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Flow through soil with initial gradient

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ABSTRACT. A short review of the phenomenon of initial gradient in irrigation, drainage and consolidation problems is made and an analysis is presented to show that for the problem of horizontal and vertical capillary saturation, nonrecognition of initial gradient would lead to serious error in the estimation of ultimate travel length and the time rate on progress of saturation line.

1. Introduction and Review

Phenomenon of water movement through clayey and loamy soils is of common occurrence in the field of irrigation and drainage engineering. It is now well recognised that for many clayey soils and to certain extent sandy soils, the seepage starts only when the hydraulic gradient exceeds a certain value called the initial or threshold gradient and the velocity gradient response for this type of soils can be represented by Egn. (1)

$$v = K(i - i_0), \quad i \geqslant i_0 \tag{1}$$

in which v is the macroscopic flow velocity, K is the permeability coefficient, i is the hydraulic gradient and i_0. is the initial or threshold gradient. Seepage of many non-Newtonian fluids, such as oil-water emulsion, bentonite suspension etc through porous media are also characterised by the initial gradient Kovacs (1969). For water flow through clayey soils, Poluberinova-Kochina (1962) states that the magnitude of initial gradient may vary from 15 to 20. Scott (1963) mentions this variation as 20 to 30 for dense clays. Experimental results of Li (1963) with Houston clay (clay content of 72 per cent with void ratio of 0·85) indicate an initial gradient of around 50. Karadi and Nagy (1961) and Kondon (1967) concludes that initial or threshold gradient increases with decreasing void ratio or decreasing water content for fully saturated soils. The increase in initial gradient with the decrease is water content as evidenced by Karadi and Nagy's experimental results on clay, is reproduced in Table 1.

The existence of initial gradient in clayey and other fine grained soils has been attributed to the predominance of surface forces over the gravity forces existing in these fine grained soil and generally, these surface forces are strong enough to counteract a certain portion of the applied gradient called initial gradient (Miller and Low 1963). So, it is natural to expect that lower the porosity and smaller the grains size higher will be the specific surface and hence higher will be the surface forces resulting an increase in initial gradient.

Consequences of the existence of initial gradient are of potential interest in several disciplines. Ground water movemnt and drainage in clayey soils soil water movement to plant roots, consolidation of clayey soils and infiltration of water into soils are some of the areas where the recognition of the role of the initial gradient is important. In drainage problems, nonrecognition of the role of initial gradient will result in inefficient layout of drains.

During last one decade or so, there is a considerable interest in solving various physical problems incorporating initial gradient in the flow relation (*i.e.*, Eqn. 1). Using unit hydrograph technique Entov (1967) has presented an analytical method for solving two dimensional problems with initial gradient. Ghorghitza (1959), Poluborinova-Kochina (1969), Valsangkar and Subramanya (1972) and Arumagam (1975) investigated the effect of initial gradient on various physical Problems of interest in the field of drainage and irrigation. Florin (1951), Roza and Kotov (1955), Girault 1960), Elnaggar, Karadi and Krizek (1971) and Parlange (1973) studied the effect of initital gradient on the one dimensional consolidation of clayey soils due to vertical drainage.

