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On the application of a primitive equation barotropic model for the prediction of storm tracks in the Indian region

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ABSTRACT. Methods of prediction of storm tracks in the Arabian Sea and the Bay of Bengal by the use of a limited area primitive equation barotropic model have been discussed with case-studies. They are (*i*) the total flow direct integration method. (*ii*) the point vortext method in which the basic flow is separated from the disturbance and integrated, treating the vortex as a point and advecting the same at each time step by the basic field and (*iii*) the modified point vortex method in which the interaction of the disturbance flow with the basic flow is taken into account in the method in (*ii*).

1. Introduction

Tropical cyclones occur in the Indian region during (*a*) pre-monsoon (April, May and early June) and (*b*) the post-monsoon (late September to early December) seasons. Due to paucity of meteorological observations in and around the storm fields,the forecasting techniques have largely so far relied on empirical rules based on the skill and experience of the synoptic forecasters. With the advent of highspeed computers and some improvement in the observing systems, objective forecasting schemes are increasingly being developed and tested in actual case studies. These schemes may be categorised as :

(*i*) The storm analogue techniques based on climatology of past storm-tracks used by Gupta and Dutta (1971), Sikka and Suryanarayana (1971) and the statistical methods using regression equations based on chosen predictors by Bansal and Dutta (1974) are in the first category. (*ii*) Use of numerical weather prediction models is another promising approach. Sikka (1974) used a non-divergent barotropical model for the prediction of storm tracks.

It is well-known that for the prediction of the movement of tropical and sub-tropical disturbances, the barotropic model is still used largely in many countries. It is perhaps the experience, that during the movement, there is little conversion of potential to kinetic energy and *vice-versa* and there is predominantly only a redistribution of kinetic energy among the different wave components. Therefore the movement of the disturbances could possibly be treated two-dimensionally by a barotropic model. In this study application of a Primitive Equation barotropic model which conserves potential vorticitiy is discussed.

2. Method of study

2.1. *Phase speed considerations*

The waves generated by the finite-difference solution of the numerical models move with a phase-speed less than that observed in the atmosphere. This is especially true of shorter wave components less than 2,000 km, found in the tropical disturbances. This underestimation is from the truncation error due to the discretization of the finite-differencing. It has been found by Bermowitz (1969) that tolerable truncation error is achieved by reducing the spatial grid interval and increasing the computational resolution. It is, however, realised that consistent with the available computational configuration of memory and speed there is always a practical limit to the number of such points. Also this demands a smaller time-step to ensure compliance with the well known stability criteria of Courant *et al.* (1928). Based on the above considerations, the methods of application discussed in this study may be categorized as below :

(*i*) Integrations with the field as a whole on (*a*) 2·5° and (*b*) 1·25° grid intervals with different initial vortex structures.

(*ii*) Point vortex advection method wherein the vortex is separated, and the basic field is integrated. The vortex considered as a point is advected by the basic field at each time-step.

(*iii*) Modified point vortex method taking into account the interactions between the vortex and basic fields in some way.

Accordingly section 3 gives brief details of the model. Section 4 gives the results of integration on a 2·5° and 1·25° grid intervals in a number of case studies. In section 5 some important factors that govern the movement of storms are examined. The point vortez method with a case study is discussed in section 6. Section 7 deals wih the attempts to incorporate the interactions between the vortex and basic fields in the method in section 6. The last section besides summing up the results, presents an outlook for future studies.

