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A software package for epicentral determination of near earthquakes

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ABSTRACT . A software package using IBM 360/44 has been developed for determining the epicentral parameters of local crustal earthquakes. The arrival times of both the direct (Pg and Sg) as well as refracted (Pn and Sn) phases recorded at the seismic stations have been utilised. The computation involves the formulation of 3×3 symmetrix matrices for fixed depths and their solution with appropriate weightage. In cases, where the reported arrival times are only for Pg and Sg phases, the depth is also floated and a 4×4 symmetrix matrix is formed and solved.

The results have been compared with those determined earlier using a large scale map (1 cm=1 km) or the U. S. Geological Survey monthly listings. It is seen that the agreement is fairly good when the data coverage is appropriate.

1. Introduction

With the opening of a close network of seismological stations around Pong and Pandoh dams large number of events are being detected which are mostly concentrated in the grid 32°-33°N, 76°-77°E. The determination of the epicentral parameters for these events using manual means has been obviously time consuming. The utility of a computer programme for this purpose was felt all the more necessary after the occurrence of the Kinnaur earthquake of 19 January 1975 which has been followed by more than 3000 aftershocks. These considerations led us to develop a computer programme using IBM 360 model 44 and utilising the near earthquake phases P_g, S_g, P_n and S_n. The seismological observatories utilised for monitoring the near earthquakes are shown in Fig. 1.

2. Description of the programme

The programme is in two parts—Main and Subroutine TIMO. In the Main programme the data are read and sorted out. Subroutine TIMO is called upon to fix up a tentative origin time of the earthquake. The location of the station reporting earliest arrival time is taken as the initial guess for the epicentre and the depth of focus is initially fixed to a certain minimum value. The guess is then optimised by iterative process. This involves formulation of 3×3 symmetric matrix and its solution, followed by calculation of variance and standard errors. In order to ascertain the depth of focus, this process is repeated for various depths, by increasing the depth in steps and finally the solution with minimum variance is selected from amongst those for various depths.

With the selected solution of epicentre, travel time residuals are calculated and based upon these, the weights for each of the individual observations are modified as necessary. The whole process is then repeated until all the residual have converged to 0·5 sec or less, to give a final solution for the epicentre. However, in case the reported arrival times are only for the P_g and S_g phases, the final result is further subjected to improvement by floating the depth as well. A 4×4 symmetric matrix is then formed and solved, which gives a more precise estimate of the depth.

Subroutine TIMO is based upon the well known empirical relation between the observed arrival times of P and S phases at a station and the origin time of the earthquake. The origin times are calculated from all the available P and S pairs of obseravations and checked for extreme values, leading to the rejection of unacceptable S-phase observations. O-time is then calculated by the least squares fit based on the lowest reported arrival time as guess and solving for the error correction. The S-phase observation of any station whose corresponding O-time is not found acceptable, is discarded for the processing of epicentre coordinates as well.

Salient features of the programme are discussed in the following sections.

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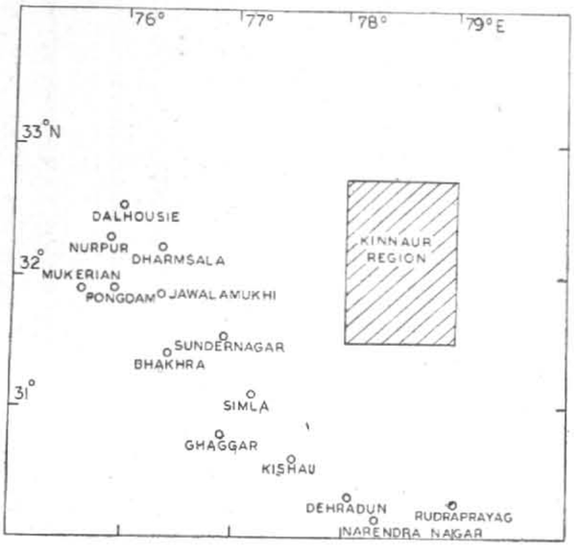

