible". The theoretical stand appears to be that degradation from a bigger to smaller scale alone is possible and *vice versa* smaller scale growing into bigger scale is theoretically not possible. Relying on experimental facts of observation, Charney and Eliassen assert that "nevertheless, conditional instability by permitting cumulonimbus convection, must play a role in the formation of hurricane" and they overcome the theoretical difficulty by parameterization which according to Ooyama (1982) is "an art of approximation by which implicit effects of the truncated small scale are put into an explicit consideration of the remaining large scale". Since 1964, a number of schemes of parameterization had been proposed with varying degrees of success in establishing that CISK contributes to cyclone intensification.

1.3. The persistence of basic difficulties encountered by Charney and Eliassen (1964) which are to some extent attributable to lack of due consideration of the kinematical characteristics of cyclone and convection, motivates the author to attempt a *de novo* attack on this problem. The author (1984) has used (*i*) L_1, L_2 and L_3 which are $L_1 = 4\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right)$, $L_2 = L_1 - (\nabla . \mathbf{V})^2$ and $L_3 = L_1 - \zeta^2$ in defining spiral, ring of maximum wind and cyclone intensification/weakening and (*ii*) vertical motion obtained from the eqation of motion

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{V} + w\frac{\partial \mathbf{V}}{\partial z} + (\mathbf{k}\times\mathbf{V})f + K\mathbf{V} = -\frac{1}{\rho}\nabla p$$

in cyclone and convective clouds. Specifically the objective of this paper is to study the role of convection in cyclone using L_1, L_2 and L_3.

Fig. 1 (a)

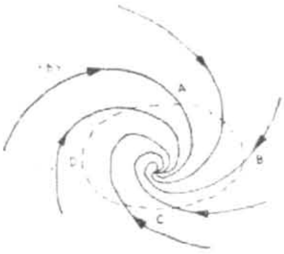

Fig. 1 (b)

2. Section 1 — Kinematical feature of cyclone

2.1. We investigate the kinematical properties of cyclone and waves and establish that the interior of a cyclone where streamlines have spiralling features will not permit streamline waves which require change of sign of curvature.

2.2. The horizontal streamlines of an atmospheric cyclone spiral inward and terminate at a point called cyclone centre. The speed at the cyclone centre is zero. Proceeding from the cyclone centre in any direction, speed increases, reaches a maximum whereafter it decreases. The locus of such points of maximum is the ring of maximum wind (RMW) enclosing the cyclone centre. The speed on the RMW need not be the same.

2.3. The kinematical features of a cyclone with spiralling feature have been established by the author in this book "*Theorems on Cyclones*" (1984) where the kinematical determinant L_1, L_2 and L_3 have been extensively used.

2.4. Let **V** be the horizontal wind vector

$$\mathbf{V} = V(\cos\psi\mathbf{i} + \sin\psi\mathbf{j}) = u\mathbf{i} + v\mathbf{j}$$

$$V^2 = u^2 + v^2$$

$$\tan\psi = \frac{v}{u}$$

Here u and v are components, V is speed and ψ is direction which is the angle, **V** makes with x-axis and is reckoned positive counterclockwise.

$$\nabla.\mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = V\frac{\partial\psi}{\partial n} + \frac{\partial V}{\partial s} = \text{divergence}$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = V\frac{\partial\psi}{\partial s} - \frac{\partial V}{\partial n} = \text{vorticity}$$

$$D_{st} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = \text{stretching deformation}$$

$$D_{sh} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \text{shearing deformation}$$

$$L_1 = 4\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right) = (\nabla.\mathbf{V})^2 + \zeta^2 - D_{st}^2 - D_{sh}^2$$

$$L_1 = 4\left(V\frac{\partial\psi}{\partial n}\frac{\partial V}{\partial s} - V\frac{\partial\psi}{\partial s}\frac{\partial V}{\partial n}\right)$$

$$L_2 = L_1 - (\nabla.\mathbf{V})^2 = -4V\left(\frac{\partial\psi}{\partial s}\right)\left(\frac{\partial V}{\partial n}\right) - \left(V\frac{\partial\psi}{\partial n} - \frac{\partial V}{\partial s}\right)^2$$

$$L_3 = L_1 - \zeta^2 = +4\left(V\frac{\partial\psi}{\partial n}\right)\left(\frac{\partial V}{\partial s}\right) - \left(V\frac{\partial\psi}{\partial s} + \frac{\partial V}{\partial n}\right)^2$$

Here $\frac{\partial}{\partial s}$ is differentiation along streamline and $\frac{\partial}{\partial n}$ is differentiation along normal to streamline.