2(a). List of symbols and their explanations

Variable	*Meaning*
u	Zonal velocity
v	Meridional velocity
m	Map scale factor
f	Coriolis parameter
g	Acceleration due to gravity
h	Height of the free surface
k	Measure of disturbance intensity, *i.e.*, the product of maximum wind speed and distance of its occurrence from the centre of the vortex
y	Distance along the longitude
x	Distance along the latitude
a, b	Constants for defining the shape of the vortex
S	Vorticity

3. The Model

The model equations in cartesian coordinates on a mercator projection are :

$$\frac{\partial u}{\partial t} + m\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial h}{\partial x}\right) - fv = 0 \quad (3.1)$$

$$\frac{\partial v}{\partial t} + m\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial h}{\partial y}\right) + fu = 0 \quad (3.2)$$

$$\frac{\partial h}{\partial t} + m\left[u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} + h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] - vh\frac{\partial m}{\partial y} = 0 \quad (3.3)$$

Explanations for symbols are givens in Sec. 2(a). For details of finite-difference scheme, boundary conditions etc reference may be made to the model used by Ramanathan and Saha (1972) for the study of western disturbances. The initial input to the model is derived from observed winds. Initialization by the balance equation (Ramanathan *et al.* 1971, Finizio 1974) was found to be adequate for the Indian region, and as in Shuman and Vandermann (1966) the forcing functions given by the balance equation (3·4).

$$\nabla^2 h = -\frac{1}{g}\frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - f\frac{v}{m}\right) - \frac{1}{g}\frac{\partial}{\partial y}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f\frac{u}{m}\right) \quad (3.4)$$

On the right side were normalized for two distinct sets of points to avoid separation of solutions from the finite-difference equations.

The forecast domain was bounded by longitudes 65°E and 105° E in the east-west and latitudes 3·75°N and 26·25°N in the north-south for 1·25° grid and by Long. 45°E and 125°E in the east-west and latitudes 0°N and 42·5°N in the north-south for 2·5°grid.

4. Results of integration

The model was applied to a number of case studies in 1970 and the results are summarized in Table 1. The grid-interval used was 2·5°.

This grid size is sufficient to resolve a wave length ~1000 km (4 grid lengths) and based on the considerations of scales, data density and available computer configurations, has been commonly used for the prediction of flow patterns in the Indian tropics. Nevertheless this grid size was too coarse for defining the initial position of the storm accurately. Also the stream function fields became shallow and vortex centres could be located only in the vorticity and potential vorticity forecast charts for 24 and 48 hours. The initial position in the vorticity chart was sometimes in error by as much as 60 km. A circle was hence drawn with 0·5° radius around the realized vortex centre and the error in the forecasts was computed, from the forecast position and the nearest point on the circle around the realised centre.

The details of storm tracks — actual and forecast are given in Figs. 1-5. The average error is (*i*) 80 km for 24 hours and (*ii*) 285 km for 48 hours. The error is no doubt considerable in some of the cases especially for 48 hours. Also there is a bias for moving the storms towards the

TABLE 1

S. No.	Date (1970)	Initial position		Forecast positions 24 hr		Forecast positions 48 hr		Actual positions 24 hr		Actual positions 48 hr		Departure (in km)	
		Lat. (°N)	Long. (°E)	Lat. (°N)	Long. (°E)	Lat. (°N)	Long. (°E)	Lat. (°N)	Long. (°E)	Lat. (°N)	Long. (°E)	24 hr positions	48 hr positions
1	3 May	15·0	88·0	17·0	89·0	19·0	89·5	18·0	90·0	19·0	91·0	110	105
2	4 May	18·0	90·0	20·0	90·0	22·0	91·2	19·0	91·0	20·0	91·5	110	170
3	5 May	19·0	91·0	20·0	92·0	—	—	20·0	91·5	—	—	Nil	—
4	29 May	15·5	71·5	18·0	70·5	21·0	71·0	17·5	71·0	19·0	68·0	30	430
5	30 May	17·5	71·5	19·5	70·0	22·5	68·0	19·0	68·0	20·0	64·0	165	455
6	31 May	19·0	68·0	20·0	65·5	21·5	64·0	20·0	64·0	20·0	60·0	110	405
7	1 Jun	20·0	64·0	20·5	61·2	—	—	20·0	60·0	—	—	75	—
8	4 Sep	23·0	82·5	23·0	81·5	24·0	80·0	23·0	80·0	24·0	75·5	85	400
9	5 Sep	23·0	80·0	23·5	76·5	26·0	73·0	23·0	75·5	23·5	72·0	80	220
10	6 Sep	23·0	75·5	24·0	71·5	27·0	74·0	23·5	72·0	26·5	71·5	20	250
11	21 Oct	16·2	86·5	20·0	86·0	23·0	85·0	17·5	88·0	21·5	89·0	300	375
12	22 Oct	17·5	88·0	21·5	88·0	24·5	88·0	21·5	89·0	24·0	91·0	60	250
13	23 Oct	21·5	89·0	25.0	91·0	—	—	24·5	91·0	—	—	Nil	—
14	9 Nov	13·5	86·5	14·5	87·5	17·0	85·5	14·5	87·0	16·5	87·5	Nil	170
15	11 Nov	16·5	87·5	18·5	88·0	21·5	89·5	17·5	88·0	23·0	92·0	75	200