Fig. 1. Seismological network near Kinnaur region

3. Details of computation

The arrival time t_i for a P_g or S_g phase at a station i ($i=1, 2 \ldots\ldots N$) with coordinates X_i, Y_i, Z_i are governed by the relation

$$(X_i-X)^2 + (Y_i-Y)^2 \times (Z_i-Z)^2 = V^2(t_i-t)^2 \quad (1)$$

where X, Y, Z are the coordinates of the focus of the earthquake and t is its origin time. V is the velocity of the concerned phase P_g or S_g as the case may be. If X_o, Y_o, Z_o and t_o are the initial guess and del X, del Y, del Z and del t are the errors we get by substitution, neglecting higher powers of errors

$$W_i(X_i-X_0)\text{ del }X + W_i(Y_i-Y_0)\text{ del }Y + W_i(Z-Z_0)\text{ del }Z - W_iV_i^2(t_i-t_0)\text{ del }t = \tfrac{1}{2}W_i[(X_i-X_0)^2 + (Y_i-Y_0)^2 + (Z_i-Z_0)^2 - V_i^2(t_i-t_0)^2] \quad (2)$$

Here W_i is the weight assigned to each individual observation. For N number of obseravations N such equations are formed (Flinn 1960).

However, for P_n and S_n phases, the equation corresponding to Eq. (1) for P_g and S_g will be

$$(X_i-X)^2 + (Y_i-Y)^2 = V_i^2(t_i - t - k)^2 \quad (3)$$

where k is the intercept of the travel time curve for P_n or S_n as the case may be for a surface source and thus the epicentre is virtually brought on the surface of the earth. The appropriate intercepts based upon the assumed structure of seismic area are calculated from the relation (say for P-phase)—

$$\text{Intercept} = 2h_1\left(\frac{1}{V_P^2} - \frac{1}{V_{Pn}^2}\right)^{\frac{1}{2}} + 2h_2\left(\frac{1}{V_{P*}^2} - \frac{1}{V_{Pn}^2}\right)^{\frac{1}{2}} - Z\left(\frac{1}{V_{Pg}^2} - \frac{1}{V_{Pn}^2}\right)^{\frac{1}{2}}$$

(with similar relationship for S-phase).

and deducted from the corresponding treveal times. Here h_1 and h_2 are the thicknesses of the granitic and basaltic layers, assuming the focus in granitic layer, V_{Pg} , V_{Pn} , and V_{P*} are estimated velocities for the assumed structural layer and Z is the depth of focus. However, the dependence of the intercept on focal depth Z, calls for prefixation of Z instead of keeping it floating. Eq. (2) for P_g and S_g observations then takes the form

$$W_i(X_i-X_o)\text{ del }X + W_i(Y_i - Y_0)\text{ del }Y - W_i V_i^2 (t_i - t_0)\text{ del }t = \tfrac{1}{2} W_i[(X_i - X_o)^2 + (Y_i-Y_0)^2 + (Z_i - Z_0)^2 - V_i^2(t_i - t_0)^2] \quad 5(a)$$

To deal with these observations in alignment with P_n and S_n observations, the equation for P_n and S_n on prefixation of Z, takes up the form :

$$W_i(X_i - X_0)\text{ del }X + W_i(Y_i - Y_0)\text{ del }Y - W_i V_i^2(t_i - t_0)\text{ del }t = \tfrac{1}{2} W_i[(X_i - X_0)^2 + (Y_i-Y_0)^2 - V_i^2(t_i - t_0 - k)^2] \quad 5(b)$$

which is equivalent to Eq. (5a), valid for P_g and S_g and can be used in conjunction with it. The possible range of Z (*i.e.*, Z_{min} to Z_{max}) is therefore initially determined depending upon the assumed structure for the region.

The N equations of the form (5a) and/or (5b) are sclved for various depths Z_{min} to Z_{max} in equal increments (say 5 km). While forming the Eqns. (5a, 5b) a small correction for the station height is also called for. If h is the height of the station above mean sea level, V is the mean crustal velocity, t is the observed arrival time and D is the epicentral distance (in km) it can be calculated from the empirical relation

$$t_{corr} = (h/V)\sqrt{1 - (V/dD/dt)^2}$$

However, in this programme we have applied a simple approximate correction in the travel time data given by

$$t_{corr} = \tfrac{3}{4}\, h/(\text{mean crustal velocity}).$$

The mean crustal velocity has been taken equivalent to that for the P_g phase, Initially a unit weight is assigned to the *P*-phases and 0·75 weight to the *S*-phases and a 3×3 symmetric matrix is formulated in the usual way by multiplying the matrix with its transpose. These equations are then solved by inverse matrix method. With the new guess for the epicentre thus obtained, the process is repeated until the errors become quite small, namely,

$$\begin{aligned} &\text{Del } t < 0\cdot 1 \text{ sec} \\ \text{and} \quad &\text{Del } X + \text{Del } Y < 1 \text{ km} \end{aligned} \tag{6}$$

when the iterations are stopped. In case the above said conditions are not visualised quickly, another check is made for the variance σ_{ITN} which is given by

$$\sigma_{ITN} = {}_{i=1} (\text{Res}_i\ {}^2 w_i) \;/\; (N—4)$$

where Res_i is obtained by substituting the computed values of the epicentral coordinates and O-time *i.e.*, X_0, Y_0 and t_0 in Eq. (5a or 5b) as the case may be.