$\frac{\partial\psi}{\partial s}$ is streamline curvature. Streamline is counterclockwise curved if $\frac{\partial\psi}{\partial s}$ is positive and is clockwise curved if $\frac{\partial\psi}{\partial s}$ is negative. $\frac{\partial\psi}{\partial n}$ is streamline diffluence/confluence. Streamlines are diffluent if $\frac{\partial\psi}{\partial n}$ is positive and they are confluent if $\frac{\partial\psi}{\partial n}$ is negative.

$\frac{\partial V}{\partial s}$ is speed divergence/covergence. It is speed divergence if $\frac{\partial V}{\partial s}$ is positive and is speed convergence if $\frac{\partial V}{\partial s}$ is negative. $\frac{\partial V}{\partial n}$ is speed shear.

2.5. The three important features of a cyclone, *viz.* spiralling feature, ring of maximum wind and intensity of cyclone are expressible in terms of L_1, L_2 and L_3. The theorems concerning these properties are given for easy reference.

(*i*) *Theorem of spiral feature*

Streamlines will have spiral features in an area where L_1, L_2 and L_3 are positive, $L_1 \neq L_2$, $L_1 \neq L_3$ and where an isolated point $\mathbf{V}=0$ exists.

(*ii*) *Theorem on ring of maximum wing* (*RMW*)

The ring of maximum wind enclosing a spiral centre is the same as the zero L_1 isopleth inside/outside which L_1 is positive/negative.

(*iii*) *Theorem on cyclone intensity*

Since speed on RMW is not the same in general, it is realistic to take $\overline{V}_{max}{}^2$ the intensity of cyclone where $\overline{V}_{max}{}^2$ is the mean of square of speed on RMW.

$$\iint L_1\, ds = 4\pi\,\overline{V}_{max}{}^2$$

where the area of integration is enclosed by RMW.

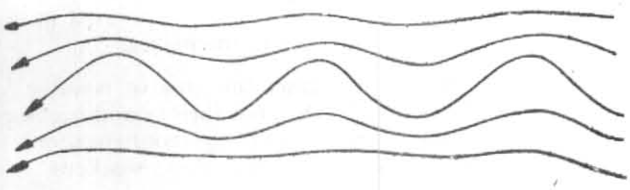

Fig. 2

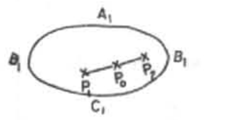

Fig. 3

Fig. 4

2.6. Streamlines spiral inward counterclockwise in northern hemispheric cyclone and they spiral inward clockwise in southern hemispheric cyclones. The cyclone centre is enclosed by RMW as schematically illustrated in Fig. 1(a) and Fig. 1(b). Here ABCDA is the RMW enclosing the cyclone centre inside which L_1 is positive, outside which L_1 is negative and on which L_1 is zero. L_2 and L_3 are negative on and outside RMW since $L_2 = L_1 - (\nabla . \mathbf{V})^2$ and $L_3 = L_1 - \zeta^2$. Hence positive areas of L_2 and L_3 must exist inside the RMW.

2.7. An essential feature of a streamline wave pattern is that the sign of curvature changes as one proceeds on any streamline. The streamlines may confluent and later to diffluent as well. Fig. 2 illustrates streamline wave pattern where sign of $\frac{\partial \psi}{\partial s}$ as well as sign of $\frac{\partial \psi}{\partial n}$ change from positive to negative *via* zero.