west. In the absence of data in and around the storm field, the vortex in almost all cases was intially defined as circular symmetric. This type of initial definition, when the storm was actually elongated north-south resulted in the reduction for $\beta=\partial f/\partial y$ value and consequent increase perhaps of the westward phase velocity of the disturbance.

Reduction of the grid size from 2·5° to 1·25° was helpful in positioning the vortex centre initially more accurately in the chart, but the real improvement in the case of the forecasts for 24 hours resulted by giving a more north-south elongation to the structure of the vortex, This was usually done by smoothing out the vortex in the initial analysis and inorporating an analytical vortex with the basic field. The rectilinear vortex used in the study may be defined as follows :

$$u = axk/(a^2x^2 + b^2y^2) \quad (4.1)$$

$$v = -bxk/(a^2x^2 + b^2y^2) \quad (4.2)$$

Figs. 6 (a) and 6 (b) give the forecast positions for the same case but with a symmetric structure and elongated orientation (*a*) straight axis and (*b*) inclined axis respectively, The improvement in the forecasts compared to Fig. 4 with 2·5° grid may be seen.

5. Assessment of factors governing the movement of tropical storms

5.1. Basic current and the disturbance intensity are two important factors that govern the movement of storms. There is no unique method by which one can objectively separate a given total flow pattern into a basic current and perturbations. The characteristics of one or the other must be subjectively specified on empirical or theoretical grounds. In the next section we use the double Fourier analysis technique to achieve this separation.

5.2. *Experiments for assessment*

Experiments were performed to assess which of the two factors — the basic current or the disturbance intensity — plays a more dominant role for prediction of storm tracks by the model.

A symmetric rectilinear vortex whose field is given by

$$u = xk/(x^2 + y^2) \quad (5.1)$$

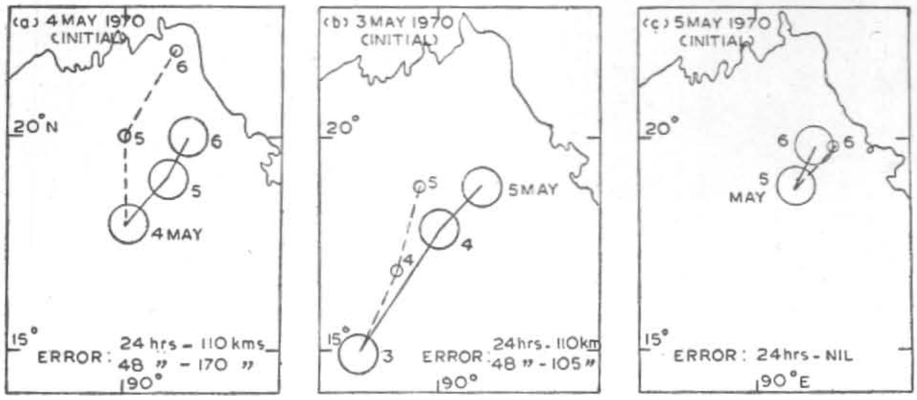
$$v = -yk/(x^2 + y^2) \quad (5.2)$$

is embedded at about the centre of a prescribed basic flow chart and the input to the model is thus constructed.

