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On aspects of updrafts in local severe storms : A control theoretic approach

P. K. DHAR and D. K. SINHA
Centre for Atmospheric Sciences, University of Calcutta, Calcutta
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सार — उर्ध्वांह में अधिकतम उर्ध्वाधर वेग के लिए अपेक्षित औसत ताप उत्प्लावकता (ताप ऊर्जा) के सर्वोत्तम वितरण का अध्ययन करने के लिए नियंत्रण सिद्धांत को प्रयुक्त करने का प्रयास किया गया है। यह परिवर्धन मूलत: एक अभिगमन प्रक्रिया है जो व्यवरोध द्वारा प्रानुकूलित की गई है। अर्थात् प्रचालन के दौरान ताप उत्प्लावकता द्वारा उत्पन्न औसत शक्ति की बढ़त को सीमित करके प्रानुकूलित की गई है। इस अभिपुष्ट निष्कर्ष पर हम उन आंकड़ों के उपयोग द्वारा पहुंचे हैं (केसलर 1974) जो कि पूर्णत: वास्तविक नहीं हैं।

ABSTRACT. In this paper, an attempt is made to apply a control theoretical approach to study the optimum distribution of average thermal buoyancy (thermal energy), required for attainment of maximum vertical velocity in the updraft. This development, basically a transport process, is conditioned by the constraint, putting stipulation on the growth of average power generated by thermal buoyancy during the time of operation. The validation has been sought on the lines (Kessler 1974) of using data, not wholly divorced from reality.

Key words — Thermal buoyancy, Thunderstorms, Vertical velocity, Mixing rate, Average power, Updraft, Nor'westers.

1. Introduction

Thermal buoyancy is found to initiate the trigger of the parcel and in a way, controls the development of the maximum vertical velocity of the parcel in case of occurrence of thunderstorms (Weisman and Klemp 1986). The magnitude of the maximum vertical velocity of the parcel does influence the development of some thunderstorms (Browning 1982); it is basically determined by the strength and lateral dimensions of the vertical draft (Browning 1982, Darkow 1982, Hill 1988).

The study of the distribution of vertical average thermal buoyancy, otherwise called thermal energy is, therefore, necessary and more so, because of the development of maximum vertical velocity of the parcel. To make the model study mathematically tractable we draw upon the concept and techniques of control theory and in particular the optimal control theory. The expression for thermal energy in terms of the maximum vertical velocity, height of the maximum vertical velocity and the rate of entrainment for the same in shortest possible time (most rapidly) is obtained. Thus one obtains, as a result, various strands of thermal energy conditioned by the constraint required for attaining the maximum vertical velocity of the parcel. The exercise of optimality also yields a quantification of buoyant energy. We have also the profiles in this paper, providing useful understanding about the process involving the parameters mentioned above.

2. Method of computation

2.1. *List of symbols used*

w	—	Velocity of the vertical wind (ms^{-1}).
$\bar{w}$	—	Average vertical velocity.
w^*	—	Maximum value of the average vertical velocity,
w_{max}	—	Maximum vertical velocity of the wind (ms^{-1}),
B	—	Acceleration identified with the thermal component of buoyancy (ms^{-2}),
H	—	Vertical extent of updraft column,
k_1	—	Assumed mixing rate for heat, momentum and water substance associated with mixing in the horizontal plane (s^{-1}),
k_2	—	Coefficient that converts condensed water to equivalent buoyancy (ms^{-2}/gm^{-3}),
T, $H/2$	—	Least time and height for attending maximum vertical velocity respectively,
P^2	—	Constraint of average power generated by thermal buoyancy,
$(\bar{\ })$	—	Average value of ( ) over $H/2$.