2.8. We will establish that sign of curvature, *viz.*, sign of $\frac{\partial \psi}{\partial s}$ will not change in an area of positive L_2 and further the sign of $\frac{\partial \psi}{\partial n}$ will not change in an area of positive L_3. We presume that components u, v and their derivatives, *viz.*,

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, V\frac{\partial \psi}{\partial s}, V\frac{\partial \psi}{\partial n}, \frac{\partial V}{\partial s}, \frac{\partial V}{\partial n},$$

$(\nabla . \mathbf{V})$, ζ, L_1, L_2 and L_3 are real, single valued, finite and continuous. Specifically we assume that they do not have *infinite values*.

Theorem 4(a)

In an area of positive L_2

(*i*) the sign of $V\frac{\partial \psi}{\partial s}$ is the same at all points,

(*ii*) the sign of $\frac{\partial V}{\partial n}$ is the same at all points,

(*iii*) the sign of $V\frac{\partial \psi}{\partial s}$ is opposite to that of the sign of $\frac{\partial V}{\partial n}$ at all points and hence sign of vertical component of vorticity ζ is the same at all points.

Theorem 4(b)

In an area of positive L_3

(*i*) the sign of $V\frac{\partial \psi}{\partial n}$ is the same at all points,

(*ii*) the sign of $\frac{\partial V}{\partial s}$ is the same at all points,

(*iii*) the sign of $V\frac{\partial \psi}{\partial n}$ is the same as the sign of $\frac{\partial V}{\partial s}$ and hence sign of horizontal divergence will be the same at all points.

Proof

$$\text{Since } L_2 = -4\left(V\frac{\partial \psi}{\partial s}\right)\left(\frac{\partial V}{\partial n}\right) - \left(V\frac{\partial \psi}{\partial n} - \frac{\partial V}{\partial s}\right)^2$$

(L_2 to be positive in an area) $\Rightarrow$ (Signof $V\frac{\partial \psi}{\partial s}$ must be opposite to that of the sign of $\frac{\partial V}{\partial n}$ and the first term must be greater than the second).

Now change of sign takes place through zero. If possible, let the sign of $V\frac{\partial \psi}{\partial s}$ change from positive to negative in an area of positive L_2 which is represented by A_1 B_1 C_1 D_1 A_1 (Fig. 3).

$V \frac{\partial \psi}{\partial s}$ is positive at P_1 and negative at P_2. Join P_1 P_2 by a line.

Proceeding along P_1 P_2, since sign of $V \frac{\partial \psi}{\partial s}$ changes from positive to negative, there must at least be a point say P_0 where $V \frac{\partial \psi}{\partial s}$ is zero. Since L_2 is positive and $V \frac{\partial \psi}{\partial s}$ is positive at P_1 and negative at P_2, the sign of $\frac{\partial V}{\partial n}$ must be negative at P_1 and positive at P_2. Proceeding along P_1 P_2, we note that $\frac{\partial V}{\partial n}$ must also change sign at P_0 and hence $\frac{\partial V}{\partial n}$ must vanish at P_0. If so, L_2 at P_0 will be a non-positive quantity, *i.e.*, either negative or zero. Since L_2 is positive at all points including P_0 by hypothesis,

(*i*) sign of $V \frac{\partial \psi}{\partial s}$ must be the same at all points,

(*ii*) sign of $\frac{\partial V}{\partial n}$ must be the same at all points,

(*iii*) sign of ζ must be the same at all points. Advancing very similar arguments, we can establish Theorem 4(b).

2.9. As a result of Theorems 1, 2, 4(a) and 4(b), it is realistic to state that the interior portion of a cyclone inside the RMW where L_2 and L_3 are positive is kinematically incompatible with perturbation requiring change of sign of vorticity ζ, horizontal divergence $(\nabla.\mathbf{V})$, change of curvature $\frac{\partial \psi}{\partial s}$ and change of sign of $\frac{\partial \psi}{\partial n}$. However, change of sign is permissible outside RMW where L_2 and L_3 are necessarily negative. Inside the RMW, there is a small zone where L_2 and L_3 are negative where also change of sign of ζ, $\nabla.\mathbf{V}$, $V \frac{\partial \psi}{\partial s}$ and $V \frac{\partial \psi}{\partial n}$ is possible but in the interior of RMW where L_2 and L_3 are positive, such changes are not permissible. If, however, perturbations move towards the interior where L_2 and L_3 are positive, there are three distinct possibilities :