(*i*) The basic October mean chart at 500 mb for the same area and grid at 2·5° interval as in section 4 was prepared and a symmetric vortex with maximum speed 30 mps at a distance of 1° from an arbitrary centre at 15°N, 92·5° E was embedded in the basic chart.

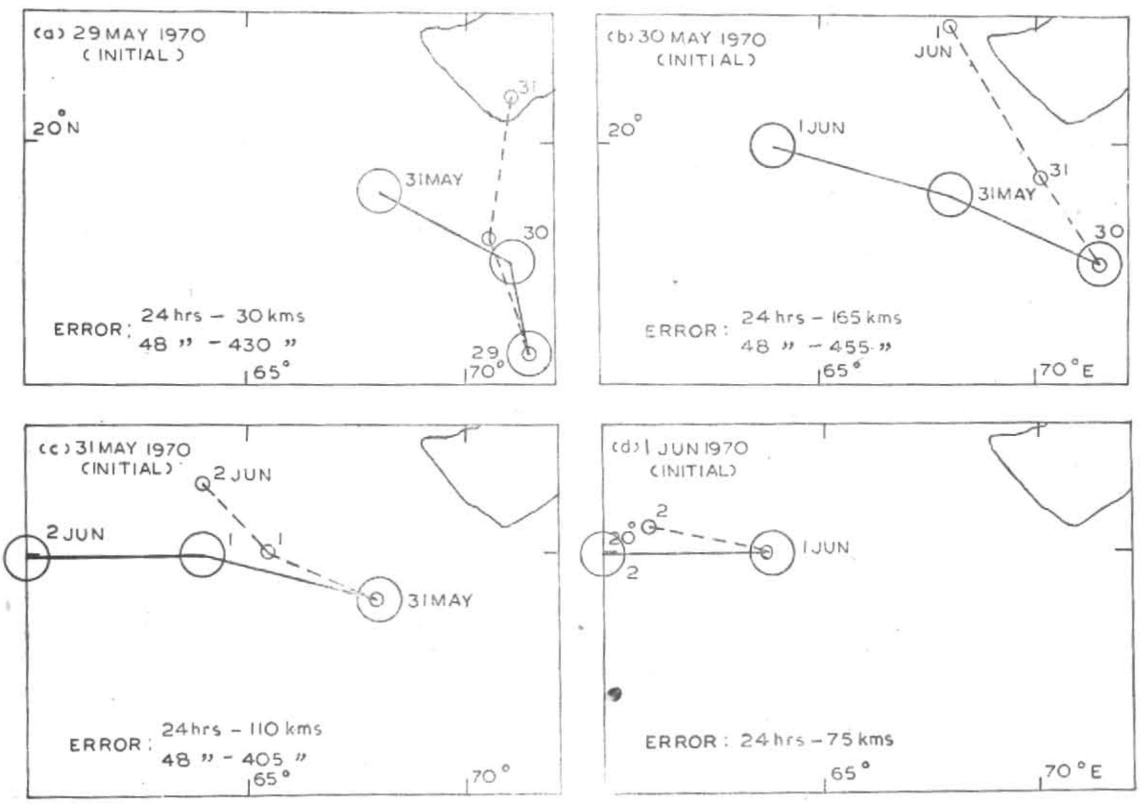
This input was integrated for 24 hours over the model.

(*ii*) The maximum speed was reduced to 20 mps and the integration was performed as in (*i*).



