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Application of split-explicit time integration scheme to a multi-level limited area model and forecast performance over Indian region

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सार — प्रचालन सम्बन्धी उपयोग के लिए विकसित किए जा रहे सांख्यीय मौसम पूर्वानुमान (एन. डब्ल्यू. पी.) मॉडल के लिए एक सस्ती समय समाकलन योजना अत्यंत महत्वपूर्ण है क्योंकि इसकी आवश्यकता सीमित आकलन समय के अन्दर दिए गए कम्प्यूटर का प्रयोग करते हुए उच्च किस्म का पूर्वानुमान देने के लिए होती है। इस शोधपत्र में चर्चित विपाटित स्पष्ट समाकलन योजना स्पष्ट योजना की तुलना में लगभग पांच गुना सस्ती है। उर्ध्वाधर तरीकों के अनुसार इस प्रणाली में मौसम पूर्वानुमानिक समीकरण रॉसबी और गुरुत्वाकर्षण तरीकों को नियंत्रित करने वाले तरीकों में विभाजित होते हैं। यह गुरुत्वाकर्षण मोड के लिए उपयुक्त संशोधन करके बृहत समय पर मॉडल के समाकलन को सुनिश्चित करता है। इस योजना का प्रयोग करते हुए सीमित क्षेत्र मॉडल (एल. ए. एम.) संहति और ऊर्जा के परिरक्षण-गुणों को दर्शाता है। इस योजना में समकारी उच्च बारम्बारता तरंगों के प्रक्रम अंतर्निहित है।

ABSTRACT. An economical time integration scheme is of vital importance for NWP model being developed for operational use as it is required to provide high quality forecasts by using the given computer within limited computing resources. Split-explicit time integration scheme discussed in this paper is near five times economical as compared to explicit scheme. In this scheme prognostic equations are splited according to the vertical modes into those governing the Rossby and gravity modes. It enables integration of the model at larger time step by incorporating suitable correction for gravity modes. The Limited Area Model (LAM) using this scheme displays conserving properties of mass and energy. The scheme has inherent mechanism of smoothing high frequency waves.

1. Introduction

Numerical solution of the atmospheric models for numerical weather prediction involves space discretization and time integration of various terms of the model equations. One of the major numerical aspect of the operational implementation of atmospheric model is to have more accurate, efficient and economical time integration scheme. Keeping in view of this fact, a multitude of time integration schemes have been developed during last two decades. Broadly these fall into four major categories, *i.e.*, explicit, semi-implicit, implicit and split-explicit time integration schemes. The detail reviews of time integration schemes are given by Lily (1965) and Kurihara (1965). Explicit schemes are simple in design and implementation in numerical weather prediction model and, therefore, a number of explicit schemes such as leapfrog, Euler, Matsuno and Lax-Wendroff have been widely used in earlier models. However, these schemes are conditionally stable, which imposes an upper limit on time step that can be used for successful model integration. Thus, an explicit time integration scheme is very much inefficient and unacceptable for fine mesh models. This led to the study and development of time integration schemes where time step could be several times larger than the conventional explicit scheme. Implicit scheme of Marchuk (1964) is the scheme which permits very large time step but during its implementation in a model, the computer time saved in using larger time step is lost in solving the Helmoltz equation. The search for economical time integration scheme led to the development of semi-implicit and split-explicit schemes. Some of the important economical time integration schemes currently used in research and operational model are semi-implicit scheme (Robert *et al.* 1972, 1984), split semi-implicit scheme (Burridge 1975), split-explicit scheme (Gadd 1978, Madala 1981) and some others.

The semi-implicit scheme integrates implicitly those terms that are primarily responsible for the propagation of gravity waves while the remaining terms are integratep in a explicit manner. Most widely used semi-implicit scheme is that of Robert *et al.* (1972, 1984). The implementation of this scheme requires solution of Helmoltz equation at each time step. The algorithm of semi-implicit scheme is easy and economical to apply while dealing with global spectral model due to the fact that the

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harmonic functions are eigen functions of two dimensional Laplacian. However, in the case of grid point model, Helmoltz equations are to be solved iteratively, which takes a lot of computer time. Further lateral boundary conditions pose additional problem in the implementation of such a semi-implicit scheme in a limited area grid point model.