(*i*) perturbations are damped and their kinetic energy is partly or wholly incorporated in cyclone feature and cyclone intensifies,

(*ii*) the perturbations enter the area of positive L_2 and L_3 where by the area shrinks and becomes zero and hence spiralling configuration is wiped out before which cyclone weakens,

(*iii*) perturbations enter from one side and the area of positive L_2 and L_3 shifts with/without shrinking. This is a combination of (*i*) and (*ii*).

Fig. 4 is schematic representation of cyclone with perturbation outside RMW.

3. Section 2 — Cyclone intensification/weakening

3.1. The maximum speed of wind is customarily used by meteorologists to specify the intensity of cyclone. Experimental evidence indicates that the speed on the ring of maximum wind is not the same at all points. Hence the mean of square of speed $\overline{V}_{max}{}^2$ can be taken as an estimate of cyclone intensity. If so, we can make use of L_1 property in the study of cyclone intensification. We note that

$$\oiint L_1\, ds = 4\pi\ \overline{V}_{max}{}^2$$

$$\left(\frac{\delta}{\delta t} \oiint L_1\, ds > 0\right) \Rightarrow \text{Cyclone intensifying}$$

$$\left(\frac{\delta}{\delta t} \oiint L_1\, ds < 0\right) \Rightarrow \text{Cyclone weakening}$$

Here the area of integration is enclosed by RMW and $\frac{\delta}{\delta t}$ is rate of change with respect to time following area enclosed by RMW.

3.2. The equation of motion is taken as

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{V} + w\frac{\partial V}{\partial z} + (\mathbf{k}\times\mathbf{V})f + K\mathbf{V} = -\frac{1}{\rho}\nabla p$$

Here w is vertical component velocity

K is coefficient of friction

f is coriolis parameter

p is pressure

ρ is density

Partially differentiating with respect to x and y and after some manipulation, we get the dynamical equation of cyclone intensification as :

$$\frac{\delta}{\delta t}\oiint\frac{L_1}{4}\,ds = \begin{cases} -2K\overline{\overline{\oiint}}\frac{L_1}{4}\,ds & \text{Friction}\\[4pt] +\overline{\overline{\left(-\frac{1}{\rho}\frac{\partial p}{\partial s}\right)}}\oiint\frac{L_1}{4}\,ds & \text{Cross isobaric flow}\\[4pt] -\bar{\bar{w}}\frac{\delta}{\delta z}\oiint\frac{L_1}{4}\,ds & \text{Vertical motion}\\[4pt] +\oiint\left|\nabla V\right|V\frac{\partial\psi}{\partial z}\frac{\partial w}{\partial n_2}\,ds & \text{Vertical motion}\\ -\oiint\left|\nabla\psi\right|V\frac{\partial V}{\partial z}\frac{\partial w}{\partial n_1}\,ds & \text{and vertical shear}\\[4pt] +\oiint V\left|\nabla\psi\right|\frac{\partial}{\partial n_1}\left(-\frac{1}{\rho}\frac{\partial p}{\partial s}\right)ds & \text{Pressure}\\ -\oiint\left|\nabla V\right|\frac{\partial}{\partial n_2}\left(-\frac{1}{\rho}\frac{\partial p}{\partial n}\right)ds & \text{and density}\\[4pt] -\oiint\beta\frac{\partial}{\partial x}\frac{V^2}{2}\,ds & \text{Rossby's parameter}\\[4pt] +\left(\frac{\delta A}{\delta t}+\bar{\bar{w}}\frac{\delta A}{\delta z}\right)\frac{\partial}{\partial A}\oiint\frac{L_1}{4}\,ds & \text{Configuration of zero isopleth} \end{cases}$$

Here (*i*) double bar represents mean over the area enclosed by RMW.