2.2. *Statement of the problem*

From the foregoing lines, the problem turns out to be one of determining the average thermal buoyancy denoted by $\bar{B}(t)$, so that the average vertical velocity $\bar{w}(t)$ can attain a maximum value, say w^*, in least possible

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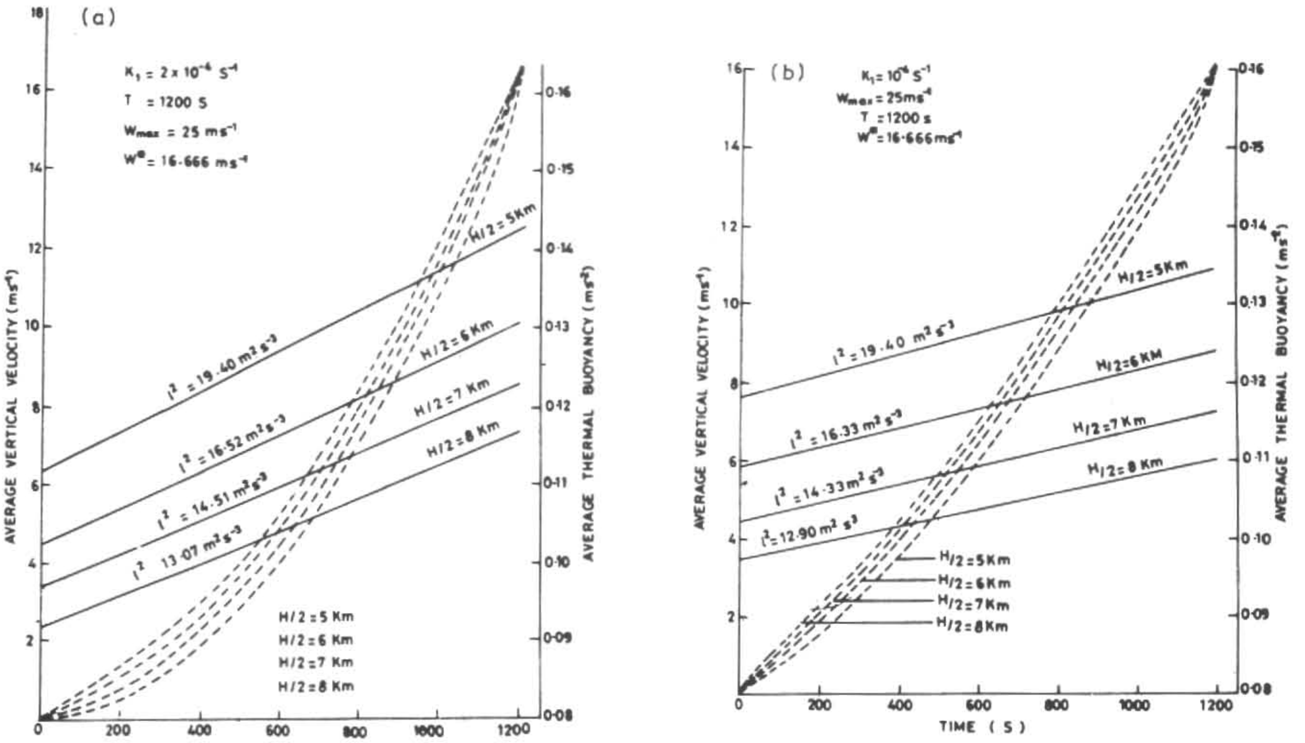

Figs. 1 (a & b). Profiles of average vertical velocity (-----) *vis-a-vis* average thermal buoyancy (——) at different heights with time

time (most rapidly) say T. This development of the updraft is conditioned by the constraint on the growth of average power vide Eqn. (1). The average thermal buoyancy $\bar{B}(t)$ is limited under the constraint.

$$\int_0^T \bar{B}^2(t)\,dt < l^2 \tag{1}$$

where, l^2 is a constant.

2.3. *Optimal control problem*

According to Butkovosky (1969), let us consider the system, given by the equation,

$$\frac{d}{dt}\,q = a(t)\,q + b(t)\,u(t) + c(t) \tag{2}$$

where, q is a state variable, $u(t)$ is the control applied in the system and $a(t)$, $b(t)$, $c(t)$ are known continuous functions of time.