Split-explicit scheme (Gadd 1978, Madala 1981) is designed to take care of the propagation of gravity waves in an explicit manner by integrating gravity wave component with suitable small time step which satisfies Courant Friedrichs Levy (CFL) criteria. The basic concept of the scheme lies in the evaluation of terms representing slow moving meteorological waves (Rossby mode) with larger time step and high frequency gravity wave with sufficiently small time step. Its implementation in model is made efficient by integrating all the modes by the same time step (large time step) and introducing suitable corrections that is for the deviations caused due to integration of fast moving gravity waves at large time step so as to arrive at the same result which would have been given by explicit integration of gravity modes at small time step.

In this paper, attempt is made to study in detail the feasibility and the performance of split-explicit time integration scheme in the framework of a limited area model (Mohanty *et al.* 1989). The details of the split-explicit scheme is given in Sec. 2 and its implementations in a limited area model is presented in Sec. 3. Various numerical experiments conducted in this study are listed in Sec. 4, the results are discussed in Sec. 5 and conclusions are summarised in Sec. 6 of the paper.

2. Split-explicit time integration scheme

2.1. *Governing equations*

The scheme is implemented in a limited area model with horizontal domain of the model bounded by 30° E-120° E and 15° S-60° N with a grid resolution of 1.875° Lat./Long. model has staggerred grid (Arakawa-c) in horizontal and sigma coordinates in the vertical. It incorporates cumulus and planetary boundary layer parameterization, fourth order diffusion and sponge boundary conditions. It has five sigma ($\sigma = p/p_s$) levels in the vertical with $\sigma=0$ as the top and $\sigma=1$ as the bottom boundary. The detail description of the model equations is given by Mohanty *et al.* (1989). In order to facilitate discussion of split-explicit scheme as applied to LAM, the closed system of equations of LAM (Mohanty *et al.* 1989) are given in the matrix form as :

$$\frac{\partial}{\partial t}(p_s u) + \frac{1}{h_x}\frac{\partial \phi}{\partial x} = A_u \qquad (1)$$

$$\frac{\partial}{\partial t}(p_s v) + \frac{1}{h_y}\frac{\partial \phi}{\partial y} = A_v \qquad (2)$$

$$\frac{\partial}{\partial t}(p_s T) + M_2 D_s = A_T \qquad (3)$$

$$\frac{\partial}{\partial t}(p_s q) = G \qquad (4)$$

$$\frac{\partial p_s}{\partial t} + N_2^T D_s = 0 \qquad (5)$$

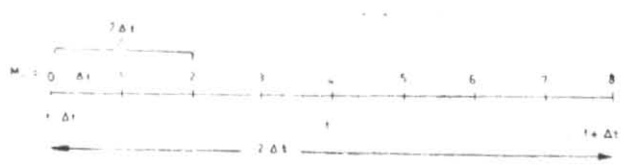

Fig. 1. Division of time interval $2\triangle t$ into m-sub-intervals for use in the split-explicit time integration scheme

$$\phi = M_1 T \qquad (6)$$

$$p_s = N_1 D_s \qquad (7)$$

where, ϕ represents geopotential height from the surface terrain. A_u, A_v, A_T and G contain non-linear terms and forcings. The detail description of each term is given Eqns. (1)-(4) by Mohanty *et al.* (1989).

Further, the symbols with matrix notation represent the following :

$$M_{1ij} = \begin{vmatrix} \alpha_{j-1} + h \text{ if } i < j \\ \alpha_j \qquad \text{if } i = j \\ 0 \qquad \text{if } i > j \end{vmatrix} \quad i, j = 1, 2, \ldots, K_T \qquad (8)$$

where,

$$\alpha_j = \begin{cases} -\dfrac{R}{2} \ln(\sigma^j/\sigma^{j+1}) \text{ if } j < K_T \\ -R \ln \sigma^{K_T} \qquad \text{if } j = K_T \end{cases}$$

$$M_{2ij} = N_3 N_1 + \frac{R}{c_p} T^* N_2^T + (T_1^*) \text{ if } i = j$$
$$0 \text{ if } i \neq j \qquad (9)$$

where,

$$N_{1ij} = -\left(\sum_{L=i+1}^{K_T} \triangle\sigma^L\right)\frac{\triangle\sigma^j}{\sigma_s - \sigma_T} + \triangle\sigma^j \text{ if } i < j$$
$$0 \text{ if } i > j$$

for $i = 1, 2, \ldots\ldots, K_T$ and $j = 1, 2, \ldots\ldots, (K_T - 1)$.

