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The EV_1 distribution for modelling extreme rain events at Bombay

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सार — इस अध्ययन में, नेमी रूप से प्रयुक्त घूर्णों और अधिकतम संभावित (MML) पद्धतियों के साथ-साथ अधिकतम एन्ट्रापी, प्रसंभाव्यतायुक्त (PWM) घूर्णों और समिश्र (MIX) घूर्णों का प्रयोग करके बम्बई में वर्षा की चरम अनुक्रमों से मॉडल EV_1 को भी लगाया गया है और उनकी सापेक्षा उपयुक्तता की तुलना की गई है। प्रेक्षित आंकड़ों से EV_1 प्राचलों और विभाजकों (T-वर्ष घटना) के आकलन में विभिन्न पद्धतियों के लिए आपेक्षित दक्षता का परिकलन यह दर्शाता है कि अधिकतम सम्भाव्यता MML अत्यंत कुशल पद्धति है।

मॉंटकार्लो तकनीक का अनुसरण करते हुए अनुकरण अध्ययन में प्राचलों और अविभाजकों के आकलन में विभिन्न पद्धतियों की दक्षता और एकांगी (बायस), मूल माध्य वर्ग त्रुटि का भी आकलन किया गया है और यह पाया गया कि सामान्यत: PWM न्यूनतम एकांगी आकलन प्रदान करते है न्यूनतम माध्य वर्ग त्रुटि मानों के साथ MML अत्यंत कुशल पद्धति है और MIX सभी प्रकार से सबसे कम सन्तोषजनक पद्धति है।

ABSTRACT. In this study, in addition to routinely used moments (MOM) and maximum likelihood (MML) methods, the EV_1 model has also been fitted to the Bombay rainfall extremal sequences using maximum entropy (MME), probability weighted moments (PWM) and mixed moments (MIX) and their relative suitability compared. Computation of relative efficiency for different methods in estimating the EV_1 parameters and quantiles (T-yr event) from observed data shows that MML is the most efficient method.

The bias, the root mean square error (RMSE) and the efficiency of different methods in estimating the parameters and quantiles have also been computed in a simulation study following the Monte Carlo technique and it is found that, in general, PWM provided least biased estimates, MML with minimum RMSE values was the most efficient method, and MIX was the least satisfactory method on all counts.

1. Introduction

The EV_1, extreme value type I (double exponential or Gumbel) (Gumbel 1958) distribution is one of the most widely used models for the probabilistic characterisation of a variety of extreme hydrometeorological sequences. Although with two parameters and a constant skew 1.1396, the model is not flexible enough to represent a large variety of extreme sequences, there are valid reasons for its extensive use (Jain and Singh 1987). After adopting this model one has to choose the most suitable method from among the various methods of fitting EV_1 model since there is no general consensus for a particular method as best method. For extreme rainfall sequences at Bombay, using estimated parameters by moments and maximum likelihood methods, Singh (1989) found EV_1 as better model than gamma. Here the EV_1 model has been fitted to the Bombay rainfall extremal sequences using five chosen methods of (1) moments (MOM), (2) maximum likelihood (MML), (3) maximum entropy (MME), (4) probability weighted moments (PWM) and (5) mixed moments (MIX). The relative efficiency based on observed data and the biasedness, the efficiency and the root mean square error (RMSE) from a set of simulated data have been computed to identify the most suitable method in order to provide most reliable estimate of the EV_1 parameters as well as quantiles (T-yr event) with the available rainfall data from Bombay.

2. Data used and its statistical properties

The annual maximum sequences of Bombay rainfall for 1, 2, 3, 6, 12, 24 and 48-hr duration have been extracted from the hourly (clock-hour) observations taken at Colaba observatory during 1924-84, details of which are given by Singh (1989). Here we would like to report that though annual rainfall series of Bombay shows increasing trend as reported by Alvi and Koteswaram (1985), the extremal rainfall sequences chosen for the present study are homogeneous and random which are ascertained after applying Mann-Kendall Rank test for randomness against trend and student's t-test for difference in the mean between two equal sub-periods (WMO 1966). The statistical properties like mean, standard deviation and coefficient of variation of different extremal sequences are given in Table 1. Each rainfall series has also been examined for normality aspect by employing Fisher's g-statistic test. The coefficients of skewness (g_1) and kurtosis (g_2) and test statistics g_1/SE(g_1) and g_2/SE(g_2) (SE denotes standard error) are also given in Table 1 which show that different extreme rainfall series are significantly (at 5% level and above) different from normal distribution.

