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An analytical model for air pollutant washout and transport due to emissions from an elevated source

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सार — सपाट भू-भाग पर एक उन्नत स्रोत द्वारा छोड़े गये प्रदूषकों के लिए एक वायु मंडलीय प्रक्षाल-वाहिका और परिवहन निदर्श के लिए विश्लेषिक विवेचन प्रस्तुत किया गया है। वर्षा की बूंदों द्वारा प्रदूषकों के अवशोषण की दर का परिकलन किया गया है। इनके परिणामों से पता चलता है कि स्रोत से सौ किलोमीटर तक की दूरी तक वर्षा की बूंदों द्वारा प्रदूषकों को अवशोषित किया जा सकता है। मध्य परिवेशी सांद्रता की प्रकृति और ऊर्ध्वाधर प्रोफाइलों का भी अध्ययन किया गया है। गैर-अवशोषित ऊपरी और निचली सीमाओं से बहुविध परावर्तन के प्रभावों का समाधान के स्वरूप से स्पष्ट पता चलता है।

ABSTRACT. An analytic treatment of an atmospheric washout and transport model is presented for pollutants released from an elevated source over flat terrain. The rate of pollutant absorption by falling raindrops is calculated. Results show that the pollutants can be absorbed by the rain drops over distances as large as hundreds of kilometres from the source. The nature of the mean ambient concentration and the vertical profiles of the ambient concentration are also studied. Effects of multiple reflection from the non-absorbing upper and lower boundaries are evident from the nature of the solution.

1. Introduction

Pollutants emitted into the atmosphere are removed by a number of natural processes. One of the most important of them is a washout process.

Precipitation scavenging is also of concern with regard to the delivery of several pollutant species (including sulphur compounds) to the surface ecosystem. Adverse effects to soils, forests and fisheries in several parts of the world are being extensively documented. While there is continuing conjecture over specific effects and mechanism, there appears little doubt at present that such negative impact can be severe under specific circumstances.

In view of the above facts various mathematical models dealing with pollutant washout have been developed. The relatively simple models assume a uniform concentration distribution throughout. In this context the work Mukherjee (1980) may be mentioned where the washout effects in an atmosphere with constant background concentration were examied. In this presentation, we have determined the rate of deposition of SO_2 on the ground, incorporating the processes of advection by the prevailing winds, diffusion by eddy motions and washout by rain.

It is worthwhile to mention that in recent years the world has witnessed an increased awareness of a potential rise in the acidity of precipitation. This phenomenon is usually referred to as 'acid rain'. It is generally recognized that acid rain is primarily the result of long-range transport and chemical transformation of combustion products from industrial and transportation sources. The present model calculations also indicate that the raindrops can absorb pollutants over distances as far as several hundreds of kilometres, and so can be effectively used for the study of long range transport of air pollutants.

2. Model formulaton

The situation to be analysed is that of pollutant dispersion and wet deposition from an elevated source emitting a pollutant continuously.

The following assumptions are made :

(*a*) The terrain is flat and the wind velocity and the eddy diffusivity are taken constants.

(*b*) Vertical dispersion is limited by an elevated inversion. A small amount of leakage will occur into the inversion and chemical reactions may take place in the aqueous phase if the inversion is capped by cloud, but the major portion of the airborne material will remain trapped in the mixing layer between the inversion and the ground. In this simple model the inversion will be taken as impenetrable and to have constant height above the ground throughout the transport process.

(*c*) Pollutants are scavenged by the falling rain and result in a fall of the ambient concentration (The concentration of the pollutants in rain being propotional to the ambient concentration).

(*d*) The average intensity of the shower is not too low so that the scavenging action may be assumed to be a continuous process.

Considering an elementary volume $\triangle v$ of the atmosphere the net advective effect due to wind of mean speed u blowing in the x-direction is $-u(\partial c/\partial x)\triangle v$.

Considering diffusion in the vertical direction only, (characterised by an eddy diffusivity D) the net diffusive effect is $D(\partial^2 c/\partial x^2)\triangle v$.

The amount of SO_2 washed out is directly proportional to the ambient concentration c and hence we express the washout term as $-\beta c\triangle v$ where β is a constant washout coefficient.