After each interation (ITN), σ_{INT} is compared with 'σ_{ITN} of the previous iteration, and on occasions when σ'_{ITN} is greater than $\sigma_{ITN\text{-}1}$ the result of (ITN-1) is stored as a 'possible' solution for the epicentre and O-time.

The process is stopped after specified number of iterations, and the solution with lowest variance is picked up from amongst all the 'possible' solutions and accepted as the appropriate solution for the respective depth. The whole process is repeated for various depths and the solution with minimum variance from amongst the appropriate solutions for various depths is taken as the final result.

Travel time residuals are then calculated *vide* Eq. (1) and the weighted residuals are checked for their convergence to 0·5 sec. In case these are higher, the following criteria have been adopted in reassigning the weights to the individual observations.

Residuals ⩾ 10 sec—reject the observation.

10 sec. > Res. ⩾ 6 sec—reject the observations provided rest of the observations do not fall short of 10 observations ; otherwise reduce weight by 1/10th.

6 sec > Res. ⩾ 3 sec reduce weight by 1/6th

3 sec > Res. ⩾ 2 sec ,, ,, ,, 1/4th.

2 sec > Res. ⩾ 1 sec ,, ,, ,, ,,1/2nd.

The idea in not rejecting outright the observation with residuals (bewtween 6 sec and 10 sec) is entirely due to paucity of observations available. The limits of 6 sec, 3 sec, 2 sec and 1 sec are subsequently reduced to 3 sec, 1·5 sec, 1·0 sec and 0·5 sec respectively. Generally even to begin with, the number of available observations is quite small and if some of the observations are to be rejected on this account, the rest of the observations become too few to give a reasonable least squares fit, with evidently such inconsistent observations. The weightage of the observation with such high residuals, however, gets rapidly reduced during the process, which in turn reduces its influence on the final result substantially. The limits chosen are based on some test runs, but are externally controlled and can be changed as required.

The whole process is repeated a specified number of times, but if, the weighted residuals converge to less than or equal to 0·5 sec earlier, processing is stopped forthwith. In case, at any stage, the majority of the residuals are found to be greater than or equal to 3 sec, processing is discontinued immediately and the result at that stage is picked up as such, presuming that the data are not consistent and no further improvement would be possible. Flow-diagram for the whole sequence is given in Fig. 2 and for sub-routine TIMO in Fig. 3.

The calculation of tentative O-time for initial guess is based upon the ratio of *P* and *S* wave velocities which is generally taken as $\sqrt{3}$ (Ichikawa 1965). The arrival times of *P* and *S* waves denoted by t_p and t_s respectively are used to compute the tenative origin time t as follows :

$$(t_S - t)/(t_p - t) = \sqrt{3}$$

or

$$t = (\sqrt{3}t_p - t_S)/(\sqrt{3} - 1)$$

Using this equation, origin time is determined for all the stations for which both *P* and *S* observations are available. Now, if t_o is the approximate O-time and dt_o its error, we have on substitution

$$dt_o = W_i[\sqrt{3}\,(t_p - t_0) - (t_S - t_0)]/(\sqrt{3} - 1) \tag{7}$$

where W_i is the weight assigned to the individual observational pair, depending upon the *P-S* interval. For *N* pairs of *P* and *S* observations, we get a set of *N* linear equations which are solved by the least squares method. For this, the lowest reported arrival time out of the full set of observations is assumed as the approximate O-time and the error correction is then calculated leading to tentative O-time.