TABLE 1

Statistical properties of extremes of 1, 2, 3, 6, 12, 24 and 48-hr rainfall recorded at Colaba, Bombay from 1924 to 1984

	Hour						
	1	2	3	6	12	24	48
Mean (mm)	57.7	83.9	102.4	137.2	181.9	238.0	322.1
S.D. (mm)	20.9	36.3	46.1	67.1	97.9	124.7	143.1
C. V. (%)	36.2	43.3	45.0	49.0	53.8	52.4	44.4
g_1	1.22	1.81	1.63	1.92	2.35	2.23	1.73
g_2	7.44	4.28	2.85	3.86	5.85	5.36	3.13
g_1/SE(g_1)	3.98	5.90	5.32	6.28	7.67	7.29	5.64
g_2/SE(g_2)	2.38	7.08	4.71	6.39	9.68	8.88	5.18

3. The EV_1 distribution

The probability density function $f(x)$ and distribution function $F(x)$ of EV_1 distribution are as follows :

$$f(x) = [\exp(-(x-u)/\alpha - \exp\{-(x-u)/\alpha\})]/\alpha \quad (1)$$

and
$$F(x) = \exp[-\exp\{-(x-u)/\alpha\}] \quad (2)$$

$$-\infty < x < \infty\,;\ -\infty < u < \infty\,;\ \alpha > 0$$

where α and u are the shape and location parameters of the distribution respectively. In terms of the reduced variate $Y\,[=(x-u)/\alpha]$ Eqn. (2) can be written as :

$$F(x) = \exp[-\exp(-Y)] \quad (3)$$

If X_T is the T-year event value of the variable x, and Y_T the corresponding value of the reduced variate Y, Eqn. (3) can be written as :

$$F(X_T) = 1 - 1/T = \exp[-\exp(-Y_T)] \quad (4)$$

then
$$Y_T = -\ln\ln[T/(T-1)] \quad (5)$$

and
$$X_T = u + \alpha\, Y_T \quad (6)$$

From among the several methods, we have chosen five competetive methods of MOM, MML, MME, PWM and MIX to fit EV_1 distribution to the extremes of Bombay rainfall. These are comparatively good methods and easily amenable to computer applications. Sometimes Gumbel's fitting method is also used for estimating the parameters of EV_1 distribution. Phien (1987) has shown that Gumbel's method was a defective version of the MOM. Hence, it is not attempted in this study. Methods of minimum chi-square and sextile (Jenkinson 1969) are also not used here as they are neither based on sound mathematical grounds nor easily amenable to computer application. The different methods used in the study are briefly described below :

3.1. *Method of moments (MOM)*

The moments estimators of the parameters α and u of the EV_1 distribution are given by :

$$\hat{\alpha} = S\sqrt{6/\pi} = 0.7797\ S \quad (7)$$

$$\hat{u} = \bar{x} - S\,v\sqrt{6/\pi} = \bar{x} - 0.45\ S \quad (8)$$

where $\hat{\alpha}$ and $\hat{u}$ are the estimates of α and u respectively; $v = 0.5772$ (the Euler's constant).

As can be seen, this method essentially requires mean $\bar{x}$ and standard deviation S of the sample.

For this method, the formula for estimating the variance of X_T is :

$$\mathrm{var}(\hat{X}_T) = \frac{\hat{\alpha}^2}{N}\left[1.168 + 0.192\,Y_T + 1.10\,Y_T^2\right] \quad (9)$$

Phien (1987) points out that in MOM parameter estimation scheme, $\hat{\alpha}$, underestimates α, $\hat{X}_T$ underestimates X_T and $\hat{u}$ overestimates u.

3.2. *Method of maximum likelihood (MML)*

Mathematical details of this method are given by Clarke (1973) and Phien (1987). The formula for estimating variance of quantiles is :

$$\mathrm{var}(\hat{X}_T) = (\hat{\alpha}/N)\left[1.109 + 0.514\,Y_T + 0.608\,Y_T^2\right] \quad (10)$$

Phien (1987) has suggested some corrections to Eqn. (10) for small samples. Ours is a sufficiently large sample size (= 61), therefore, Phien's corrections are not considered.