$$N_{2i} = \frac{\triangle\sigma^i}{\sigma_s - \sigma_T} \text{ for } i = 1, 2, \ldots, K_T \qquad (10)$$

and

$$\left(\frac{\sigma_i}{\sigma_o}\right)^{\kappa} \frac{1}{\triangle\sigma^i} \left| \theta^{i+1} \triangle\sigma^i + \theta^i + \triangle\sigma^{i+1} \right|$$
$$\Big/ \left(\triangle\sigma^i + \triangle\sigma^{i+1}\right) \text{ if } i = j$$

$$N_{3ij} = -\left(\frac{\sigma_i}{\sigma_o}\right)^{\kappa} \frac{1}{\triangle\sigma^i} \left| \theta^{i-1} \triangle\sigma^i + \theta^i \triangle\sigma^{i-1} \right|$$
$$\Big/ \left(\triangle\sigma^i + \triangle\sigma^{i-1}\right) \qquad \text{if } i - 1 = j \qquad (11)$$

otherwise

$$= 0$$

where, $\kappa = R/c_p$

Here p_s is the surface pressure, u and v the zonal and meridional wind components respectively, f the coriolis parameter, T the temperature, q the specific humidity $\eta = d\sigma/dt$ the vertical velocity in sigma coordinate, R the gas constant for dry air, ϑ the potential temperature, c_p the specific heat capacity of dry air at constant pressure, D the divergence.

2.2. *Split-explicit method*

Governing equations of motion can be split into two categories of motion, the Rossby modes and the gravity modes. The Rossby modes are non-linear, slow moving and contain most of the atmospheric energy whereas gravity modes are linear, fast moving and contain only small fraction of total energy. The RHS of Eqns. (1)-(6) represent the Rossby modes while on LHS, pressure gradient and divergence terms give rise to gravity modes. The phase speed of the Rossby mode is at least four times slower than the external gravity modes.

Based on these facts, attempts have been made by various workers to carry out time integration with different time steps each satisfying CFL criteria of respective mode. In this category different schemes are : split semi-implicit scheme [Burridge (1975)] and split-explicit schemes [(Gadd (1978), Madala (1981)]. In the scheme devised by Burridge, gravity modes are treated implicitly and Rossby modes explicitly. This significantly reduces the number of elliptical equations which need to be solved. However, its application in model leads to time truncation errors. In Gadd's scheme both the Rossby wave and gravity wave contributions are treated explicitly using two time steps, smaller one for gravity modes and larger one for the Rossby modes. This scheme also suffers from variable time truncation errors. Madala's scheme (1981) provides substantial improvement over Gadd's scheme. In the Gadd's method all the gravity wave contributions are integrated with the same time step while in Madala's scheme each gravity mode is treated separately. In this scheme various tems contribute additively instead of multiplicatively as in ordinary time-splitting methods. This provides a more accurate solution than the earlier two methods. We define CFL time steps $\triangle tg$ and $\triangle tm$ for external gravity mode ($\simeq$ 300 ms^{-1}) and fastest Rossby mode (20 ms^{-1}) respectively and choose a time step $\triangle t$ such as $\triangle t < \triangle tm$. It enables slower meteorological modes (15 & 5 ms^{-1}) to be integrated explicitly with time step $\triangle t$ instead of $\triangle tg = \triangle t$ (in Fig. 1). Integrating Eqns. (1) — (7) from $t - \triangle t$ to $t + \triangle t$ we obtain :

$$p_s u(t+\triangle t) - p_s u(t-\triangle t) + 2\triangle t \frac{1}{h_x}\frac{\partial \overline{\phi}}{\partial x} = 2\triangle t\, A_u(t) \quad (12)$$

$$p_s v(t+\triangle t) - p_s v(t-\triangle t) + 2\triangle t \frac{1}{h_y}\frac{\partial \overline{\phi}}{\partial y} = 2\triangle t\, A_v(t) \quad (13)$$

$$p_s T(t+\triangle t) - p_s T(t-\triangle t) + 2\triangle t\, M_2 \overline{D}_s = 2\triangle t\, A_T(t) \quad (14)$$

$$p_s q(t+\triangle t) - p_s q(t-\triangle t) = 2\triangle t\, G(t) \quad (15)$$

$$p_s(t+\triangle t) - p_s(t-\triangle t) + 2\triangle t\, N_2^T \overline{D}_s = 0 \quad (16)$$

where, the operator (—) is defined as

$$\overline{\beta} = \frac{1}{2\triangle t}\int_{t-\triangle t}^{t+\triangle t} \beta\, dt \quad (17)$$

Since the RHS terms vary slowly over the time scale of Rossby modes, these can be computed once every other $\triangle t$ step, *i.e.*, at time 't'. Thus the RHS terms can be rewritten as $2\triangle t\, A_u(t)$, $2\triangle t\, A_v(t)$, $2\triangle t\, A_T(t)$ and $2\triangle t\, G(t)$ respectively in Eqns. (12)-(17).

