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Solution to the problem of flow in wells in confined aquifer of variable thickness

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ABSTRACT. An analytical solution to the problem of steady state flow in a fully penetrating well placed in a confined aquifer of variable thickness is presented. The thickness of the aquifer is assumed to either decrease or increase linearly from the face of the well. It is shown that the available solution for uniform aquifer thickness is a particular case of the solution presented. The percentage error in discharge prediction in assuming the aquifer thickness to be uniform, is calculated and presented.

1. Introduction and the definition of the problem

The problem of steady state flow in a fully penetrating well placed in a confined aquifer of uniform thickness is available in any standard text book on hydrogeology (Davis and De Wiest 1966). It is hard to meet a practical situation where the aquifer thickness is uniform in the entire zone of influence. Hence two cases are visualised where the aquifer thickness are either increasing or decreasing linearly from the well face. The geometrical configuration of the aquifer is assumed to be symmetrical with respect to the well. Fig. 1(a) and Fig. 1 (b) give the definition sketches of the problem considered, where,

q = Steady state discharge

h_w = Water level at the well face

H = Piezometric head at the radius of influence

h = Head at any distance r.

R = Radius of influence

D_0 = Thickness of the aquifer at the well face

D_R = Thickness of the aquifer at the radius of influence.

The problem is to find out the steady state discharge and piezometric pressure distribution for the problems explained.

2. Solution

If D_r is the depth of aquifer at any distance r then assuming Darcy's law and Dupuit's assumptions to be valid and from continuity, one can write

$$\frac{q}{2\pi r D_r} = K \frac{dh}{dr} \tag{1}$$

where K is the permeability coefficient of the aquifer. Again D_r can be expressed as

$$D_r = D_0 + \left(\frac{D_R - D_0}{R}\right) r \tag{2}$$

For decrease in thickness of the aquifer, right hand bracketed portion of Eqn. (2) will be negative. Combining Eqns. (1) and (2)

$$\frac{q}{2\pi r \left(D_0 + D_0 \left(\frac{D_R}{D_0} - 1\right) r/R\right)} = K \frac{dh}{dr} \tag{3}$$

Non-dimensionalising the above expression, one can write

$$\frac{q_*}{x[1 + (G-1)x]} = \frac{H}{R} \frac{dy}{dx} \tag{4}$$

where,

$$q_* = \frac{q}{2\pi R K D_0} \tag{5}$$

Non-dimensional discharge

$$x = r/R \tag{6}$$

$$y = h/H \tag{7}$$

$$G = D_R/D_0 \tag{8}$$

Termed as 'geometry parameter'

The boundary conditions of the problem are given by Eqns. (9) and (10) :

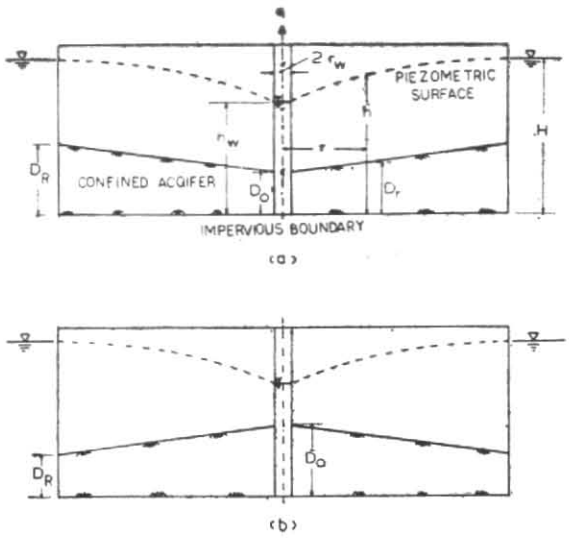

Fig. 1. Definition sketches for the problem

$$\left.\begin{aligned} r &= r_w \\ &\text{or} \\ h &= h_w \end{aligned}\right\} \qquad \left.\begin{aligned} x &= x_0 = r_w/R \\ &\\ y &= y_0 = h_w/H \end{aligned}\right\} \tag{9}$$

and

$$\left.\begin{aligned} r &= R \\ &\text{or} \\ h &= H \end{aligned}\right\} \qquad \left.\begin{aligned} x &= 1 \\ &\\ y &= 1 \end{aligned}\right\} \tag{10}$$

Integrating Eqn. (4) along with the boundary condition (9), one gets the piezometric pressure distribution as

$$y - y_0 = \frac{q_*}{H/R}\left[\log\left(\frac{x}{x_0}\,\frac{1+x_0(G-1)}{1+x(G-1)}\right)\right] \tag{11}$$

Eqn. (11) gives the piezometric head distribution for any geometry parameter $G(=D_R/D_0)$. For aquifer of uniform thickness $G = 1$ and the insertion of $G=1$ in Eqn. (11) leads to the well known piezometric head distribution for uniformly thick confined aquifer.