The processes of advection, diffusion and washout combined together give the following differential equation :

$$D\frac{\partial^2 c}{\partial z^2}-u\frac{\partial c}{\partial x}-\beta c=0 \qquad (1)$$

The mathematical description of the problem is completed by the boundary conditions which accompany Eqn. (1). They are :

(*i*) A continuous source at (0, h) of constant strength Q gives rise to the boundary condition at $x=0$

$$c(0,z)=(Q/u)\,\delta(z-h) \qquad (1a)$$

(*ii*) No flux of pollutants across impenetrable lower and upper boundaries imply

$$D(\partial c/\partial z)=0 \quad \text{at} \quad z=0,\ H \qquad (1b)$$

Applying the method of separation of variables, we find the following solution of Eqn.(1) satisfying the boundary contidions (1a & b) :

$$c(x,z)=\frac{Q}{uH}e^{-\beta x/u}\left[1+2\sum_{n=0}^{\infty}\cos\frac{n\pi h}{H}\times\right.$$

$$\left.\times\cos\frac{n\pi z}{H}\,e^{-D\frac{n^2\pi^2}{H^2}\frac{x}{u}}\right] \qquad (2)$$

Eqn. (2) was used for the study of the mean ambient concentration and the rate of absorption of SO_2 by the falling raindrops.

For the study of the vertical profiles of the ambient concentration the solution in the above form posed certain computational difficulties particularly in the near field regimes ($x<10$ km). It was found that while the vertical profiles were obtained fairly easily at the far field ($x>10$ km), the summation term in the RHS of (2) converged very slowly in the near field regions and involved a large number of iterations.This meant an unnecessary expenditure of expensive computer time.

To overcome this difficulty Eqn. (2) had to be recast into a modified form. This was done as follows :

TABLE 1

Basic parameters and their values adopted in this model

Parameter		Value
*Source strength,	Q	7.5×10^{-4} kg s^{-1} m^{-1}
Effective stack height	h	75, 150, 225 and 375 m
Wind velocity	u	10 m s^{-1}
Eddy diffusivity	D	15 m^2 s^{-1}
$\triangle$scavenging coefficient	β	10^{-4} s^{-1}
Height of elevated inversion H		1000 m, 750 m, 500 m

*The units of Q are expressed in kg m^{-1} s^{-1} in order to have the units of $c(x,z)$ in kg m^{-3}. This had to be done because the present model is 2—d. The value of the source strength is typical of a stack at the Badarpur Thermal Power station at Delhi. We might consider Q to be the cross-wind integrated source strength. Strictly, for a point source Q should be expressed as gs^{-1}, then in that case $c(x,z)$ should be considered at the cross-wind integrated concentrations. $\triangle$*After* Chamberlain (1960), β can also be calculated by usings a distribution function for the size distributed raindrops, the mass transfer coefficient and the number density of the rain drops.

Introducing the parameter $\sigma^2=2Dx/u$ and the substitutions

$$v=\tfrac{1}{2}\left(\frac{\pi\sigma}{H}\right)^2,\quad \tau=\frac{\pi}{2H}\left(z\pm h\right) \text{ in the relation}$$

$$1+2\sum_{n=1}^{\infty}e^{-n^2 v}\cos 2n\tau=\sqrt{\frac{\pi}{v}}\sum_{n=-\infty}^{+\infty}e^{-(\tau-n\pi)^2/v}$$

$$[\text{with } R_e(v)>0],$$

Eqn. (2) could be easily expressed in the form (Whittaker and Watson 1958] :

$$c(x,z)=\frac{Q}{\sqrt{2\pi}\,u\sigma}e^{-\beta x/u}\left[\sum_{n=-\infty}^{+\infty}e^{-(z+h-2nH)^2/2\sigma^2}+\right.$$

$$\left.+\,e^{-(z-h-2nH)^2/2\sigma^2}\right] \qquad (3)$$

The solution (3) can be interpreted as the contribution from the source term located at $z=h$ and its infinite images due to reflection from the two parallel boundaries ($z=0$ and H).

The exponential terms in the summation in (3) now converged very rapidly even in the near field regions and thus the vertical profiles of the ambient concentration could be obtained easily by using Eqn. (3).