The u momentum Eqn. (12) can now be rewritten as :

$$p_s u(t+\triangle t) + 2\triangle t \frac{1}{h_x}\frac{\partial}{\partial x}[\,\overline{\phi} - \phi(t)\,] = p_s u(t-\triangle t) - 2\triangle t \frac{1}{h_x}\frac{\partial}{\partial x}\phi(t) + 2\triangle t\, Au(t) \quad (18)$$

The RHS terms in Eqn. (18) are equal to explicit evaluation of $p_s u(t+\triangle t)$. Thus,

$$p_s u(t+\triangle t) = p_s u^{exp}(t+\triangle t) - 2\triangle t \frac{1}{h_x}\frac{\partial}{\partial x}[\,\overline{\phi} - \phi(t)] \quad (19)$$

Thus split-explicit time integration is equivalent to explicit time integration at a larger time step and $-2\triangle t \frac{1}{h_x}\frac{\partial}{\partial x}\left[\overline{\phi} - \phi(t)\right]$, which is known as correction term. It arise due to integration of gravity modes at large time step. The term is computed from smaller time step.

Similarly, the other prognostic equations can be written in the same manner, where the superscript 'exp' denotes values computed using explicit time integration over $2\triangle$t.

3. Implementation of the split-explicit scheme in LAM

The implementation of split-explicit time integration scheme in a model is carried in four steps. These are :

(*i*) Splitting of equations and computation of vertical modes,

(*ii*) Determination of time steps corresponding to different phase speeds based on CFL criteria,

(*iii*) Estimation of correction terms in eigen space, and

(*iv*) Transformation from eigen space to grid space for integration at large time step.

Of the above four, first two are dependent only on the structure of model (number of levels) and basic state of the atmosphere. Therefore, these are computed only once in the beginning before first time step of the model integration.

3.1. Splitting of equations and computation of vertical modes

While discussing split-explicit method in earlier section, we have seen that RHS of Eqns. (1)-(6) having non-linear terms represent the Rossby modes and on LHS pressure gradient and divergence terms give rise to gravity modes. By combining thermodynamic equation, surface pressure tendency eqation and hydrodynamic equation, we arrive at following set of governing equations:

$$\frac{\partial p_s u}{\partial t} + \frac{1}{h_x}\frac{\partial \phi}{\partial x} = A_u \tag{20}$$

$$\frac{\partial p_s v}{\partial t} + \frac{1}{h_y}\frac{\partial \phi}{\partial y} = A_v \tag{21}$$

$$\frac{\partial \phi}{\partial t} + M_3 D_s = A_\phi \tag{22}$$

where, $M_3 = M_1 M_2 + N_2^T (RT^* - \phi^*)$

$\phi = p_s(\phi - \phi_s) + (RT^* - \phi^*)p_s$

We linearize above equations by setting RHS equal to zero to get the natural modes (eigen modes) of the system :

$$\frac{\partial p_s u}{\partial t} + \frac{1}{h_x}\frac{\partial \phi}{\partial x} = 0 \tag{23}$$

$$\frac{\partial p_s v}{\partial t} + \frac{1}{h_y}\frac{\partial \phi}{\partial y} = 0 \tag{24}$$

$$\frac{\partial \phi}{\partial t} + M_3 D_s = 0 \tag{25}$$

Eqns. (23) and (24) display no vertical coupling as there are no vertical derivatives in these equations. However, Eqn. (25) has vertical coupling because of M_3 matrix. The M_3 matrix represents the thickness of the different layers of the model and their mean temperature structure (please refer Eqns. 25, 8 & 9).

If E represents the eigen vector matrix of M_3 (with each column representing an eigen vector), the variations of dependent variable can be expressed as the linear combination of the structure functions.

Further, diagonal matrix λ is defined as

$$\lambda = E^{-1} M_3 E \tag{26}$$

where, diagonal elements of λ are the eigen values of M_3. Since M_3 contains vertical coupling, the eigen values give the phase speeds of the vertical modes of the model.