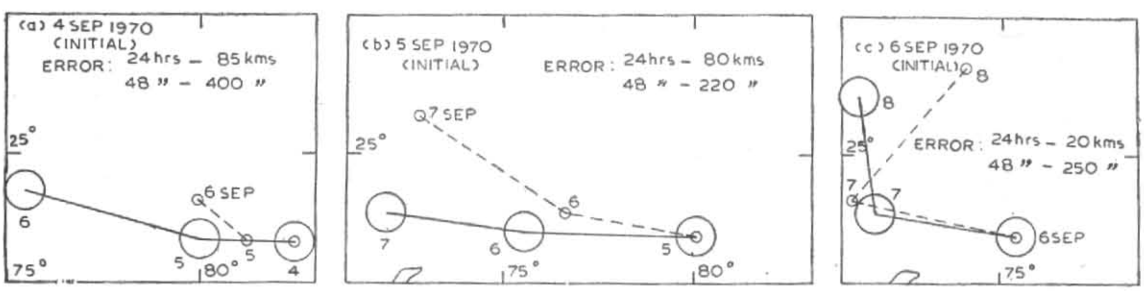
— ACTUAL TRACK, --- FORCAST TRACK

Fig. 1. Storm track prediction P. E. Barotropic model at 500 mb



— ACTUAL TRACK, --- FORECAST TRACK

Fig. 2. Storm track prediction P. E. Barotropic model at 500 mb



— ACTUAL TRACK, --- FORECAST TRACK

Fig. 3. Storm track prediction P .E. Barotropic model at 500 mb

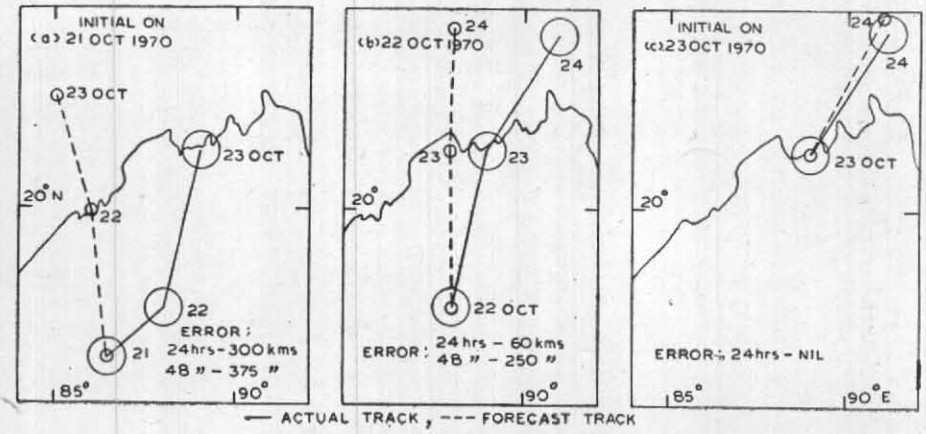


Fig. 4. Storm track prediction P.E. barotropic model at 500 mb

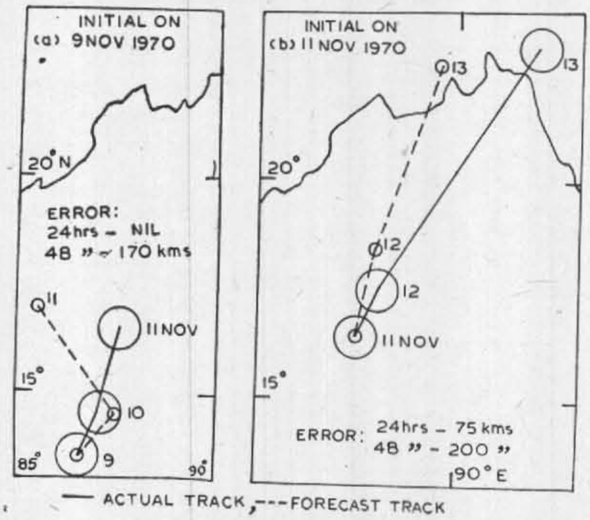


Fig. 5. Storm track prediction P. E. barotropic model at 500 mb

It was found that the movement of the storm was practically the same in both cases except that in (*ii*) the wind field around the vortex was weaker.