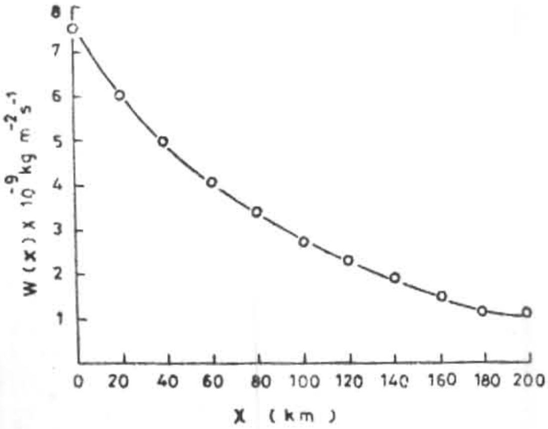

Fig. 1. Rate of absorption of SO_2

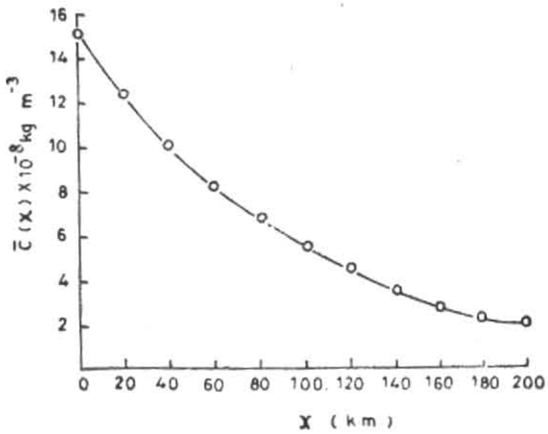

Fig. 2. Variation of mean ambient concentration

It is also interesting to note that in the limit $H \rightarrow \infty$ only the term $n=0$ contributes to the summation in (3) which represents the solution of the Gaussian plume model (elevated source) with washout effect :

$$c(x,z) = \frac{Q}{\sqrt{2\pi}} \frac{e^{-\beta x/u}}{u\sigma} \left[e^{-(z-h)^2/2\sigma^2} + e^{-(z+h)^2/2\sigma^2} \right] \quad (3a)$$

3. Results and discussions

The model was evaluated with the parameters as indicated in Table 1 and was executed on the ICL 2960 system available at I.I.T., Delhi.

The model formuated was found capable of simulating various aspects of atmospheric phenomena, some of which are examined below.

3.1. *The rate of absorption by the falling rain drops*

An expression for the rate of absorption $W(x)$ by the falling rain drops at any downwind distance x is derived as follows :

The flux $F(x)$ of the pollutants across a vertical section (from the ground to the inversion layer) at any downwind distance x is given by

$$F(x) = \int_0^H u\, c\,(x, z)\, dz$$

upon using (2) we find

$$F(x) = Q\, e^{-\beta x/u}$$

Hence the amount of pollutants that are absorbed over the distance between 0 to x is $Q\,(1-e^{-\beta x/u})$. Thus, the average rate of absorption between distances x_0 and x is

$$\frac{Q\left(1-e^{-\beta x_0/u}\right) - Q\left(1-e^{-\beta x/u}\right)}{x - x_0}$$

From the above expression it directly follows that the rate of absorption $W(x)$ by the rain drops at any downwind distance x is :

$$W(x) = \frac{\beta Q}{u} e^{-\beta x/u} = \frac{\beta}{u} Q(x) \quad (4)$$

where $Q(x)$ can be thought of as the source strength effective at distance x.

From Eqn. (4) it is seen that the rate of absorption $W(x)$ at any down-wind distance is an exponentially decaying function. The results shown in Fig. 1 indicate that $W(x)$ decreases from 7.5×10^{-9} kg $m^{-2}s^{-1}$ at apoint close to the source to about 1.0×10^{-9} kg m^{-2} s^{-1} at 200 km downwind. Thus, very close to the source $W(x)$ is given by $\beta Q/u$, whereas further downwind the source is depleted by the absorbing rain drops.

The source depletion approach is described in detail by Pasquill (1962) and essentially treats deposition by washout as a perturbation to the general plume dispersion without washout. The shape of the vertical plume profile is assumed to be unaltered by the washout process and the constant source strength is replaced by a virtual decreasing source strength. The result is a plume which diminishes exponentially with downwind distance while retaining the original shape of the undepleted plume.