3.3. *Method of maximum entropy (MME)*

Jowitt (1979) has provided a method of parameter estimation for EV_1 distribution based on the principle of maximum entropy. Phien (1987) has given an alternate solution for MME estimates of the EV_1 parameters.

The formula for estimating variance of quantiles is (Phien 1986) :

$$\mathrm{var}(\hat{X}_T) = (\hat{\alpha}^2/N)\left(1.115 + 0.545\,Y_T + 0.645\,Y_T^2\right) \quad (11)$$

3.4. *Method of probability weighted moments (PWM)*

The concept of PWM was introduced by Greenwood *et al.* (1979) as a general method of parameter estimation for those distributions whose distribution function $F = F(x)$ and its inverse form $x = x(F)$ are explicitely defined. The EV_1 is one such distribution, its distribution function $F = F(x)$ is given by Eqn. (2) and $x = x(F)$ as :

$$x = u - \alpha\ln(-\ln F) \quad (12)$$

In this study $\hat{\alpha}$ and $\hat{u}$, the PWM estimates of α and u, respectively, are obtained following the method of Landwehr *et al.* (1979) :

$$\hat{\alpha} = (2\,b_1 - b_0)/\ln 2$$

$$\hat{u} = b_0 - v\,\hat{\alpha} \quad (13)$$

where, $b_0 = E(x) = \bar{x}$ and $b_1 = (1/N)\,\Sigma\,(i-1)\,x_i/(N-1)$

$i = 1, 2, \ldots\ldots\ldots\ldots, N$ the rank of x_i after arranging in ascending order.

TABLE 2

Efficiency of different methods in estimating parameters and quantiles with respect to MML method based on observed data

Method	α	U	X_2	X_5	X_{10}	X_{20}	X_{50}	X_{100}	X_{200}	X_{500}
					1-hour					
MOM	0.47	0.81	0.85	0.71	0.64	0.60	0.57	0.55	0.54	0.53
MML	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MME	0.89	0.94	0.93	0.91	0.91	0.90	0.90	0.90	0.90	0.90
PWM	0.65	0.87	1.13	1.24	1.10	1.00	0.92	0.88	0.85	0.81
					12-hour					
MOM	0.30	0.51	0.54	0.45	0.40	0.38	0.36	0.36	0.34	0.33
MML	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MME	0.80	0.85	0.84	0.83	0.81	0.81	0.81	0.81	0.81	0.81
PWM	0.55	0.73	0.96	1.05	0.93	0.85	0.78	0.74	0.71	0.69
					48-hour					
MOM	0.40	0.70	0.72	0.60	0.54	0.51	0.48	0.47	0.40	0.46
MML	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MME	0.87	0.92	0.91	0.89	0.88	0.88	0.88	0.88	0.88	0.88
PWM	0.62	0.83	1.09	1.19	1.05	0.96	0.88	0.84	0.81	0.78

The PWM estimators of the variance of the parameters and quantiles are as follows (Phien 1987):

$$\mathrm{var}\,(\hat{\alpha}) = (\hat{\alpha}^2/N)\,(0.8046\,N - 0.1855)/(N-1),$$

$$\mathrm{var}\,(\hat{u}) = (\hat{\alpha}^2/N)\,(1.1128\,N - 0.9066)/(N-1)$$

and $\mathrm{var}\,(\hat{X}_T) = (\hat{\alpha}^2/N)\,[(1.1128\,N - 0.9066) - (0.457\,N - 1.1722)\,Y_T + (0.8046N - 0.1855)\,Y_T^2]/(N-1)$ (14)

3.5. *Method of mixed moments* (*MIX*)

Details of this method have been provided by Jain and Singh (1987). In this method first moment of the EV_1 and ln EV_1 distributions are used. On simplification, the MIX estimators of the parameters are as follows :

$$\hat{a} = 1.28255\,/\,S_x$$

and $\exp\,(\hat{a}\hat{u}) = 1 + \hat{a}\bar{x} + \hat{a}^2/2\left[\left(S_x^2 + \bar{x}^2\right)\right]$ (15)

where $\bar{x}$ and S_x^2 are mean and variance of x respectively and $\hat{a} = 1/\hat{\alpha}$.