(*iii*) In the next series of experiments, a symmetric vortex of known intensity was embedded on a basic uniform westerly flow of known strength. By varying the strength of the basic flow but keeping the intensity of the vortex the same in each case, a number of input charts for the model were constructed. Integrating the various inputs for 24 hours, it was found that the movement of the vortex given by the model was roughly proportional to the strength of the prescribed basic flow.

These findings led us to the development of the point vortex method given in the next section.

6. Point vortex method

6.1. *Details*

Sikka and Ramanathan (1973) reported that the wind analyses on a normal set of observations in the Indian region by different analysts differed only in the representation of a smaller scales. They concluded that it was only the scales greater than 3000 km wave length that could be reasonably assessed with the present netwok of observations.

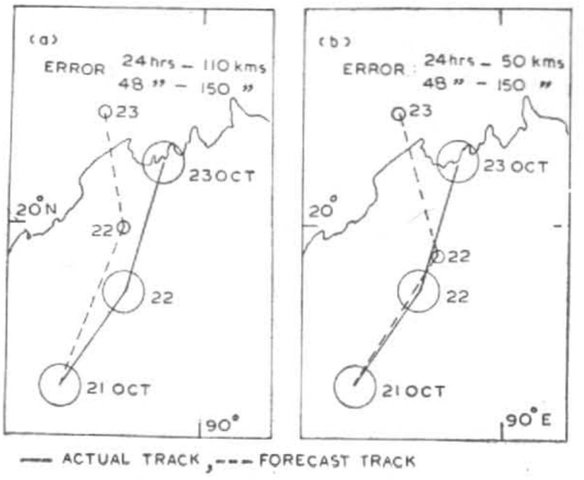


Fig. 6. Storm track prediction P.E. model at 500 mb (Fine mesh)
(a) Elongated vortex with vertical axis
(b) Elongated vortex with axis inclined at 45°

In this method, we subjected the input u and v component fields to double Fourier analysis. For details refer to Ramanatham and Sikka (1972). All the wavelengths less than 3000 km were considered as that due to the vortex and removed. The residue was taken as basic field and then integrated for periods up to 48 hours. Treating the vortex as a point vortex, the centre was advected each hour by the value of the new u and v of the basic field during the integration at the advected centre points. The forecast position of the centre of the vortex was thus computed.

6.2. *Case study*

Results of the application of the method in a typical situation of a storm centred at 88·0°E and 17·5°N on 22 October 1970 are given below :

Time	*Actual position*	*Forecast position*	*Error (km)*
Initial	88·0°E, 17·5°N		
24 hr	88·5°E, 21·5°N	88·5°E, 20·5°N	60
48 hr	90·0°E, 24·0°N	91·0°E, 23·5°N	75

Though this method gave reasonable forecasts in many situations, a difficulty arose when it was applied to the case of the movement of the storm centre at 92·5°E, 15·5°N on 8 Septmber 1972. When the vortex was removed by the method of Fourier analysis, the residual basic flow was very weak during the integration, and the vortex centre advection was practically nil. In such cases non-linear interactions between the basic field and the disturbance field become very dominant and the basic field perhaps could not be separated once and for all from the disturbance field and integrated. The modified point vortex advection method for taking into account the interactions in some way, is described in the next section.

7. Modified point-vortex advection method

7.1. *Assumption for prescribing the vortex field*

In this section we make use of the method proposed by Sasaki (1955). While the vortex was identified by the minimum contour height value in their studies, we use the maximum cyclonic vorticity value. The following assumptions are involved in specifying the characteristics of the vortex field :

(*i*) The vortex was assumed as symmetric,

(*ii*) The central lowest pressure was prescribed from ship's reports and synoptic experience. The maximum wind speed was then determined by Fletcher's formula (Fletcher 1955).

