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DESIGN OF JOINT PROBABILITY MODEL OF RAINFALL DEPTH AND DURATION DURING HEAVY RAINSPELL OVER CALCUTTA

Many authors have studied the rainspell characteristic through different statistical (stochastics) methods. Sivaramakrishnan (1987) has studied the rainspell characteristic over Mohanbari from self-recording raingauge charts data and analysed the intensity and duration of the spell alongwith their time of occurrence. A comparative study of rainfall spells in Bangalore was conducted by Ganesan and Rao (1986) by analysing the autographic rainfall charts by dividing the period of rainfall in a month into three phases. The method suggested by Eagleson (1970) to handle the problem of rainstorm depth and duration, is applied here for the study of heavy rainfall spell over Calcutta. The density function of duration is fitted with the Weibull exponential distribution and the density function of rainfall depth for each class interval of spell duration is fitted by the two-parameter gamma distribution. The usefulness of fitting the two-parameter gamma function is due to the fact that the shape of the density distribution function is changed with the spell duration.

The rainfall data over Calcutta has been obtained and processed from the self-recording rainfall charts of short duration rainspell having criteria fulfilling the condition according to meteorological definition of heavy rainfall analysis (intensity of rainspell 12 mm/hr). 698, in number of such spells have been considered for this study over Calcutta (Alipore) during the period, from 1981 to 1986. The statistical parameters of the distribution have been calculated through analytical and graphical methods.

2. *Methodology* — The design of heavy rainfall spell over a certain place can be understood by the modelling from the joint probability distribution function $\phi\ (d,\ t_r)$ of rainfall depth d and the duration of rainfall t_r. This distribution is expressed as the product of two distributions functions, namely, the density function of depth for given duration $\psi\ (d/t_r)$ and the density function of duration $f(t_r)$.

Mathematically it can be expressed as :

$$\phi\ (d,\ t_r) = \psi\ (d/t_r)\, f\,(t_r) \qquad (1)$$

The probability density function of rainspell duration $f\ (t_r)$ can be fitted by Weibull exponential distribution and is given by :

$$f\,(t_r) = \frac{1}{l}\, e^{-(t_r\, -m)/l} \qquad (2)$$

where, the parameters l and m are found out graphically from the probability of occurrences of the duration of the class interval.

The conditional probability density function of rainfall depth for a particular duration $\psi\,(d/t_r)$ is fitted by the two-parameter incomplete gamma distribution, and is given by :

$$\psi\ (d/t_r) = \frac{d^{a-1}\ e^{-d/b}}{b^a\ \Gamma\ (a)} \qquad (3)$$

TABLE 1

Probability distribution of duration class interval of rainspell over Calcutta

Duration t_r (min)	Frequency (f)	Probability (P)	ln (P)
0-10	276	0.395	—0.929
10-20	211	0.302	—1.197
20-30	87	0.125	—2.079
30-40	39	0.056	—2.882
40-50	34	0.049	—3.016
50-60	27	0.039	—3.244
60-90	15	0.021	—3.863
90-120	5	0.007	—4.962
120-150	4	0.006	—5.116

l=33.33 and m=80.66

where, a & b are shape and scale parameters respectively and depend on the type of rainspell and duration. These parameters are determined by the statistical method of mean and variance :

$$\text{Mean} = ba \quad \text{and} \quad \text{Variance} = b^2a$$

3. *Procedure and discussion* — The frequencies of duration of each of the class intervals have been obtained from 698 rainspells over Calcutta using the IMD statistical criteria of heavy rainfall spell. The probabilities of such spell of each of class interval of duration have been found out (Table 1). Class intervals of duration are taken for each 10 minutes interval up to 60 min. and afterwards each class is taken into account 30 min. interval up to 150 min. duration.

The logarithmic probabilities of the class intervals of duration have been plotted against the duration of the corresponding classes and the parameters of the Weibull distribution are calculated from the graph (Fig. 1) with the Eqn. (2) and the probability density of such distribution is given by :

$$f(t_r) = 0.337\ e^{-0\cdot 03}\ t_r \qquad (4)$$

The parameters of the incomplete gamma distribution have been found out for each of the class intervals with the aid of the rainfall amounts of all the rainspells in the class interval of duration. The parameters a & b thus obtained (Table 2) for the duration ranges are used for determining probability density function ψ of the duration range.