The formulae for the variance of the parameters and quantile estimates for this method are not yet available.

4. Results

The χ^2 and K-S tests of goodness-of-fit have shown that EV_1 distribution fitted by five different methods gave, by and large, equally satisfactory fit to the extremes of Bombay rainfall. The estimated parameters as well as quantiles up to the return period of 500-yr for extreme rainfall of 1, 2, 3, 6, 12, 24 and 48-hr durations by different methods are given in Table 3. The corresponding figures given by the different methods are quite comparable. The problem now is to choose the most suitable unified method of fitting EV_1 model to the extreme rainfall of different durations at Bombay.

4.1. *Relative efficiency*

A particular method is treated as more efficient when compared to others in estimating the EV_1 parameters and its quantiles if the variances of the parameters and quantiles estimated by the method are relatively low. When more than two methods are to be compared, the method with the least variance is chosen as the base method for comparison. The efficiency of a particular method over the base method in estimating a given parameter $\hat{\alpha}$ (for instance) is calculated as follows :

$$E_{ff} = \frac{\left[\mathrm{var}\,(\hat{\alpha})\right]_b}{\left[\mathrm{var}\,(\hat{\alpha})\right]_p} \qquad (16)$$

where subscripts b and p denote the base method and particular method, respectively. If the efficiency, E_{ff} is greater than unity, the particular method is deemed more efficient than the base method.

An examination of the standard errors (not given here) of the parameters and quantiles revealed that they were least in case of MML. Taking MML as the base method the E_{ff} of different methods has been calculated. The E_{ff} for parameters and quantiles of different methods are given in Table 2 for chosen rainfall series of 1, 12 and 48-hr durations. It is seen from the table that, in general,

TABLE 3

Estimates of the EV_1 parameters and quantiles (in mm) obtained by different methods for different rainfall series

Method	α	U	X_2	X_5	X_{10}	X_{20}	X_{50}	X_{100}	X_{200}	X_{500}
					1-hour extreme rainfall					
MOM	16.5	48.1	54.2	72.9	85.3	97.2	112.5	124.1	135.5	150.7
MML	15.3	48.5	54.1	71.3	82.8	93.8	108.0	118.7	129.3	143.3
MME	15.7	48.6	54.4	72.1	83.9	95.2	109.8	120.7	131.6	146.0
PWM	16.3	48.2	54.2	72.7	85.0	96.8	112.0	123.4	134.8	149.8
MIX	16.4	40.1	46.0	64.6	76.9	88.7	103.9	115.3	126.7	141.7
					2-hour extreme rainfall					
MOM	28.5	67.4	77.9	110.2	131.6	152.1	178.7	198.6	218.5	244.7
MML	24.5	68.8	77.8	105.5	123.9	141.5	164.3	181.4	198.4	220.9
MME	25.5	69.2	78.5	107.4	126.5	144.8	168.5	186.3	204.0	227.4
PWM	26.8	68.4	78.2	108.6	128.8	148.1	173.1	191.9	210.5	235.2
MIX	28.3	62.7	73.1	105.2	126.4	146.8	173.1	192.9	212.6	238.5
					3-hour extreme rainfall					
MOM	36.2	81.5	94.8	135.9	163.0	189.1	222.8	248.1	273.3	306.6
MML	30.7	83.2	94.5	129.2	152.3	174.4	202.9	224.2	245.7	273.9
MME	32.3	83.8	95.6	132.2	156.5	179.7	209.8	232.4	254.8	284.5
PWM	34.4	82.6	95.2	134.2	160.0	184.8	216.9	240.9	264.8	296.4
MIX	35.9	77.9	91.0	131.7	158.7	184.6	218.0	243.1	268.1	301.1
					6-hour extreme rainfall					
MOM	52.8	106.7	126.0	185.8	225.4	263.4	312.6	349.4	386.1	434.6
MML	41.6	110.3	125.6	172.7	203.9	233.8	272.5	301.5	330.5	368.6
MME	44.6	111.4	127.8	178.3	211.8	243.9	285.6	316.7	347.8	388.7
PWM	48.0	109.4	127.0	181.5	217.5	252.1	296.9	330.4	363.8	407.9
MIX	52.3	108.0	127.2	186.5	225.8	263.5	312.2	348.8	385.2	433.8
					12-hour extreme rainfall					
MOM	76.9	137.5	160.7	253.0	310.8	366.2	437.1	491.7	545.2	615.9
MML	56.5	145.0	165.7	229.8	272.2	312.8	365.5	404.9	444.2	496.0
MME	61.3	146.6	169.1	238.5	284.5	328.6	385.7	428.4	471.0	527.3
PWM	65.8	144.0	168.1	242.7	292.0	339.4	400.7	446.6	492.4	552.8
MIX	76.4	149.1	177.1	263.6	320.9	375.8	447.0	500.3	553.4	623.5
					24-hour extreme rainfall					
MOM	98.0	181.4	217.4	328.5	402.1	472.7	564.0	632.5	700.7	790.7
MML	73.1	190.4	217.1	300.0	354.8	407.4	470.5	526.5	577.3	644.4
MME	79.1	192.3	221.3	311.0	370.4	427.4	501.1	556.3	611.4	684.0
PWM	84.8	189.1	220.2	316.2	379.9	440.9	519.9	579.1	638.1	715.9
MIX	97.2	192.8	228.5	338.7	411.7	481.7	572.3	640.2	707.8	797.1
					48-hour extreme rainfall					
MOM	112.5	257.2	298.4	425.9	510.4	591.3	696.2	774.7	852.9	956.2
MML	95.4	263.3	298.3	406.5	478.1	546.8	635.7	702.3	768.7	856.3
MME	99.4	264.8	301.2	403.9	448.4	559.9	652.5	721.9	791.1	882.3
PWM	104.5	261.8	300.1	418.6	496.9	572.2	669.6	742.5	815.2	911.1
MIX	111.6	243.6	284.5	411.0	494.7	575.0	679.0	756.9	843.5	936.9