(*ii*) $\frac{\partial}{\partial n_1}$ differentiation along isogon,

(*iii*) $\frac{\partial}{\partial n_2}$ differentiation along isopleth of speed,

(*iv*) $\frac{\delta}{\delta z}$ differentiation with respect to z following the area enclosed by RMW,

(*v*) β is Rossby's parameter,

(*vi*) A is the area enclosed by RMW which is the same as zero isopleth.

For derivation, see Chapter 11 "*Theorems on Cyclones*" (Lakshminarayanan 1984).

3.3. We will discuss the contribution of the different terms.

(1) *Friction* contributes to exponential weakening.

(2) *Cross isobaric flow* means that streamline cuts isobar.

$\frac{\partial p}{\partial s}$ negative $\Longleftrightarrow$ streamline is directed into a relatively lower pressure as illustrated in Fig. 5.

$\frac{\partial p}{\partial s}$ positive $\Longleftrightarrow$ streamline is directed into a relatively higher pressure as illustrated in Fig. 6.

Cross isobaric inflow into a lower pressure contributes to exponential intensification. Since low pressure is

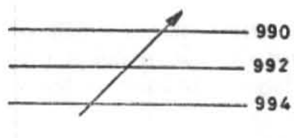

Fig. 5

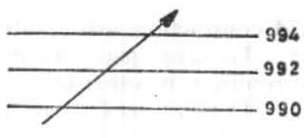

Fig. 6

associated with spiralling-in streamlines, cross isobaric flow contributes to cyclone intensification.

3.4. *Vertical motion*

If cyclone intensity decreases with height (and it is so, in the case of tropical cyclones), $\frac{\delta}{\delta z}\oiint\frac{L_1}{4}\,ds$ is negative and hence mean vertical upward motion will contribute to cyclone intensification.

3.5. *Vertical motion and vertical shear*

Isogons are nearly radial lines and vertical motion in the case of cyclone is minimum at the centre and increases as we proceed outwards and hence it is realistic to take $\frac{\partial w}{\partial n_1}$ to be positive. Tropical cyclones are characterised by decrease of speed with height and hence $\frac{\partial V}{\partial z}$ is negative. Taking $\frac{\partial V}{\partial z}$ as negative and $\frac{\partial w}{\partial n_1}$ as positive, contribution due to this term is to intensify. The term containing $\frac{\partial\psi}{\partial z}$ and $\frac{\partial w}{\partial n_2}$ can be ignored if the cyclone is vertical and streamline pattern unchanged with height, *i.e.*, $\frac{\partial\psi}{\partial z}=0$. If the cyclone is tilted, $\frac{\partial\psi}{\partial z}$ will be positive and negative. Actual data will be required to establish its contribution and no easy rule of thumb is possib

Pressure and density and Rossby's parameter

The lengthy discussion which the term containing pressure and density requires is given in the author's book. We do not propose using these terms in the present study.

Configuration of zero L_1 isopleth

Positive $\delta A/\delta t$.	Expansion of RMW at a level with respect to time.	Positive $\delta A/\delta z$: Expansion of RMW with height at a time
Negative $\delta A/\delta t$.	Contraction of RMW at a level with respect to time	Negative $\delta A/\delta z$: Contraction of RMW with height at a time

Shea and Gray (1973) show that smaller/bigger radius of ring of maximum wind is associated with higher/lower values of V_{max}. It is, therefore, realistic to take $\frac{\partial}{\partial A}\oint\frac{L_1}{4}\,ds = \pi\frac{\partial}{\partial A}V_{max}^2$ as negative. Hence very rapid contraction will result in spectacular intensification. To save space, we do not discuss in detail the contributions by other terms. They are given in author's book *"Theorems on Cyclones"*.

Summing up, the three important factors contributing to cyclone intensification are (*i*) vertical upward motion, (*ii*) cross isobaric inflow and (*iii*) contraction of ring of maximum wind.