3.2. *The mean ambient concentration*

The mean ambient concentration, $\bar{c}(x)$, is given by

$$\bar{c}(x) = \int_0^H \frac{c\,(x, z)\, dz}{H}$$

and from (2) we get

$$\bar{c}(x) = \frac{Q}{uH} e^{-\beta x/u} \quad (5)$$

This function is plotted in the Fig. 2. As is evident from the form of $\bar{c}(x)$ we find that the mean ambient concentration falls off exponentially from a value of 15×10^{-8} kg m^{-3} very close to the source to about 2×10^{-8}kg m^{-3} at 200 km downwind.

The fall of the mean ambient concentration over increasing downwind distanes clearly demonstrates the efficacy of the washoutc mechanism.

We also find that the larger the value of β and the smaller the value of u the more effective is the washout process.

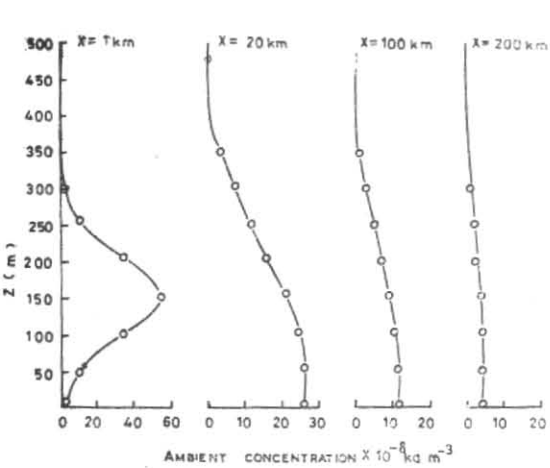

Fig. 3. Vertical profiles of the ambient concentration

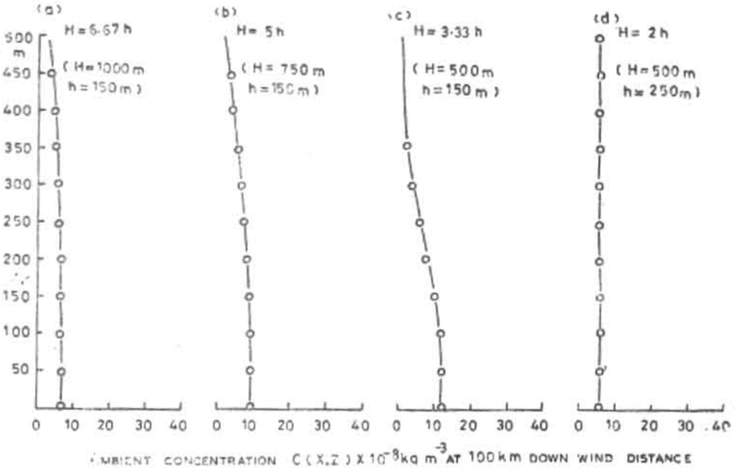

Fig. 4. Effects of source position on the vertical profiles of the ambient concentration

Also the mean ambient concentration is independent of the effective stack height and is inversely propotional to the height of the inversion layer.

The case with $\beta \to 0$, corresponds to the situation without washout and $\bar{c}(x) = Q/uH$. However, the ambient concentration $c(x,z)$, with $\beta \to 0$ does not tend to zero as $x \to \infty$. On the contrary the result one obtains from a Gaussian model shows that the ambient concentration does tends to zero as x approaches infinity. This apparent paradox can be resolved by considering the fact that in the absence of sink mechanisms, the pollutants that are emitted, get reflected from the impenetrable upper and lower boundaries thereby contributing to the ambient concentration levels even far away in the downwind direction. However, when $\beta \neq 0$, the process of absorption by the rain drops in operative and the ambient concentration does tend to zero as $x \to \infty$.

3.3. *The vertical profiles of the ambient concentration*

Fig. 3 depicts the vertical profiles of the ambient concentration (with $h = 150$ m, $H = 500$ m). In the near field one observes the familiar Gaussian profile with the maximum concentration at the effective stack height.

As one proceeds further downwind the effects of ground reflection are felt (since the effective stack height is closer to the ground than to the inversion level, the reflection at the latter boundary is comparatively small). In the profile at $x = 20$ km the effect of ground reflection is clearly apparent. The effect of ground reflection is to increase the ground level concentration (GLC) well above that anticipated without reflection.

Thus in all the far field profiles we find a preponderance of pollutant matter confined to the ground and very little of them at the upper boundary. In general the ambient concentration falls off exponentially upwards and attains a zero value asymptotically at the upper boundary. The reverse will be the trend of the pollutant distribution in case the source is close to the inversion layer in comparison to the ground.