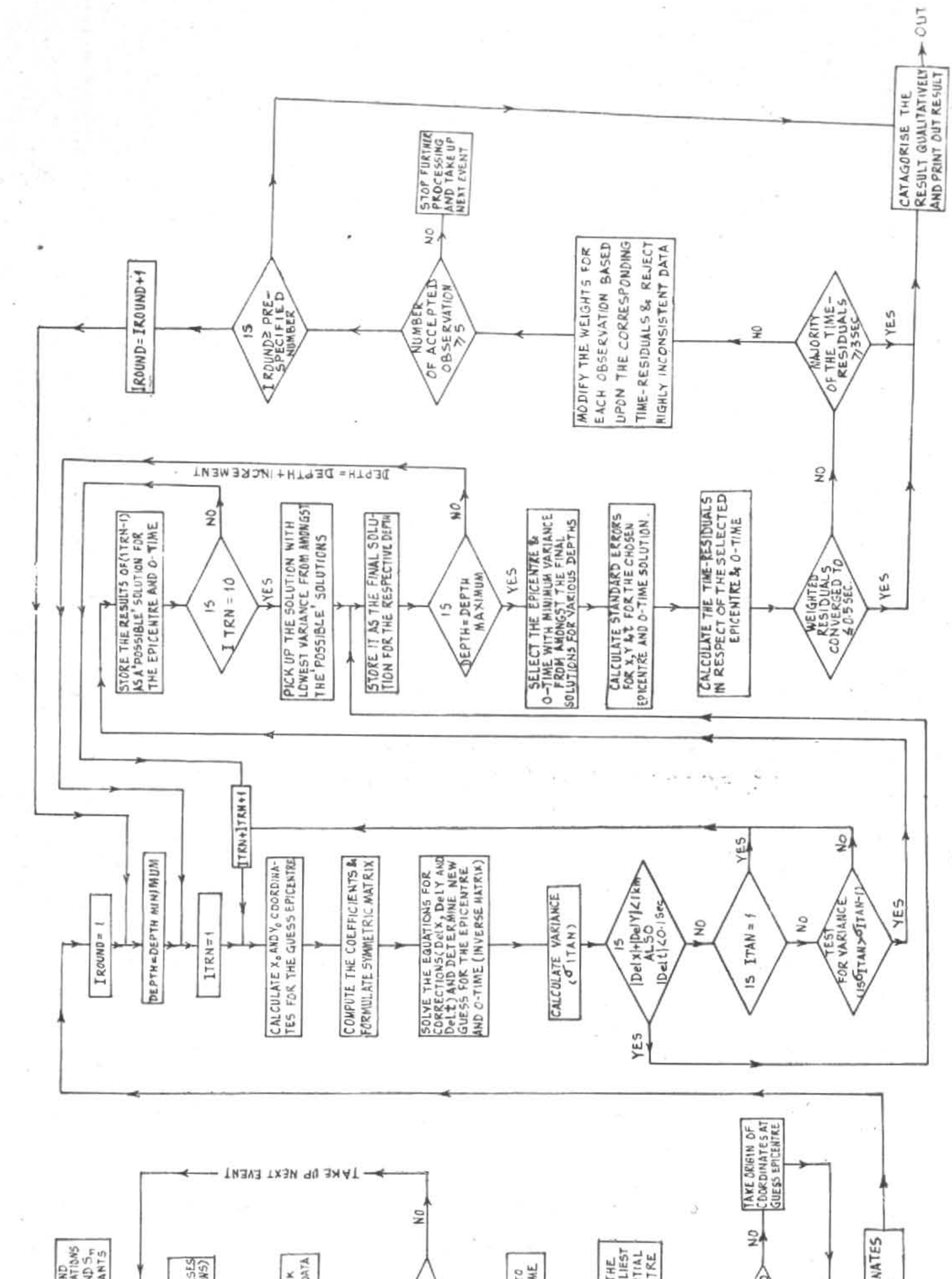

Fig. 2 Main Programme

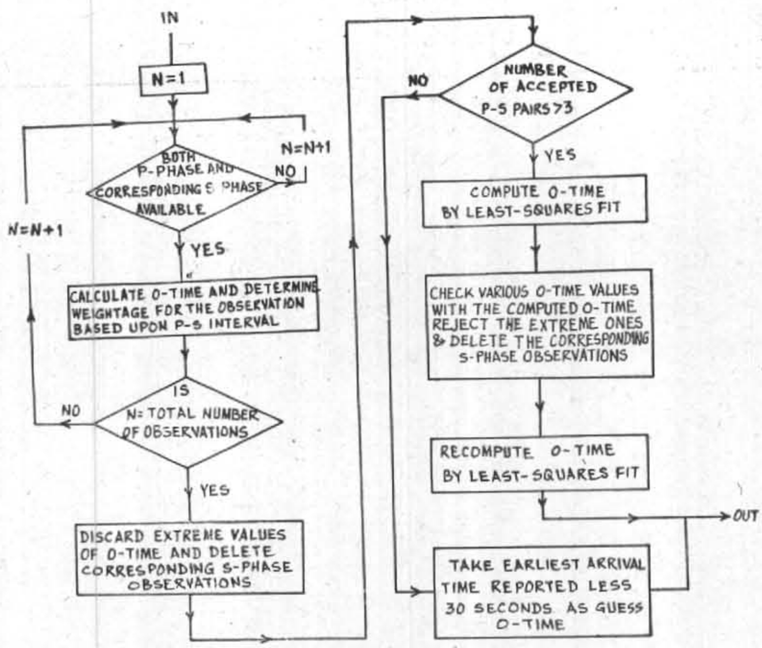

Fig. 3 Sub-routine TIMO

The various O-times pertaining to the individual pairs of observations calculated earlier are now compared with the tentative O-time and corresponding to the extreme values, the S-phase observations are once again rejected for further computation of the epicentre coordinates or the O-time. This screening of the S-phase observations, however, very much depends upon the limit of tolerance chosen, which has to be flexible depending upon the quality of the observations expected in the seismic region under study. Finally, the tentative O-time is improved upon by redetermining it based on the accepted pairs of P and S observations.

If the number of pairs of accepted P, S observations falls short of 3, O-time is not calculated and instead, the earliest arrival time reported is taken as the tentative O-time after arbitrarily deducting 30 seconds from it.

If all the observations belong to only P_g and S_g phase (*i. e.*, the reported observations do not carry any P_n/S_n phase and consequently, the problem of intercept deduction from travel time does not arise) the programme has the provision to further improve the results by floating the depth as well. Eq. (2) is then followed instead of Eqs. (5a, 5b) and 4×4 symmetric matrix is formed and solved instead of the 3×3. The convergence test (Eq. 6) also gets modified to

$$\text{del } Z + \text{del } Y + \text{del } Z < 1 \text{ km}$$

With error corrections for X, Y, Z and t thus incorporated, the final result is more precise for depth.

Provision has also been kept in the programme to directly attempt the floated Z solution in case all the observations are only P_g and S_g.

Conversion of latitude and longitude of stations into rectangualr coordinates and of the epicentral coordinates back to latitude and longitude are carried out with the help of the following mapping functions :

$$X = R_n \ (\Psi - \Psi_o) \ \cos\lambda$$
$$Y = R_m(\lambda - \lambda_o) \ + X^2 \tan \ \lambda_o/2R_n$$