4. Section 3 — Convection

4.1. Since atmospheric convection with cloud formation is conditioned by buoyancy effects, the kinematical characteristics and energy changes in fluid motions affected by buoyancy forces are to be categorically specified. Take the simple experiment of cork submerged initially in water and later allowed to rise by buoyancy force. Cork rises upward and adjacent water has vertical upward, downward and horizontal motion. The potential energy of water decreases and is converted into kinetic energy of water and cork and it also increases the potential energy of cork. The downward and horizontal motion of water are in fulfilment of continuity requirement. Assuming atmospheric convection is very similar to cork motion in water, it would be realistic to specify kinematical characteristics and energy changes in atmosphere as :

(*i*) both upward and downward motion must occur and horizontal motion must undergo changes,

(*ii*) conversion of potential energy into changes in kinetic energy resulting in both upward and downward motion and horizontal motion.

4.2. Convective clouds are formed in atmospheric layers which are conditionally unstable. If an unsaturated parcel of air is pushed upward, its temperature falls dry adiabatically, relative humidity increases, reaches saturation and if upward ascent continues, the moisture excess of saturation condenses, releases latent heat energy and the rate of fall of temperature will correspond to moist adiabatic lapse which is less than dry adiabatic lapse. Since environmental lapse is generally less than dry adiabat, we require (*i*) some mechanism for ascent of unsaturated air to the level of condensation : in this layer, buoyancy effect is negative, (*ii*) environmental lapse must be greater than moist adiabatic lapse : in this layer, buoyancy effect is positive. Atmospheric layer with lapse greater than the moist adiabatic lapse is said to be conditionally unstable. Hence convective cloud formation can be said to have the following features :

(*i*) a forcing mechanism in the lower layer so that unsaturated air ascends and becomes saturated. In this layer, buoyancy effect is negativet,

(*ii*) environmental lapse must be greater than moist adiabatic lapse. In this layer, buoyancy effect is positive,

(*iii*) both upward and downward motion must occur associated with changes in horizontal motion. Downward motion and changes in horizontal motion are necessitated by continuity,

(*iv*) heat of the parcel due to release of latent heat energy of condensation, increases 'potential energy which gets converted into changes of vertical upward, downward and horizontal motion.

5. Section 4 — Vertical motion

5.1. Cyclone intensification/weakening has been shown to be associated with vertical motion. Vertical motion is also an important feature of convection. Even though atmospheric motion is largely horizontal and vertical component is relatively weak, vertical component plays an important role in cyclone as well as convection. To study vertical motion, we make use of the fact that vertical motion w is explicitly present in the equation of motion for horizontal wind vector, *viz.*,

$$\partial\mathbf{V}/\partial t + (\mathbf{V}.\nabla)\ \mathbf{V} + (\mathbf{k}\times\mathbf{V})\,f + K\mathbf{V} + w\,\partial\mathbf{V}/\partial z = -(1/\rho)\,\nabla p \tag{1}$$

Performing a scalar multiplication on (1) with $(\mathbf{k}\times\nabla p)$, we get

$$w = \frac{-\sqrt{\left(f + V\frac{\partial\psi}{\partial s} + \frac{\partial\psi}{\partial t}\right)^2 + \left(K + \frac{\partial V}{\partial s} + \frac{\partial \log V}{\partial t}\right)^2}\ \cos(\theta_1 + \theta_{is})}{\sqrt{\left(\frac{\partial\psi}{\partial z}\right)^2 + \left(\frac{\partial \log V}{\partial z}\right)^2}\ \cos(\theta_2 + \theta_{is})} \tag{2}$$

where,

$$\tan \theta_{is} = \frac{-\left(\frac{\partial p}{\partial s}\right)}{-\left(\frac{\partial p}{\partial n}\right)}$$

$$\tan \theta_1 = \frac{\left(f + V\frac{\partial \psi}{\partial s} + \frac{\partial \psi}{\partial t}\right)}{\left(K + \frac{\partial V}{\partial s} + \frac{\partial \log V}{\partial t}\right)}$$

θ_{is} is the angle between isobar and streamline

θ_1 is the angle between streamline and $\partial \mathbf{V} / \partial t + (\mathbf{V} \cdot \nabla) \mathbf{V} + (\mathbf{k} \times \mathbf{V}) f + K\mathbf{V}$

θ_2 is the angle between streamline and vertical shear $\partial \mathbf{V} / \partial z$

5.2. *Vertical motion in cyclone*

A study of vertical motion in a cyclone with spiral features using a mathematical model $\mathbf{V} = A \nabla \phi + B \nabla \phi \times \mathbf{k}$ where $\phi = \exp\{-(x^2+y^2)/2\sigma^2\}$ and σ is the radius of ring of maximum wind shows that the cyclone centre is characterised by :

(*i*) Vertical upward motion with a positive minimum at the cyclone centre when cyclone is relatively weak.

