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Sea level isobaric analysis by the method of optimum interpolation*

M. C. SINHA, V. S. TIWARI, R. N. GUPTA and B. R. ARORA

Meteorological Office, New Delhi

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सार – गान्डिन अनुकूलतम अन्तवेषण विधि (बालोउसेव इत्यादि 1968 और गांडिन 1969) से समुद्र तल के दाब का विश्लेषण करने के लिए एक प्रोग्राम विकसित किया गया है। इस लेंख में अनुकूलतम अंतवेषण विधि का वर्णन किया गया है तथा अपनाई गई एलगोरिथम को दिया गया है। अन्ततः समुद्रतल दाब क्षेत्र के विश्लेषण के परिणाम दिए गये हैं। यह समायोजनीय ग्रिड आकार वाला सुवाह्य प्रोग्राम है।

ABSTRACT. A programme has been developed to obtain analysis of sea level pressure by the Gandin's optimum interpolation method (Balousev *et al.* 1968 and Gandin 1969). In this paper the method of optimum interpolation is described and the algorithm followed is given. Finally the result of analysis of sea level pressure field are presented. The programme is portable with adjustable grid size.

1. Introduction

There has been a great need for the analysis of isobaric field on sea level by machine method. Surface data distribution is highly asymmetric with high density over habited land areas and very scanty network over deserts, mountains etc. Very few ship observations are received from the sea areas other than shipping lanes. The methods for objective analysis are very sensitive to asymmetry of data. Gandin's optimum interpolation method (Balousev *et al.* 1968 and Gandin 1969) is, however, suited for such a condition. Experiments were made to obtain analysis by optimum interpolation methods. The details of the method, the algorithm followed and the results obtained are discussed in the following sections.

2. Optimum interpolation method

The analysis is so obtained that the root mean square error of the analysis is minimum about the normal values of the field in a statistical sense. In this case normal monthly pressure field is taken and anomaly of all the surface pressure observations are obtained

$$p_i = P_i - \overline{P_i} \tag{1}$$

where i represent the observing stations. The anomaly at the required grid point is expressed as the weighted mean of anomalies at the neighbouring observing stations as :

$$\Sigma\, w_i\, p_i$$

The root mean square error of this estimate will be

$$E = \overline{(p_g - \Sigma\, w_i\, p_i)^2} \tag{2}$$

Assuming that p_i contains δp_i as observational error. The RMS error will be

$$E = \bar{p}_g{}^2 - 2 \sum_{i=1}^{n} w_i\, \overline{p_g p_i} + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j\, \overline{p_i p_j} + \sum_{i=1}^{n} w_i^2\, \delta p^2 \tag{3}$$

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TABLE 1

Distance $\times 10^{-3}$ km	0	0.1	0.2	0.3	0.4	0.5	0.7
μ_{ij}	1.0	0.990	0.970	0.945	0.905	0.876	0.770
Distance $\times 10^{-3}$ km	1.1	1.7	1.9	2.3	2.5	2.7	3.0
μ_{ij}	0.550	0.215	0.045	−0.019	−0.060	−0.090	−0.120
Distance $\times 10^{-3}$ km	3.3	3.7	4.5				
μ_{ij}	−0.120	−0.100	0.000				

$$\text{or } \epsilon = 1 - 2\sum_{i=1}^{n} w_i \frac{\overline{p_g\, p_i}}{\overline{p_g^2}} + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j \frac{\overline{p_i\, p_j}}{\overline{p_g^2}} + + \sum_{i=1}^{n} w_i^2\, \sigma_i^2 \tag{4}$$

considering the variance of p to be same every where we get :

$$\epsilon = 1 - 2\sum_{i=1}^{n} w_i\, \mu_{gi} + \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_{ij}\, w_i\, w_j + + \sum_{i=1}^{n} w_i^2\, \eta_i \tag{5}$$

The values of weights w_i such that ϵ is minimum are obtained by putting partial differentials of ϵ with respect to w_i equal to zero

$$\sum_{j=1}^{n} \mu_{ij}\, w_j + \eta_i\, w_i = \mu_{gi}\,;\; i = 1,\; 2 \ldots\ldots \eta \tag{6}$$