(*iii*) The maxiumum wind occurs at about 10 or 60 nautical miles from the storm centre in the Indian Seas as determined by Koteswaram and Gasper (1956) from climatological studies.

7.2. *Description of the method*

Let the point (x_0, y_0) be the centre of the storm. The initial cyclonic vorticity S at the vortex centre (x_0, y_0) is an extremum and hence

$$\frac{\partial}{\partial x} S\,[\,0, x_0\,(0), y_0\,(0)\,] = 0 \qquad (7.1)$$

$$\frac{\partial}{\partial y} S\,[\,0, x_0\,(0), y_0\,(0)\,] = 0 \qquad (7.2)$$

After a displacement of the centre in $\triangle t$ secs to the point $[x_0(\triangle t), y_0(\triangle t)]$, since the new point is also an extremum, we have

$$\frac{\partial}{\partial x}\left[S\left\{\triangle t, x_0(\triangle t), y_0(\triangle t)\right\}\right] = 0 \quad (7.3)$$

$$\frac{\partial}{\partial y}\left[S\left\{\triangle t, x_0(\triangle t), y_0(\triangle t)\right\}\right] = 0 \quad (7.4)$$

We now define the velocity of displacement of the extremum point as (C_{x0}, C_{y0}), where,

$$C_{x_0} = \lim_{\triangle t \to 0} \frac{\triangle x_0}{\triangle t} \quad (7.5)$$

$$C_{y_0} = \lim_{\triangle t \to 0} \frac{\triangle y_0}{\triangle t} \quad (7.6)$$

where, $\triangle x_0 = x_0(\triangle t) - x_0(0)$ (7.7)

$$\triangle y_0 = y_0(\triangle t) - y_0(0) \quad (7.8)$$

We take $\triangle t$ a small time interval and hence $\triangle x_0$ and $\triangle y_0$ are also of a small order. Expanding in Taylor's series

$$S_x(\triangle t) = S_x(0) + S_{tx}(0)\,\triangle t + \ldots \quad (7.9)$$

where the suffixes denote differentiation.

$$S_x(0, x_0 + \triangle x_0, y_0 + \triangle y_0) = S_x(0, x_0, y_0) + S_{xx}(0, x_0, y_0)\,\triangle x + S_{xy}(0, x_0, y_0)\,\triangle y + \ldots \quad (7\cdot10)$$

So that

$$S_x[\triangle t, x_0(\triangle t), y_0(\triangle t)] = S_x(0, x_0 + \triangle x_0, y_0 + \triangle y_0) + S_{tx}(0)\,\triangle t + \ldots = S_x(0, x_0, y_0) + S_{xx}(0, x_0, y_0)\,\triangle x + S_{xy}(0, x_0, y_0)\,\triangle y + S_{tx}(0)\,\triangle t + \ldots \quad (7\cdot11)$$

Similarly,

$$S_y[\triangle t, x_0(\triangle t), y_0(\triangle t)] = S_y(0, x_0, y_0) + S_{yy}(0, x_0, y_0)\,\triangle y + S_{yx}(0, x_0, y_0)\,\triangle x + S_{ty}(0)\,\triangle t + \ldots \quad (7\cdot12)$$

From (7·11) and (7·12) neglecting higher order terms we have,

$$S_{xx}\,\triangle x + S_{xy}\,\triangle y + S_{tx}\,\triangle t = 0 \quad (7\cdot13)$$

$$S_{xy}\,\triangle x + S_{yy}\,\triangle y + S_{ty}\,\triangle t = 0 \quad (7\cdot14)$$

and

$$C_x = \triangle x/\triangle t = -(1/k)\,(S_{tx}S_{yy} - S_{ty}S_{xy}) \quad (7\cdot15)$$

$$C_y = \triangle y/\triangle t = -(1/k)\,(S_{ty}S_{xx} - S_{tx}S_{xy}) \quad (7.16)$$

where, $k = S_{yy}S_{xx} - S_{yx}^2$ (7.17)

7.3. *Method of computation and case study*

The input grid-point u and v fields were subjected to double Fourier analysis, as before, to remove all the scales whose wavelengths were less than 3,000 km. We get thus a basic $\bar{u}$ and $\bar{v}$ fields and so the basic S field. The area of forecast in all cases was so chosen to have the initial observed storm centre at about the middle of the forecast domain. The storm characteristics were prescribed as in section (7·1.)