3.2. Determination of time step

It is seen from above that the eigen values of M_3 give phase speeds of the vertical modes (natural modes) of numerical model and there are as many natural modes as there are layers in the model. For example, for a five level P.E. model in tropics Eqns. (1)-(7), the five eigen modes have characteristic phase velocities of approximately 300,70,30,15 and 5 ms^{-1}, while Rossby modes moves at about 20 ms^{-1}. Thus we can determine the gravity modes moving faster than the Rossby mode requiring much smaller time steps and the remaining modes moving slower than Rossby mode requiring larger time step. Since the terms on the left hand side of the Eqns. (12)-(16), namely the pressure gradient and the divergence terms, vary over the time steps determined by all the modes, time interval $2\triangle t$ is sub-divided into m sub-intervals of length $2\triangle\tau$ (Fig. 1). For gravity waves with phase speeds (300, 70, 30 ms^{-1}) the value of m for five level model is 8, 4, 2 respectively. Within these sub-intervatls (ime step $2\triangle\tau$) the time integration is carried out explicitly. Then we have

$$\beta = \frac{1}{m}\sum_{n=1}^{m} \beta^n \tag{27}$$

where, $m = \dfrac{\triangle t}{\triangle \tau}$ and $\beta^n = \beta\,(t - \triangle t + 2n\triangle\tau)$

Further it is seen from Eqn. (19) that the results of the future state variables obtained by integrating the equations of motion by using the split-explicit finite difference technique differ from the results obtained by explicit time integration scheme by a deviation term equal to

$$-2\triangle t\,\frac{1}{h_x}\frac{\partial}{\partial x}\left[\bar{\phi} - \phi(t)\right]$$

Similarly for v momentum equation, thermodynamic equation and the surface pressure tendency equation the correction terms are $-2\triangle t\,\dfrac{1}{h_y}\dfrac{\partial}{\partial y}\,[\bar{\phi} - \phi\,(t)]$, $2\triangle t\, M_2(\bar{D} - D(t))]$ and $2\triangle t\, N_2^T\,[\bar{D} - D(t)]$ respectively. Thus in order to use the split-explicit integration scheme we have to evaluate $\bar{\phi}$ and $\bar{D}$.

3.3. Estimation of correction terms in eigen space

The momentum Eqns. (1) & (2) can be combined to give the equation for $\dfrac{\partial D}{\partial t}$ as follows :

$$\frac{\partial}{\partial t} D + \nabla^2 \phi = \frac{\partial}{\partial x} A_u + \frac{\partial}{\partial y} A_v \tag{28}$$

Similarly, the thermodynamic, the surface pressure tendency and the hydrostatic Eqns. (3)-(5) can be combined to get an equation for $\partial\phi/\partial t$ as follows :

$$\frac{\partial \phi}{\partial t} + M_3 D_s = M_1 A_T \tag{29}$$

where,

$$M_3 = M_1 M_2 + N_2^T\,(RT^* - \phi^*).$$

However, there is a practical difficulty in implementing the split-explicit method using the above equation in grid point space. Since different gravity modes of the model will satisfy different CFL criteria, the magnitude of small time step $\triangle\tau$ will differ. Thus, the values of D and $\bar{\phi}$, hence the deviation terms will be different corresponding to different gravity modes. In order to integrate Eqns. (28) and (29) for different gravity modes separately, we have to transform grid point variables to the eigen space variables, where the modes can be treated independently.

The homogeneous system of Eqns. (1)-(5) contain only the gravity waves. These are the natural gravity modes (eigen modes) of the model and their number is equal to the number of vertical layers in the model. These natural gravity modes form a complete set of eigen functions which satisfy the boundary conditions of the model. For obtaining the correction terms for different modes, we express the dependent variables as linear combinations of the structure functions (the respective columns of the eigen vector matrix E) as follows :

$$
\begin{aligned}
u &= E.a \\
v &= E.b \\
T &= E.c \\
D &= E.d \\
\phi &= E.e
\end{aligned} \tag{30}
$$

where, the elements of the coefficient vector a, b, c, d and e are the amplitudes of the eigen modes. We multiply the prognostic equations by E^{-1} and using the relationships between dependent variables and the amplitude of the eigen modes, we obtain the following spectral equations for the wave amplitudes for the ith mode :

$$\frac{\partial}{\partial t}(p_s a_i) + \frac{1}{h_x}\frac{\partial}{\partial x} e_i = (E^{-1} A_u)_i \tag{31}$$