The weights w_i depend on the autocorrelation function μ_{ij} as given by Eqn. 7, on η_i the mean square of the relative error of observations and the dispersion of the station around the grid point. The distances between each of the observational points and the grid point are calculated first. These distances are converted into correlation matrix. The values of w_i can then be calculated by solving the following equation :

$$\begin{bmatrix} \mu_{11}+\eta_1 & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22}+\eta_2 & \cdots & \mu_{2n} \\ \mu_{31} & \mu_{32} & \cdots & \mu_{3n} \\ \vdots & \vdots & & \vdots \\ \mu_{n1} & \mu_{n2} & \cdots & \mu_{nn}+\eta_n \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} \mu_{g1} \\ \mu_{g2} \\ \mu_{g3} \\ \vdots \\ \mu_{gn} \end{bmatrix} \tag{7}$$

Having evaluated w_i the values of the pressure at the grid points can be calculated as

$$p_g = \sum_{i=1}^{n} w_i\, p_i \tag{8}$$

It is assumed that the pressure field is statistically homogeneous and isotropic. Therefore, the correlation functions are independent of the position and direction of location of the points.

The value of correlation function μ_{ij} are interpolated from a table (Table 1 for values of autocorrelation function. The values are taken as Gandin (1969).

The values for auto-correlation functions refer to geopoetntial of isobaric surfaces, but have been used for surface pressure field in this study for the time being. The improvement of these correlation functions will be taken up subsequently.

3. Computation procedures

Monthly normal pressure fields are picked out at all the grid points from normal charts. Thereafter, the normal values are interpolated at all the observational points.

All surface data are read for pressure observations. All high level stations reporting geopotentials are rejected. All pressure values from the ship observations are also taken into account.

Before the actual analysis is done at the grid points the following checking procedure is adopted. At observed data points the pressure value is calculated from observed data at neighbouring stations by the present method of optimum interpolation. If these derived values differ from the actual observed pressure values, reported at the station, by more than a certain pre-assigned amount the observation is rejected. This value was 4.0 mb in the present study.

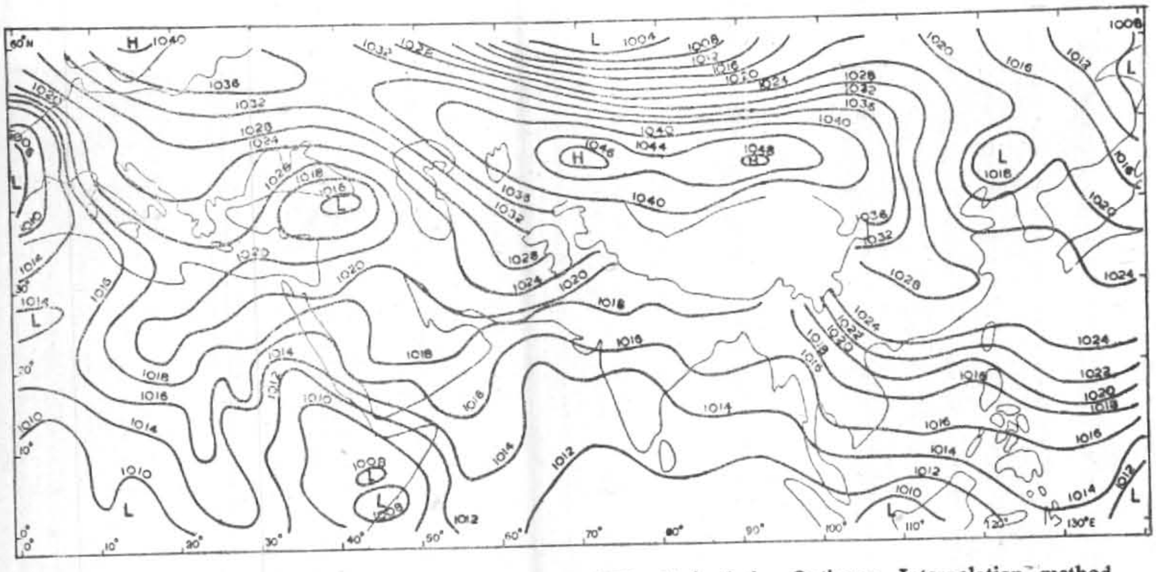

Fig. 1. Sea level isobars for 00 GMT of 21 January 1979 obtained by Optimum Interpolation method