(*ii*) Vertical downward motion at and near the cyclone centre where a negative minimum occurs when cyclone is intense. The downward motion is enclosed by a zero isopleth of w outside which strong vertical upward motion takes place.

Fig. 7 gives the radial profile of vertical motion when the cyclone is weak and also when the cyclone is intense. The noteworthy point so far as this paper is concerned is that vertical upward motion continues to exist outside the ring of RMW whether it is weak or intense. This upward motion provides the necessary vertical upward lift for moist air to form convective clouds.

5.3. *Vertical motion in streamline wave*

We model streamline wave as

$$u = u_m(z)$$
$$v = v_m(z)$$

where,

$\mathbf{V}_m = u_m \mathbf{i} + v_m \mathbf{j}$ is the mean current which is a function of z

$A \exp\left(-\frac{y^2}{2\sigma^2}\right) \sin\left[\frac{2\pi}{\lambda}(x - ct)\right]$ is the perturbation

A is amplitude

$\exp(-y^2/2\sigma^2)$ laterally damps the perturbation

λ is wave length

c is phase velocity

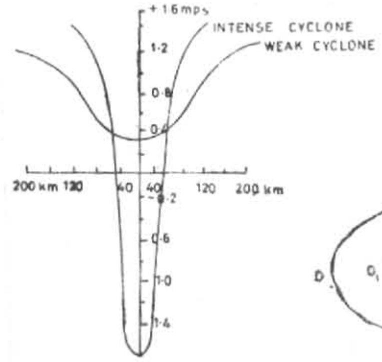

Fig. 7

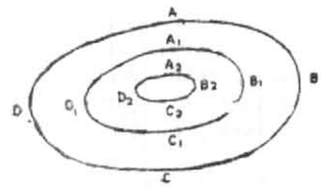

Fig. 8

Introducing the streamline wave into Eqn. (2) vertical motion w was studied for various values of λ, c cross isobaric angles θ_{is} and θ_2. The study shows :

(*i*) Vertical upward and downward motion takes place only when the wave length is relatively small (10 km or less),

(*ii*) Cross isobaric flow facilitates occurrence of upward and downward motion. The amplitude of vertical motion increases as the cross isobaric angle increases,

(*iii*) Amplitude of vertical motion increases for larger values of θ_2, *i.e.*, if the vertical wind shear is characterised by large values of change of direction with height and/or small values of change of speed with height.

5.4. Modelling convection as characterised both by *upward and downward* motion and estimating convection intensity with the help of vertical motion, we note that convection must be associated with perturbation of *short horizontal wave length* and hence convection must be a *small scale phenomenon* only. The causative factor for *change of sign* of w in cyclone as well as convection is the change of sign of $\partial V/\partial s$ which is speed convergence or *vice versa* and magnitude of *coefficient of friction* determines the *size of convection*. For the occurrence of good convection, following factors are required :

(*i*) Sufficient moisture,

(*ii*) Environmental lapse to be greater than moist adiabat,

(*iii*) Upward motion in the layer where air is unsaturated,

(*iv*) Horizontal perturbation of short wave length,

(*v*) Cross isobaric flow,

(*vi*) Direction change with height to be relatively large and/or change of speed with height to be relatively small,

(*vii*) Phase velocity not to be the same as the mean current.

5.5. To save space, derivations of vertical motion in cyclone as well as in streamline waves are not presented. They are given in Chapters 12 and 14 of "*Theorems on Cyclones*" (1984).