Combining Eqns. (10) and (11), one gets

$$\frac{H}{R}(1-y_0) = q_* \log\left\{\frac{1+x_0(G-1)}{G_{x0}}\right\} \tag{12}$$

$$\text{But } \frac{H}{R}(1-y_0) = \frac{H}{R}(1-h_w/H)$$

$$= (H-h_w)/R = I_{av} \tag{13}$$

where I_{av} may be called as average gradient. Hence Eqn. (12) can be re-written as

$$q_* = \frac{I_{av}}{\log\left\{\dfrac{1+x_0(G-1)}{G_{x0}}\right\}} \tag{14}$$

Eqn. (14) gives the non-dimensional discharge as a function of average gradient (I_{av}), non-dimensional well radius ($x_0=r_w/R$) and the geometry parameter ($G=D_R/D_0$). This relation is plotted in Fig. 2 for $G=0{\cdot}1$, $1{\cdot}0$ and $10{\cdot}0$. For aquifer of uniform thickness ($G=1{\cdot}0$), Eqn. (14) reduces to the well known q_* *vs* I_{av} relationship as shown in Eqn. (15) below.

$$q_{*n} = \frac{I_{av}}{\log(1/x_0)} \tag{15}$$

where q_{*u} is the non-dimensional discharge for uniform aquifer thickness ($G=1$).

The percentage error (ϵ) in discharge prediction in assuming the thickness of the aquifer to be uniform, can be obtained for various values of geometry parameter G from Eqn. (16) and is plotted in Fig. 3.

$$\epsilon = \left(\frac{q_{*u} - q_*}{q_{*u}}\right) \times 100 \tag{16}$$

3. Discussion and Conclusions

From the foregoing analysis, the following conclusions can be drawn :

(*i*) Available solutions for piezometric head distribution and the variation of discharge with

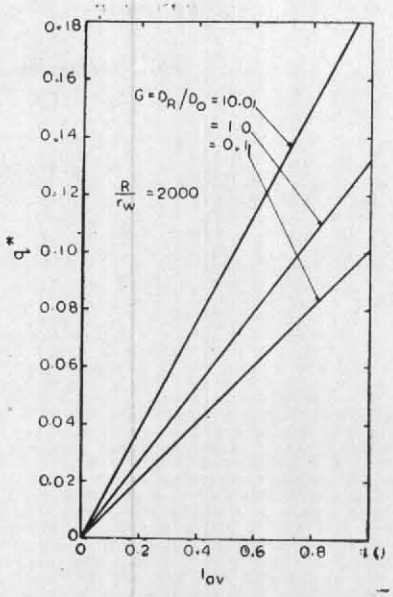

Fig. 2. Variation of non-dimensional discharge with average gradient for different values of geometry parameter

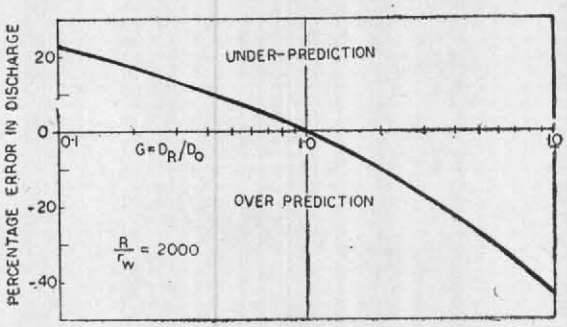

Fig. 3. Error in discharge prediction as a function of geometry parameter

average gradient for uniform aquifer thickness are particular cases of the solutions for variable aquifer thickness as derived in Eqns. (11) and (14).

(*ii*) The discharge-average gradient (q_* *versus* I_{av}) relationship is always linear for all values of geometry parameter G. For $G>1$, the discharge is always more than the discharge predicted on the basis of uniform aquifer thickness and for $G<1$, it is just the reverse.

(*iii*) The error diagram as shown in Fig. 3 gives the magnitude of percentage error involved in discharge prediction based on uniform aquifer thickness. Over prediction by as much as 43 per cent is resulted when $G=D_R/D_0=10$ and an underprediction of 23 per cent results when $G=0{\cdot}1$.

If the geometrical configuration of the aquifer in the field is similar to the one, dealt in this paper, it is hoped that the analysis presented can be used for correct and better insight to the physics and engineering of the problem.

REFERENCE

Davis, S. N. and DeWiest, J. M. 1966 *Hydrogeology*, John Wiley and Sons, Inc., New York.