For the problem, which is to find a control $u(t)$ such that the state of the system q of Eqn. (2) arrives at the point $q^*(t)$ in the smallest possible time T under the constraint $\int_0^T u^2(t) \leqslant l^2$ (l is given positive number), the optimal control $u(t)$ can be written as :

$$u(t) = l^2\,G(t)\,\xi \tag{3}$$

under the condition (Butkovosky 1969, p. 197, 230)

$$\alpha\,\xi = 1 \tag{4}$$

where, ξ is a number, and

$$\alpha(t) = \psi(t)q^*(t) - \int_0^t \psi(t)c(t)dt - q_0,\ \ G(t) = \psi(t)b(t).$$

$$\frac{1}{l^2} = \int_0^T \xi^2 G^2(t)dt,\ \ \psi(t) = \phi(-t),\ \ q(0) = q_0 \tag{5}$$

and $\phi(t)$ is the solution of the homogeneous linear differential Eqn. (2).

2.4. *Basic equations of the system*

In one dimensional cloud models, the vertical equation of motion (Kessler 1974) is

$$\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z} = B - k_1 w \tag{6}$$

For updrafts, we integrate Eqn. (6) between $z = 0$ and $z = H/2$ which yields (*vide* Kessler 1974)

$$\frac{d}{dt}\bar{w} + \frac{w^2(H/2)}{H} = \bar{B} - k_1\bar{w} \tag{7}$$

where,

$$(\bar{\ }) = \frac{2}{H}\int_0^{H/2} (\ )dz \text{ and } w\left(\frac{H}{2}\right) = w_{max} \tag{8}$$

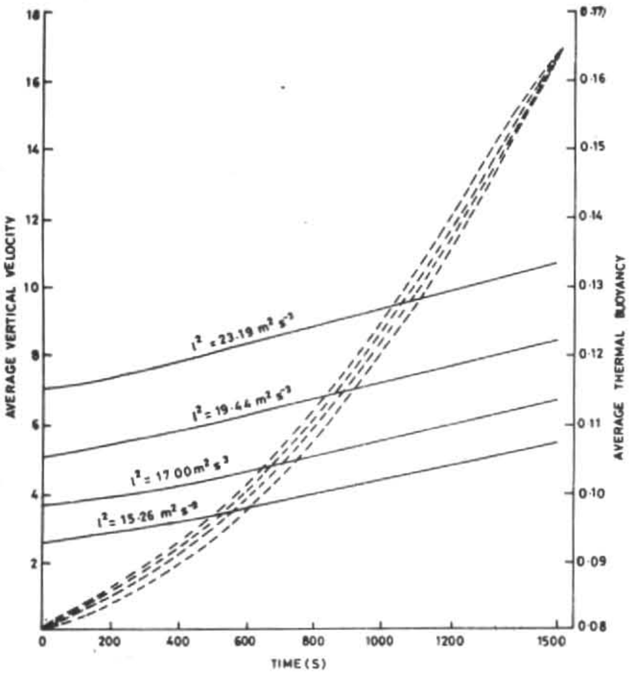

Fig. 2. Profiles of average vertical velocity (— — —) *vis-a-vis* average thermal buoyancy (——) at different heights with time

According to (Kessler 1969), let us assume the parabolic profile as :

$$w = 4\frac{w_{max}}{H}\left(z - \frac{z^2}{H}\right) \quad (9)$$

Thus

$$w(H/2) = w_{max} \text{ and } \frac{2}{3} w_{max} = w^* \quad (10)$$

2.5. *Solution of the problem*

The solution of Eqn. (7) under initial condition $w = 0$ at $t = 0$ gives,

$$w(t) = e^{-k_1 t} \int_0^t e^{k_1 \eta} \left\{\bar{B}(\eta) - \frac{w^2_{max}}{H}\right\} d\eta \quad (11)$$