TABLE 4

Results of the simulation study for the chosen rainfall series of 1, 12 and 48-hr duration

Parameter	Bias (%)					RMSE (%)					EFFI (%)				
	MOM	MML	MME	PWM	MIX	MOM	MML	MME	PWM	MIX	MOM	MML	MME	PWM	MIX
1-hour extreme rainfall															
α	0.7	1.6	0.8	0.3	1.7	15.1	10.9	11.7	12.6	14.8	52.5	100.0	87.7	75.7	54.5
u	−0.5	−0.2	−0.4	0.2	9.2	5.1	4.6	4.9	4.9	12.2	79.9	100.0	89.3	88.5	14.2
x_{100}	0.2	0.9	0.5	0.3	4.1	9.7	7.3	7.8	8.4	12.9	55.9	100.0	86.5	75.6	31.4
x_{500}	0.3	1.0	0.5	0.3	3.7	10.6	7.9	8.4	9.1	13.3	54.6	100.0	86.6	75.2	35.1
12-hour extreme rainfall															
α	1.3	1.6	1.4	0.3	1.6	14.2	11.3	11.1	12.6	15.1	63.7	100.0	104.5	80.5	56.1
u	−0.7	−0.4	0.2	0.2	−2.3	8.3	5.9	6.1	6.9	9.7	49.7	100.0	93.4	71.0	36.3
x_{100}	0.7	0.9	1.0	0.3	0.2	10.8	8.1	8.1	9.3	13.2	56.5	100.0	99.8	75.2	37.5
x_{500}	0.8	1.0	1.1	0.3	0.5	11.4	8.6	8.6	9.9	13.6	57.3	100.0	100.6	75.9	40.5
48-hour extreme rainfall															
α	0.7	2.2	1.0	−0.5	2.6	14.5	11.1	11.3	12.5	14.1	58.5	100.0	95.9	78.7	62.3
u	−0.3	−0.1	−0.4	−0.1	3.9	6.7	5.3	5.9	5.9	9.3	63.6	100.0	83.0	82.5	33.1
x_{100}	0.4	1.3	0.5	−0.4	3.0	10.1	7.9	8.1	8.8	12.3	59.9	100.0	94.3	80.1	40.6
x_{500}	0.5	1.5	0.6	−0.4	2.9	10.9	8.4	8.6	9.4	12.7	59.0	100.0	94.9	79.4	44.0

MML is the most efficient method. Based on efficiency the four methods in order of decreasing suitability can be arranged as MML, MME, PWM and MOM. MIX method could not be tested for its efficiency since variance formula for its estimates is not known.