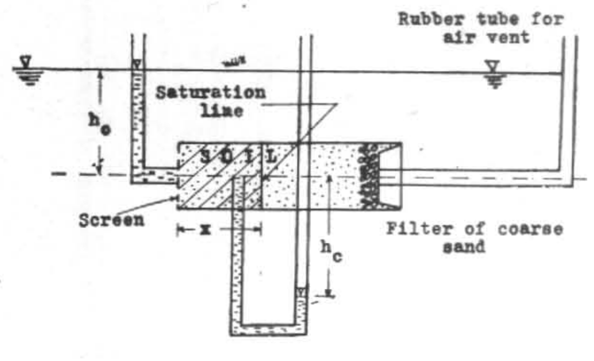

Fig. 1. Definition sketch for horizontal capillary movement

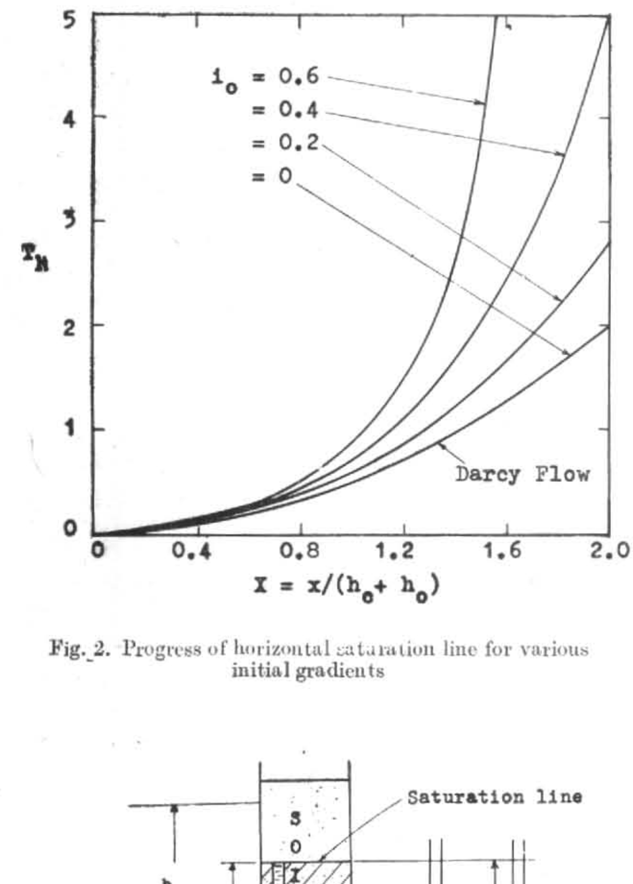

Fig. 2. Progress of horizontal saturation line for various initial gradients

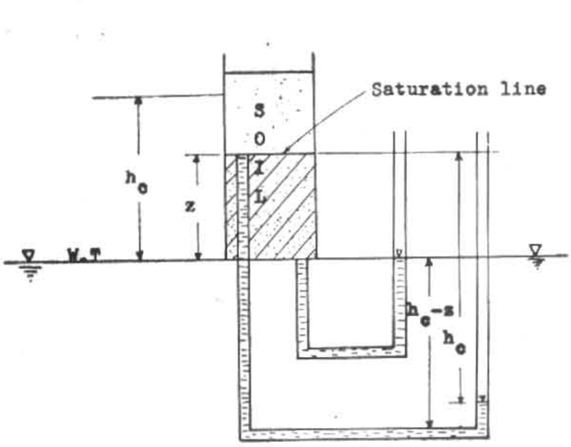

Fig. 3. Definition sketch for vertical capillary movement

TABLE 1

Variation of initial gradient of clayey soils with water content (After Karadi and Nagy 1961)

Water content in per cent	Initial or threshold gradient (i_o)
29·1	22·8
32·2	12·8
34·5	7·3
39·3	3·1
42·1	0

Here in this paper, exact analytical solution or the problems of horizontal and vertical water movement in capillary zones incorporating flow equation $v = K\ (i-i_0)$ are presented. The qualitative and the quantitative effect of the existence of initial gradient on these problems are brought out and discussed.

2. The problem, analysis and conclusions

The movement of water front or saturation line in the capillary zone is of interest in earth dams and other similar structures. Assuming the flow to be one dimensional (either horizontal or vertical), theoretical expressions for the movement of saturation line as a function of time, in the absence of initial gradient, is available (Taylor 1948). The extension of the same in the presence of initial gradient follows.

(i) *Rate of horizontal capillary saturation*

To assure a truly one dimensional horizontal flow, a system as shown in Fig. 1 is considered. The system, as shown, is used many times for the capillarity-permeability test (Taylor 1948). Here, the soil sample has been brought to state of dry powder and then packed into a glass tube having a screen over one end and a vented stopper at the other end. The tube is then immersed in shallow depth in a horizontal position and the water proceeds into the interior of the soil by capillary action. The distance of saturation x as a function of time t is to be found out for flow law represented by Eqn. (1).

For the system as shown in Fig. 1 the flow Eqn. (1) can be written as

$$S n_e\, v_s = K \left(\frac{h_c + h_0}{x} - i_0\right) \tag{2}$$

where,

S = degree of saturation
n_e = effective porosity
v_s = seepage velocity

h_c = capillary head which is constant for a porosity

h_0 = depth of immersion (as shown in Fig. 1)

Eqn. (2) can further be written as

$$Sn_e \frac{dx}{dt} = K\left(\frac{H}{x} - i_0\right) \quad (3)$$

Separating the variable and integrating it with the initial condition (4)

$$t = 0,\ x = 0 \quad (4)$$

one gets

$$T_h = \frac{\log \dfrac{1}{1 - i_0 X} - i_0 X}{i_0^2} \quad (5)$$

where,

$$T_h = \frac{Kt}{HSn_e} = \text{nondimensional time, called time factor} \quad (6)$$

$$H = h_c + h_0 \quad (7)$$

$$X = \frac{x}{H} \quad (8)$$

Eqn. (5) gives the position of water front as a function of time and the initial gradient i_0 in nondimensional form. In the limit $i_0 \to 0$, Eqn. (5) converts back into the well known x *vs* $\sqrt{t}$ relation as given by Eqn. (9) or (9a)

$$X = \sqrt{2\,T_h} \quad (9)$$

or

$$x = \left[\sqrt{\frac{2\,KH}{Sn_e}}\right] \sqrt{t} \quad (9\text{ a})$$

The nondimensiona travel length X as a function of time factor T_h as given by Eqns. (5) and (9) are plotted for various values of initial gradient i_0 and is shown in Fig. 2.