where λ and Ψ are the general latitude and longitude, λ_o and Ψ_o are the latitude and longitude of the origin of coordinates. R_m and R_n are radii of curvature of the earth in the plane of the meridian and in the prime vertical respectively at the latitude of the origin of coordinates. R_m and R_n pertain to the zone of activity under study and can easily be picked up from Geodesy tables.

In the end, the results have also been categorised as A, B, C and D as follows :

Category A : Residuals converged within half the number of rounds of weight modification.

Category B : Residuals converged within the final round of weight modification.

TABLE I

Comparison of Results

Date	Computer Lat. (°N)	Computer Long. (°E)	Computer Depth (km)	Time *h*	Time *m*	Time *s*	Manual Lat. (°N)	Manual Long. (°E)	Manual Depth (km)	Time *h*	Time *m*	Time *s*	USGS Lat. (°N)	USGS Long. (°E)	USGS Depth (km)	Time *h*	Time *m*	Time *s*
17-1-69 (*a*)	32·61	76·50	0·0	19	42	31·7	32·66	76·51	20·00	19	42	31·1						
,, (*b*)	32·62	76·52	0·0	19	42	31·8	,,	,,	,,	,,	,,	,,						
24-1-69 (*a*)	32·40	76·25	15·0	04	10	11·5	32·46	76·30	10·0	04	10	11·2						
,, (*b*)	32·40	76·26	13·4	04	10	11·7	,,	,,	,,	,,	,,	,,						
18-1-70 (*a*)	32·68	76·32	20·0	00	20	41·5	32·74	76·34	15·0	00	20	41·2						
,, (*b*)	32·70	76·32	20·0	00	20	41·4	,,		,,	,,	,,	,,						
20-1-75	32·07	78·46	0·0	13	24	10·1	31·96	78·49	30·0	13	24	12·0	32·13	78·55	N	13	24	12·7
23·1-69 (*a*)	32·51	76·29	0·0	15	16	22·7	32·60	76·35	15·0	15	16	22·4	..	..	..	..	..	..
,, (*b*)	32·59	76·35	0·0	15	16	22·6	,,	,,	,,	,,	,,	,,	..	..	..	..	..	..
27-1-75	32·16	78·57	30·0	8	10	20·8	32·15	78·30	30·0	8	10	21·0	32·46	78·73	N	08	10	18·7
27-1-75	31·94	78·44	5·0	9	23	35·3	32·05	78·67	30·0	9	23	37·0	32·03	78·64	N	09	23	37·7
27-1-75	32·49	78·51	15·0	13	41	38·1	32·45	78·41	30·0	13	41	41·0	32·32	78·50	N	13	41	38·8
27-1-75	32·52	78·25	5·0	15	58	5·5	32·65	78·43	30·0	15	58	06·0	32·72	78·48	N	15	58	06·1
29-1-75	31·88	78·41	10·0	15	49	24·8	31·82	78·29	15·(	15	49	27·0	31·88	78·48	N	15	49	25·9
31-1-75	32.00	78.25	0.0	14	04	12.2	32.45	78.57	30.0	14	04	10.0	32.42	78.48	45	14	04	11.0
10-3-75	32·24	78·54	0·0	3	06	59·9	32·13	78·68	30·0	03	07	02·0	32·21	78·78	N	03	07	3·4
1-4-75	31·84	78·18	0·0	16	18	43·4	..	..	..	..	..	..	32·88	78·50	N	16	18	44·0
6-9-75	32·65	76·44	30·0	13	20	16·9	32·57	76·70	10	13	20	16·5	..	..	..	..	..	..
2-10-75	32·78	76·14	5·0	21	12	34·0	32·88	76·10	10	21	12	32·0	..	..	..	..	..	..