6. Section 5 — Cyclone and convection

6.1. The streamline curvature sign is unchanged in a spiral pattern whereas a streamline wave pattern requires change of sign of curvature. Obviously these two patterns are mutually incompatible. The question is "Can spiral and wave patterns coexist ? If so, under what circumstances" has been investigated theoretically. Spiral pattern and wave patterns are mutually incompatible inside the ring of maximum wind where an area of positive L_2 and L_3 exists but can coexist outside it, especially outside ring of maximum wind.

6.2. In Fig. 8, ABCDA is the ring of maximum wind and is the same as zero L_1 isopleth inside which L_1 is positive, outside which L_1 is negative. $A_1B_1C_1D_1A_1$ is the zero L_2 isopleth inside/outside which L_2 is positive/negative. $A_2B_2C_2D_2A_2$ is the zero L_3 isopleth inside/outside which L_3 is positive/negative.

In the case of atmospheric cyclones, $|\nabla \cdot \mathbf{V}| \ll |\zeta|$ and hence zero L_3 isopleth will be inside the zero L_2 isopleth.

Outside $A_1B_1C_1D_1A_1$, streamlines without change of sign of curvature and streamlines with changes of sign of curvature can exist.

6.3. It is realistic to take that (*i*) atmospheric convection is associated with short wave horizontal wind vector perturbation requiring changes in sign of curvature, *i.e.*, $\partial\psi/\partial s$ and changes in sign of speed divergence $\partial V/\partial s$, (*ii*) that these perturbations propagate from outside the RMW. Such convections are feasible especially outside the ring of maximum wind but not possible in the area of positive L_2 and L_3. We may conclude that the cyclone centre will be characterised by either no cloud if the cyclone axis is vertical or with relatively smaller amount of convective cloud if the axis is tilted.

6.4. Since the flow spirals inward towards the cyclone centre, the small-scale perturbations associated with convective cloud formation move towards cyclone centre in which case, there are two possibilities :

(*i*) perturbations shrink the area of positive L_2 and L_3 which ultimately becomes zero, thereby wiping out the spiral configuration,

(*ii*) the perturbations are damped out and the spiral configuration is preserved in which case the kinetic energy of perturbation will be transformed into:

(*a*) heat energy which will increase temperature,

(*b*) potential energy which will require the air to proceed upwards,

(*c*) increase in the kinetic energy of horizontal motion which will increase the horizontal speed.

We have already noted that the vertical upward motion inside RMW contributes to cyclone intensification.

6.5. The two factors which are conspicuously common in cyclone and convective intensification are :

(*i*) Cross isobaric flow

(*ii*) Vertical motion

The upward motion which spiralling configuration of a cyclone necessitates provides the initial ascent of unsaturated moist air to form clouds. The perturbations due to cloud formation move into the interior and vanish. While vanishing inside RMW, they provide increased amount of vertical upward motion inside the ring of maximum wind and hence contribute to cyclone intensification. We note that convection and cyclone intensification are mutually co-operative in the sense vertical and perturbation due to convections vanishing inside the RMW contributes to cyclone intensification.

7. Conclusion

(*i*) L_2 and L_3 are positive in the interior of cyclone inside the ring of maximum wind.

(*ii*) Perturbation in horizontal wind field requiring change of sign of curvature $\partial\psi/\partial s$ and change of sign of speed divergence $\partial V/\partial s$ are incompatible in an area where L_2 and L_3 are positive.

(*iii*) Convective clouds and horizontal wind field perturbations of short wave length are associated.

(*iv*) If perturbations caused by convective clouds are prevented entry into the cyclone interior where L_2 and L_3 are positive, then perturbation kinetic energy either contributes to increase the kinetic energy of cyclone wind field and/or increases the potential energy by making the air go up and such vertical upward motion is one of the factors contributing to cyclone intensification.

(*v*) Convection is not the determining factor in cyclone intensification unless other factors are relatively weak but can be important at some stage in a cyclone's life.

(*vi*) Small scale turbulence can contribute to cyclone intensification and this is facilitated by the inherent kinematical stability feature of cyclone interior where L_2 and L_3 are positive *vide* Theorems 4(a) and 4(b).

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