The total flow

$$u = \bar{u} + v^*$$
$$v = \bar{v} + v^* \quad (7.18)$$
and so $S = \bar{S} + S^*$

where u^*, v^* and S^* due to the vortex field were kept constant during the forecast period. Initially S_{xx}^*, S_{xy}^* and S_{yy}^* were computed at the centre point and then preserved for use in each time-step.

Hence $S_{ty} = \overline{S_{ty}}$ and $S_{tx} = \overline{S_{tx}}$ (7.19)

The basic field was integrated and at each time-step, new $\overline{S}_{yy}$, $\overline{S}_{xx}$ and $\overline{S}_{xy}$ values were computed. By adding the preserved S_{yy}^*, S_{xx}^* and S_{xy}^* due to the vortex we get S_{yy}, S_{xx} and S_{xy} at the centre point for the time step. The values of S_{ty} and S_{tx} in equation (7·19) were also computed for each time-step. Thus knowing all the terms in the right-hand side of equations (7·15), (7·16) and (7·17) the vortex point speed (C_x, C_y) was computed at each time-step. The vortex centre was now advected with respect to C_x and C_y to get a new centre point for each time-step. The movement of the point vortex for forecast periods upto 48 hours was thus computed.

This method was applied to the storm centred at 15·5°N and 92·5°E on 8 September 1972. This was the case study for which the method in last section failed.

The forecast statistics were as below :

Time	*Actual position*	*Forecast position*	*Error*
Initial	15·5°N, 92.5°E		
24 hr	16.5°N, 88.5°E	16.2°N, 88.5°E	20 n. mile
48 hr	18.0°N, 85.5°E	17.0°N, 87.0°E	110,,

Though the forecasts by this method improved over the point vortex method for this case study, it was noted in other cases that the vortex movement was generally slow compared to the actual cases. One difficulty usually experienced during the application of this method was that the values of S_{xx}, S_{xy} and S_{yy} in the equations (7·15), (7·16) and (7·17) become very small for some time-steps. As the equations in such cases be come indeterminate for the computation of C_x and C_y the vortex at these time steps were moved according to the values of C_x and C_y for the previous time step.

Summing up and outlook for the future

The use of a limited-area primitive equation barotropic model for the prediction of storm tracks was examined. The grid used was 2·5° and the reduction to 1·25° enabled more accurate positioning of the vortex centre and also improved the forecasts. In most of the cases the storm had to be treated as a circular symmetric vortex due to data sparsity in the vicinity of the storm centre. These affected the forecasts, especially when the storm was actually moving more towards the north than west. From experiments with analytical symmetrical vortex embedded on prescribed basic flows, it was realized that the basic flow could be more vital for the movement than the disturbance intensity. The basic flow was then separated from the disturbance flow by double Fourier analysis techniques, and integrated treating the vortex as a point and advecting the point along with the basic flow. Though there were successful forecasts, by this method, soon it became apparent that the interaction of the disturbance flow with the basic flow had to be taken into account during integration. The modified method was more successful but forecasts were still underestimated in some cases.

It is perhaps necessary to reduce the grid-size to 55 km which will reduce the truncation error and increase the accuracy of the phase speed. Also the initial structure of the storm needs to be more accurately defined. Aircraft reconnaissance data and special observational network during some storm seasons will be helpful. The possibility of developing satisfactory methods for separating out the basic flow from the disturbance needs to be explored.

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