Here, according to the given notations in Eqns. (2)-(5), one can write,

$$\phi^{(t)} = e^{-k_1 t}; \quad G(t) = e^{k_1 t} \quad (12)$$

$$\alpha(T) = \frac{2}{3} e^{k_1 T} w_{max} + \frac{w^2_{max}}{H}\left(\frac{e^{k_1 T} - 1}{k_1}\right)$$

$$\frac{1}{l^2} = 2\left(\frac{e^{2k_1 T} - 1}{2k_1}\right) \quad (13)$$

$$\frac{1}{\xi} = \frac{2}{3} e^{k_1 T} w_{max} + \frac{w^2_{max}}{H}\left(\frac{e^{k_1 T} - 1}{k_1}\right) \quad (14)$$

and

$$\bar{B}(t) = l^2 \xi e^{k_1 t} \quad (15)$$

where,

$$w_{max} = \frac{3}{2} w^*$$

2.6. *Numerical computation*

To obtain an idea about the behaviour pattern of average thermal buoyancy, a few graphical representations are also drawn. We have chosen, for reasons of simplicity, the average thermal buoyancy as a measure of control in the process of attaining maximum velocity of updraft (Kessler 1974). We computed numerically l^2, $\bar{B}(t)$ and $\bar{w}(t)$ with the help of Eqns. (11)-(15) and for better understanding of the dynamics, we draw upon the data provided by Kessler (1974, 1982), given below :

$$w_{max} = 25 \text{ ms}^{-1}; \quad k_1 = 10^{-4} \text{ and } 2 \times 10^{-4} \text{ s}^{-1},$$
$$T = 1200 \text{ and } 1500\text{s}; \quad k_2 = .01 \text{ ms}^{-2}/\text{gm}^3.$$

3. Discussion

It is worthwhile to explore some of the important features from the graphs. In particular, Figs. 1-2 represent profiles, which indicate the development of both average thermal buoyancy and average vertical velocity of the parcel during the period necessary for reaching maximum vertical velocity. Figs. 1 (a & b) are basically intended to give an idea about the behaviour pattern of mixing rate in regard to average thermal buoyancy. In fact, keeping the other parameter fixed, one finds an increase of mixing rate augmenting the total average thermal buoyancy which is so essential for generation of maximum velocity during the entire time span. One may provide a physical interpretation in that an increase of mixing rate as mentioned by Kessler (1974) and Priestley (1953) decreases the diameter of the parcel and hence, increases the lapse rates in case of absolute buoyancy (which exists in case of super-adiabatic lapse rate) is in agreement with observation by Priestley (1953). This leads to the increase of the average thermal buoyancy and small increase of average power l^2 during the whole time. Secondly, keeping the least possible time and all other parameters fixed, as the height of the updraft column increases, we find that the initiating average thermal buoyancy decreases. This is, otherwise, consistent physically because of the fact that the continuity of horizontal convergence is associated with the vertical divergence or stretching of the layer; the increase of layer thickness increases the lapse rate of the layer and the stability of the layer decreases, which gives the decrease of thermal buoyancy. The same possible explanation can be given in discussing the decrease of the average power I^2 during the time of operation (whole interval) with the increase of height of the layer when all other parameters except the height $H/2$ are fixed.

Let us now discuss few other distinguishing features between two graphs. From Figs. 1 (b) and 2 we note that the maximum velocity of the parcel may be attained by a slow updraft over a long period of time or possibly by a stronger updraft operating for proportionately shorter period of time. This signifies the decrease of initiating thermal buoyancy with the increase of least time for attainment of maximum velocity of the parcel; this is reasonably in accordance with physical realities. Thus, with the increase of time of operation T, the average power I^2 increases, which we note from the graphs in Figs. 1 (b) and 2. This can also be seen from physical considerations.

It may be remarked, in passing, that even though the foregoing analysis stems from a simple model approach, there are few facets that come close to realities.

4. Conclusion

The authors are essentially an attempt to use a fairly idealised control theoretic model (Kessler 1974, 1982), enabling them to focus few striking features, which are otherwise brought out in the investigation of this kind. From the theoretical exercises, there emerge some specific values of the thermal buoyancy, depending, of course, on the environmental parameters which, in a way, are the thresholds for attaining a maximum vertical velocity of the parcel.

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