The joint probability density function $\phi\,(t_r,\ d)$ of the rainfall amount for each of the duration ranges have been found out from Eqn. (1). The distribution

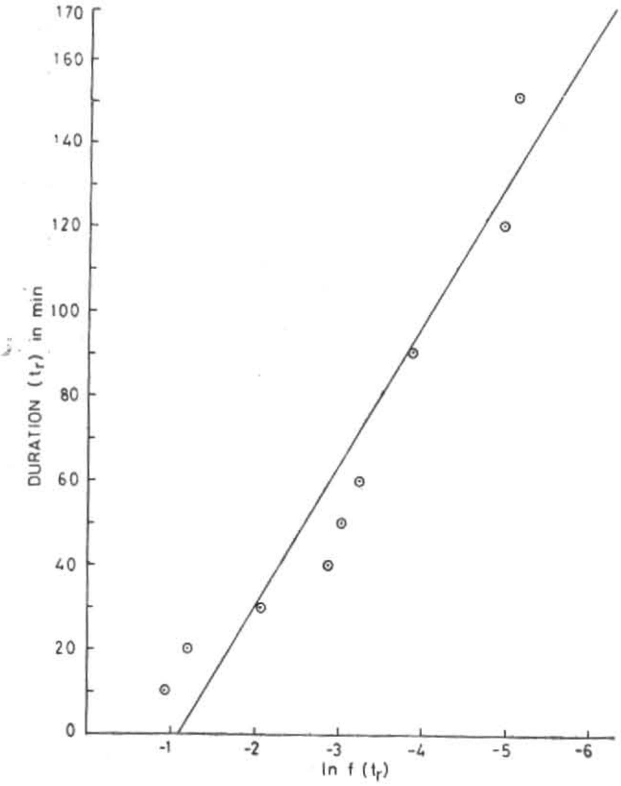

Fig. 1. Weibull distribution of rainfall duration over Calcutta

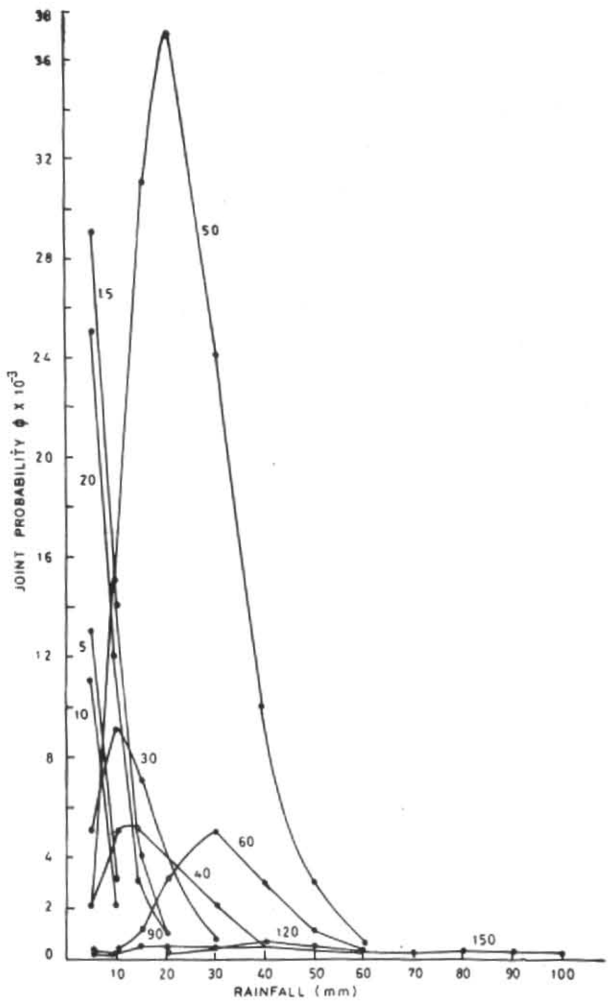

Fig. 2. Joint probability distribution of rainfall depth and duration over Calcutta