Wherever the observations are confined to *Pg* and *Sg* phases only, the result with floating depth has also been indicated (*b*).

Category C : Residuals did not converge.

Category D : Majority of the residuals $\geqslant 3$ sec at any stage, implying inconsistent data.

Computations for data falling short of 5 observations (P or S) initially or due to deletion of highly inconsistent data during processing are not done and the case is categorised as X.

4. Results and Discussion

In order to test the computer programme developed by us, the epicentres of some of these events whose first arrivals were P_g phases at most of the observations, in the Beas Project region were determined on the basis of the model given by Kamble *et al.* (1974) for Kangra district. The results given in Table 1 show good agreement. Solving the matrix directly for the depth also gave very satisfactory results.

Next, the events with their epicentres in Kinnaur region were selected which were determined earlier(Chaudhury and Srivastava—*see* Ref.) using a different velocity model for the region worked out on the basis of larger aftershocks. For such events, although P_n was the first arrival at most of the observatories, P_g was also first arrival at Sundernagar, Kishau and Rudraprayag for some cases. Thus the programme could be given a test using most of near earthquake phases. Of these, the epicentres for 9 earthquakes were also reported by U. S. Geological Survey using Jeffreys Bullen's travel time tables. The results obtained by different methods were fairly comparable.

The above programme was run with the weight for P_n or P_g phase as one while different weights were assigned ranging from zero to one for S_n or S_g phases. A critical examination of the results showed that a weight of $0{\cdot}75$ for secondary phases may be adopted satisfactorily for the computations. It was noted that if the weights of S-phase were made zero, it was difficult to obtain a reliable solution due to the drastic reduction of the total number of available observations.

Besides, some other limitations of the programme have been noted. The accuracy in the determination of epicentral parameters is highly dependent on the timings of different phases reported by the observatories and their azimuthal distribution. Inspite of several checks incorporated in the programme, reliable solutions may not be available if the data is discrepant and the phases have been incorrectly identified. Thus an error of misinterpretation of S_n as S_g phase, may sometimes lead to inaccurate results if the number of observations is small. In the manual method, if the absolute times of P and S phases are incorrect, $P-S$ interval can be utilised which could not be incorporated in this programme. Similarly attempts should not be made to extend the programme for distances larger than about 400 km because of the limitations inherent in the conversion from spherical to cartesian coordinates. Although more trials are being undertaken for the precise application, it is advisable to try the programme when the data is available for at least five suitably distributed stations.

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REFERENCES

Chaudhury, H. M. and Srivastava, H. N.	—	Proc. 4th World. Conf. Earthq. Engr. (In press)
Flinn, E. A.	1960	*Bull. Seismol. Soc. America*, **50**, *p.* 467.
Inchikawa, M.	1965	*Papers in Met. & Geophys.* (Tokyo), **26**, *p.* 90.
Kamble, V. P., Verma R. K. and Chaudhury, H. M.	1974	*Indian J. Met. Geophys.*, **25**, pp. 229-238.