4.2. *Simulation study*

A simulation study has also been undertaken to evaluate bias, root mean square error (RMSE) and efficiency (E_{ff}') of different methods in estimating the parameters and quantiles of the EV_1 distribution. In order to avoid more computer time only 1000 data sets, each set of sufficiently large size 50, were generated. The formula for generating EV_1 random variables is as follows :

$$x = u - \alpha \ln [- \ln U] \tag{17}$$

where U is the uniformly distributed random number in the interval 0 to 1. The Fortran library function of NEC-S-1000 computer system of the National Informatics Centre, Pune (India) is used for generating U. The values u and α are considered to be the population parameters which have been estimated from observed hourly extreme rainfall data by different methods and are used to generate x.

From the generated sample of size 50, $\hat{\alpha}$ and $\hat{u}$ have been estimated by different methods for each of the $N(= 1000)$ replications. The bias and RMSE have been calculated using the following formulas (as an example for parameter α) :

$$\text{Bias} = (1/N) \sum (\alpha - \hat{\alpha})/\alpha$$

and
$$\text{RMSE} = \left[\left(1/N \right) \sum \left\{ \frac{\alpha - \hat{\alpha}}{\alpha} \right\}^2 \right]^{\frac{1}{2}} \tag{18}$$

The bias and RMSE of different methods in estimating the parameters and quantiles have been calculated for each of the rainfall series under investigation. For brevity, these indices for parameters u and α and T-yr events of x_{100} and x_{500} are given in Table 4 as percentage of the population value for chosen series of 1, 12 and 48-hr durations.

A summary of the results is as follows :

(*i*) In general, the bias is minimum for PWM and maximum for MIX. In terms of the bias, the five methods can be arranged as PWM, MOM, MME, MML and MIX from best to worst.

(*ii*) Based on RMSE values, the five methods in order of decreasing suitability can be arranged as MML, MME, PWM, MOM and MIX.

Besides bias and RMSE, in this simulation study mean square error (MSE) of the parameters and quantiles of different methods are also computed for evaluating efficiency of the methods. The relative efficiency (E_{ff}') for the parameter α is given by :

$$E_{ff}' = \frac{\left[\text{MSE} (\hat{\alpha}) \right]_b}{\left[\text{MSE} (\hat{\alpha}) \right]_p} \tag{19}$$

where b and p denote the base and particular methods, respectively, and MSE= RMSE^2, the root mean square error.

Taking MML as the base method, the efficiency of different methods in estimating the parameters and quantiles has been calculated for different rainfall series. For the chosen rainfall series of 1, 12 and 48-hr durations, the percentage efficiency of different methods for $\hat{\alpha}$, $\hat{u}$, $\hat{x}_{100}$ and $\hat{x}_{500}$ are given in Table 4. In terms of efficiency

MML is the best method, the other four methods can be arranged as MME, PWM, MOM and MIX in order of decreasing suitability.

5. Discussion and conclusions

The highest efficiency as indicated by the minimum RMSE value is shown by MML, followed by MME as the second best. MIX is the least satisfactory method on all counts. Computation of the efficiency indices E_{ff} and E_{ff}' has led to the same conclusion, which suggests that E_{ff} based on variances and covariances of the parameters could well be used to evaluate efficiency of different methods in estimating EV_1 parameters and quantiles rather than E_{ff}' whose computation involves a cumbersome Monte Carlo simulation process.

On the whole, performance of the MML seems to be the best, though MME can also be used with comparable efficiency. Lowery and Nash (1970), however, indicated that MML estimates were biased. If one wishes to improve upon bias of MML estimates Fiorentino and Gabriele (1984) have suggested corrections and called it as Corrected Maximum Likelihood (CML) estimates. The expressions for correcting bias of the MML parameters are as follows :

$$\alpha^* = N\,\hat{\alpha}/(N-0.8)$$

$$u^* = \alpha^* \ln\left[\, N/\Sigma \exp(-x/\alpha^*)\,\right] - 0.7\,\alpha^*/N \quad (20)$$

where α (α^*) and $u(u^*)$ are the MML (CML) estimates of the parameters α and u respectively. This correction can reduce the bias significantly without producing any significant change in the RMSE of the estimates.

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