TABLE 2

Conditional probability density function of mean rainfall depth for particular class interval duration

Duration t_r (min.)	Mean rainfall depth in the class d (mm)	Standard deviation (σ)	Variance (σ^2)	a	b	$\psi(\bar{d}/t_r)$
0-10	2.63	1.63	2.66	2.60	1.01	0.079
10-20	6.92	4.45	19.80	2.42	2.86	0.111
20-30	13.32	6.66	44.35	4.00	3.33	0.055
30-40	18.66	9.24	85.38	4.08	4.57	0.045
40-50	24.21	10.19	103.84	5.64	4.29	0.445
50-60	31.87	10.42	108.58	9.35	3.41	0.090
60-90	41.17	24.55	602.70	2.81	14.64	0.012
90-120	45.98	14.27	203.63	10.38	4.43	0.054
120-150	81.37	15.90	252.81	26.16	3.11	0.048

functions $\phi\ (t_r,\ d)$ of each of the duration ranges are given by :

$$\phi_{10} = 0.164\, d^{1.60}\, e^{-(0.93d+c)} \quad \text{for } 0<t_r\leqslant 10 \text{ min.}$$
$$\phi_{20} = 0.027\, d^{1.42}\, e^{-(0.35d+c)} \quad \text{for } 10<t_r\leqslant 20 \text{ ,,}$$
$$\phi_{30} = 0.438\times10^{-3}\, d^{3.0}\, e^{-(0.30d+c)} \quad \text{for } 20<t_r\leqslant 30 \text{ ,,}$$
$$\phi_{40} = 1.146\times10^{-4}\, d^{3.08}\, e^{-(0.22d+c)} \quad \text{for } 30<t_r\leqslant 40 \text{ ,,}$$
$$\phi_{50} = 1.517\times10^{-5}\, d^{4.64}\, e^{-(0.23d+c)} \quad \text{for } 40<t_r\leqslant 50 \text{ ,,}$$
$$\phi_{60} = 0.087\times10^{-9}\, d^{8.35}\, e^{-(0.29d+c)} \quad \text{for } 50<t_r\leqslant 60 \text{ ,,}$$
$$\phi_{90} = 0.876\times10^{-4}\, d^{1.31}\, e^{-(0.07d+c)} \quad \text{for } 60<t_r\leqslant 90 \text{ ,,}$$
$$\phi_{120} = 1.820\times10^{-13}\, d^{9.38}\, e^{-(0.23d+c)} \quad \text{for } 90<t_r\leqslant 120 \text{ ,,}$$
$$\phi_{150} = 0.028\times10^{-37}\, d^{25.16}\, e^{-(0.32d+c)} \quad \text{for } 120<t_r\leqslant 150 \text{ ,,}$$

where, $c = 0.03\, t_r$

Now, the probabilities of certain amount of rainfall of certain duration have been calculated by using the above appropriate ϕ-function of the particular class of duration range. Probabilities thus obtained for each of the particular duration are plotted against the rainfall amounts of desired values and curves are obtained for the particular durations, as shown in Fig. 2.

4. *Conclusion* — From the graph of joint probability distribution function (Fig. 2), it is seen that the probabilities of such distribution up to 20 min. duration are decreasing with the increase of rainfall amount. Beyond 20 min. duration, the probabilities of such distribution follow exponential curves with the increase of rainfall amount. The peakedness of the curve decreases for increase of duration-range-curve and the peakedness shifted towards the increasing rainfall amount with the increase of duration-range-curve. The curves almost flatened for the high value of duration-range-curves beyond 60 min. duration where probabilities are very small. The highest probabilities attain in 50 min. duration curve where peak value reaches around the rainfall amount 20 mm for the duration range between 40 min. and 50 min.

By this probability distribution ϕ, the probability of any desired amount of rainfall of any given duration can be calculated by using the above appropriate ϕ-function equation.

The study of this probability distribution of depth for any rainfall amount of a given duration is very important for many engineering-design purposes like installation of plant and structure.

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