The striking feature of Eqn. (5) is that in the presence of initial gradient ultimate travel length (*i.e.*, X at $T_h = \infty$) is finite and is equal to $1/i_0$ whereas nonrecognition of initial gradient would lead to prediction of infinite ultimate travel length.

An seen from Fig. 2 progress of saturation line becomes increasingly slower with the increase in initial gradient. Typically, for a nondimensional travel length (X) of 1·5, and for a initial gradient (i_0) of 0·6 the time factor is 3·896 and the non-recognition of initial gradient for this travel length would result in an error of 246 per cent in the time factor estimation. Eqns. (9 a) and (9) suggest that many experimental reports published earlier exhibiting a nonlinear relation between x *versus* $\sqrt{t}$ may be explained in terms of the presence of initial gradient i_0.

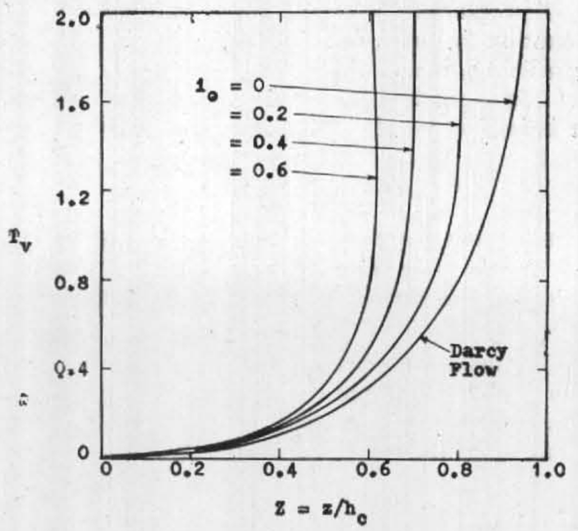

Fig. 4. Progress of vertical saturation line for various initial gradients

(ii) Rate of vertical capillary saturation

For this case, the system along with heads acting on it is schematically presented in Fig. 3.

At any instant of time, if z is the height of saturation line, then by the flow law assumed

$$Sn_e\, v_s = K\left[\frac{h_c - z}{z} - i_0\right] \quad (10)$$

where h_c is the height of capillary rise (constant for a particular porous media system at a particular porosity) and z being the travel length in the vertical direction as explained in Fig. 3. Other parameters in Eqn. (10) are explained earlier. The seepage velocity v_s in this case is equal to dz/dt and hence Eqn. (10) can be written as

$$\frac{dZ}{dT_v} = \left(\frac{1 - Z}{Z} - i_0\right) \quad (11)$$

where,

$$Z = \frac{z}{h_c} \quad (12)$$

and

$$T_v = \frac{Kt}{h_c\, Sn_e} = \text{nondimensional time factor} \quad (13)$$

Integrating Eqn. (11) with nitial condition (14)

$$t = 0,\ z = 0 \quad (14)$$

one obtains

$$T_v = \frac{\log \dfrac{1}{1 - (1 + i_0)\,Z}}{(1+i_0)^2} - \frac{Z}{(1 + i_0)} \quad (15)$$

Eqn. (15) gives the position of water front or saturation line as a function of time and the initial gradient i_0, in non-dimensional form. In the absence of initial gradient, Eqn. (15) reduces to the known relation between T_v and Z as given by Eqn. (16)

$$T_v = \log \frac{1}{1-Z} - Z \tag{16}$$

The relation between non-dimensional travel height Z and the time factor T_v as given by Eqn. (15) is plotted for various values of i_0 and is shown in Fig. 4.

It is interesting to note that the form of equation for both vertical and horizontal capillary rise as a function non-dimensional time factor is very much similar except that i_0 in Eqn. (5) is replaced by $(1+i_0)$ in Eqn. (15). In the absence of initial gradient the travel height equals to capillary hight (*i.e.*, $Z=1$ at $T=\infty$), whereas the presence of initial gradient makes the ultimate travel height less than capillary height and the ultimate nondimensional travel height ($Z=z/h_c$) equals to $1/(1+i_0)$. As seen from Fig. 4 and Eqn. (15), similar to horizontal capillary saturation, progress of vertical saturation line also becomes increasingly slower with the increase in initial gradient. Typically, for a non-dimensional travel height ($Z=z/h_c$) of 0·6 and for an initial gradient (i_0) of 0·6, the time factor (T_v) is 0·8823 and the non-recognition of this initial gradient in the analysis would incur an error of 179 per cent in the prediction of time factor for the same travel height.

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