In Fig. 4 the vertical profiles of the ambient concentration (at 100 km downwind distance) are shown for a a few special cases. In Figs. 4 (a-c), the effective stack height is 150 m and the height of the inversion level is varied from 500 metres to 1000 metres. It is observed that pollutant concentration decreases upwards in all the three cases. However, in Fig. 4(d) we observe an almost flat profile. This is because the stack is exactly midway between the upper and lower boundary and as a result the pollutants released from the stack suffer reflection from both boundaries equally effectively.

When the stack is tall enough to be closer to the upper boundary we get a higher concentration at the upper boundary because of greater reflection from the upper boundary in comparison to the lower boundary. These 'top heavy' profiles are not shown explicitly because they are merely mirror images of the corresponding profiles with short stacks which have been already depicted. This feature can also be demonstrated analytically

TABLE 2

Variation of ground level concentration (GLC)

GLC × 10^{-8} kg m^{-3}

H(m)	h(m)	x=20 km	x=100 km	x=200 km
500	75	27.7	12.5	4.6
	150	23.8	10.7	3.9
	225	18.5	8.3	3.1
	375	5.0	2.3	0.8
750	75	51.6	23.2	8.5
	150	21.4	9.6	3.5
	225	16.0	7.2	2.6
	375	8.2	3.7	1.6
1000	75	50.2	22.6	8.3
	150	13.8	6.2	2.3
	225	12.0	5.4	2.0
	375	2.1	0.96	0.35

by replacing z by $z = H - \xi$ in Eqn. (2), whereby one gets the concentration distribution in the x, ξ system as :

$$c(x, \xi) = \frac{Q}{uH} e^{-\beta x/u} \left[1 + 2 \sum_{n=1}^{\infty} \cos \frac{n\pi}{H}(H-h) \times \times \cos \frac{n\pi\xi}{H} e^{\frac{-Dn^2\pi^2}{H^2} \frac{x}{u}} \right]$$

the effective stack height now being $(H-h)$ instead of h.

Finally the nature of the ground level concentration (GLC) must also be discussed. GLC values are shown in Table 2, for various values of the effective stack height and inversion layer heights at downwind distances of 20 km, 100 km and 200 km respectively. It is observed that the GLC decreases, as is expected with increasing downwind distances from the source. For a given value of the inversion level the GLC decreases as the effective stack height increases, for all the three downwind distances. For moderately tall stacks (h being 150 m and 225 m) the GLC decreases with increasing values of the inversion level, at all downwind distances. For a very short stack ($h = 75$ m) the GLC at first increases as the inversion level increases and then slightly decreases with further increase of the inversion level. For a very tall stack ($h = 375$ m) a similar trend is observed. However, the decrease in GLC when H changes from 750 to 1000 m is much more pronounced than in the corresponding case of a very short stack.

4. Limitations and advantages

A major limitation of this model in common with most simple dispersion models is its inability to allow for the change in wind velocity with height above the surface.

The assumption of D being constant throughout is not very realistic either.

Non-inclusion of the lateral diffusion term is also another limitation. The other major assumption of steady state conditions is also a limitation. One can expect a well developed quasi-static boundary layer for atmost about 6 hours in a day. With a mean wind speed of 10 ms^{-1}, the corresponding travel distances would be about 200 km (Maul 1977). Chemical transformation processes and dry deposition effects have been disregarded altogether.

Consideration of these observations reaffirm the conclusion that the model is most useful for the study of mesoscale transport where most box and Gaussian models fail.

Acid precipitation studies can be undertaken by this model because acidity in precipitation is observed at places far away from the point of release. An acid rain model must be able to deal with long range transport. As has been observed the present model is ideally suited for this purpose. Finally, being an analytic model it has several advantages. Computer storage requirements are well below those incurred by using numerical solutions.

5. Conclusions

An atmospheric transport and wet deposition model has been presented for pollutants emitted from an elevated source over flat terrain. This two dimensional analytical model incorporates the processes of advection, diffusion and washout simultaneously and with relative case. Within the limitations of most simple analytic models, the model described is extremely useful for the study of long range transport of gaseous pollutants. It is sufficiently flexible to provide a viable alternative to numerical solutions of the diffusion equation and can produce significant savings in computer costs.

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