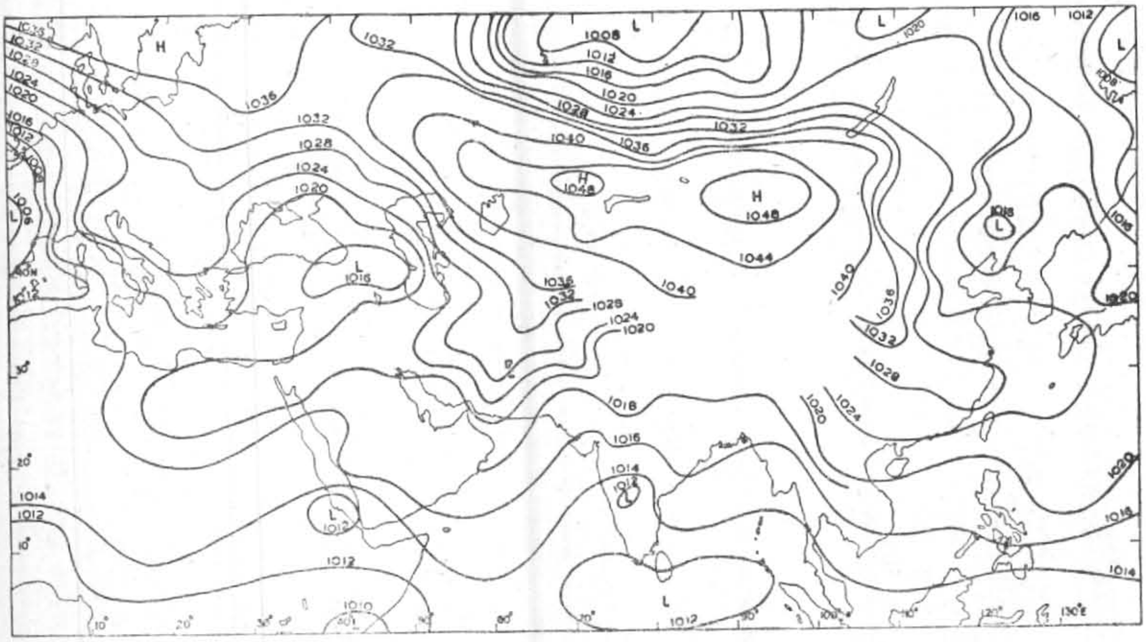

Fig. 2. Sea level isobars for 00 GMT of 21 January 1979 obtained by manual analysis

All stations situated within a distance of 10° (1110 km approx.) from the grid points are scaned and nearest eight stations are taken for the calculations of grid point pressure value. Finally the pressure field obtained is smoothed by nine point smoothing operator (Shuman 1957).

The programme is perfectly portable and can be used anywhere on earth. It is on latitude longitude grid with adjustable resolution. The parameters for area, size and resolution are read as data at object time.

4. Conclusions

The sea level pressure analysis obtained for the 00 GMT of 21 Jan 1979 for the area equator to 60° N and Greenwich meridian to 140° E is given in Fig.1. The correspondence between the machine analysis and the manual analysis given in Fig. 2 is remarkable. All features like lows, troughs are brought out nicely and the central values of the lows and highs are also comparable. The study is in progress and detailed statistical verification of the analysis products will be presented in a subsequent paper.

5. List of Symbols used

E	Root mean square error
ϵ	Mean square of the relative optimum interpolation error
P_i	Actual pressure at observing stations
$\overline{P_i}$	Normal pressure at observing stations
p_i	Pressure anomaly at observing station
p_g	Pressure anomaly at grid point
w_i	Weight factor applied to anomaly at station i
μ_{ij}	Auto correlation function
η_i	Mean square of the relative error of observations
σ_i	Root mean square error of observation at point i
Σ or $\sum$	Summation over nearest neighbouring observations considered for influence

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