$$\frac{\partial}{\partial t}(p_s b_i) + \frac{1}{h_y}\frac{\partial}{\partial y} e_i = (E^{-1} A_v)_i \tag{32}$$

$$\frac{\partial}{\partial t}(p_s c_i) + (M_2 d)_i = (E^{-1} A_T)_i \tag{33}$$

$$\frac{\partial}{\partial t}(p_s) + (\overset{T}{N_2} d)_i = 0 \tag{34}$$

Now, the correction terms for the system of equations (for $p_s a$, $p_s b$, $p_s c$, $p_s q$ and p_s) are :

$$\sum_i 2\triangle t \frac{1}{h_x}\frac{\partial}{\partial x}[\bar{e}_i - e_i(t)],$$

$$\sum_i 2\triangle t \frac{1}{h_y}\frac{\partial}{\partial y}[\bar{e}_i - e_i(t)],$$

$$\sum_i 2\triangle t\, M_2\, [\bar{d}_i - d_i(t)],$$

and $\sum_i 2\triangle t \overset{T}{N_2}[\bar{d}_i - d_i(t)]$

respectively. The summation over i is carried out for those modes for which the split-explicit technique is used. The above correction terms can be obtained from the prognostic equations for e and d. The equations for e and d are obtained from Eqns. (28) and (29) as :

$$d_i(t+\triangle t) - d_i(t-\triangle t) + 2\triangle t(\delta x^2 + \delta y^2)$$
$$\chi[\bar{e}_i - e_i(t)] = \overset{ex}{d_i}(t+\triangle t) - d_i(t-\triangle t) \tag{35}$$

and $e_i(t+\triangle t) - e_i(t-\triangle t) + 2\triangle t\, \lambda_i [\bar{d}_i - d_i(t)]$
$$= \overset{ex}{e_i}(t+\triangle t) - e_i(t-\triangle t) \tag{36}$$

For the first step of integration of sub-interval $\triangle\tau$, the Euler-backward time integration method is used to is march from time $t-\triangle t$ (*i.e.*, point 0 in Fig. 1) to time $t-\triangle t+\triangle\tau$ (point 1). Then a leapfrog scheme used to march each successive time step $\triangle\tau$ until time $t+\triangle t$ (over $2m_i\triangle\tau_i$ sub-intervals). In the present study, the time interval $2\triangle t$ is sub-divided into 8 (m_i=8) su-2 intervals for the first (external) gravity mode and 4 and sub-intervals respectively for the second and third modes respectively for a five layer model (K_T=5). The fourth and fifth modes travel sufficiently slow enough to be incorporated into the large Rossby time step($\triangle t$).

3.4. *Transformation to grid space*

The correction terms computed above are applied to prognostic Eqn. (19) for u component (similarly for v, T, p_s) to get prognostic variable at grid point by split-explicit method. The computation of non-linear processes such as advection and physics is carried out in usual grid space at large time step :

4. Numerical experiment with the integration schemes

4.1. *Efficacy of split-explicit method*

In order to determine the appropriate time steps for integration by explicit and split-explicit methods and to demonstrate the efficiency of split-explicit method over explicit method a number of experiments were conducted with initialized data of 22 May 1979. Model integrations were carried out with time steps of 120, 180, 240 and 300 seconds for explicit (leapfrog) and with time steps of 600, 900, 1200, 1500 and 1800 seconds for split-explicit method. In the case of explicit scheme time step for the grid length of 1.875° at 45° N lies between 240 and 300 seconds. However, during numerical experiements model blow up after 6 hours and 2 hours with 240 and 300 seconds time steps respectively. The model could be integrated successfully up to 48 hour with 180 seconds only with explicit method. On the other hand, the model integration was successful up to a period of 48 hours with a time step as high as 1200 seconds with the split-explicit method of integration.

Fig. 3 depict the time variations of mean absolute divergence of σ=0.5, for the split-explicit case for time steps of 1200, 1500 and 1800 seconds respectively. It is seen from Fig. 3 that the model integrations failed after approximately 7 and 9 hours with time steps of 1500 and 1800 seconds respectively. With a time step of 1200 seconds the model integrations were successful up to 48 hours.

The comparative behaviour of explicit and split-explicit method has been studied with the help of following experiments :

(a) Explicit method with time step of 180 seconds (E_1),

(b) Split-explicit method with time step of 180 seconds (E_2),

(c) Split-explicit method with time step of 900 seconds (E_3).

The results of above experiements are discussed in Sec. 5.1 & 5.2.

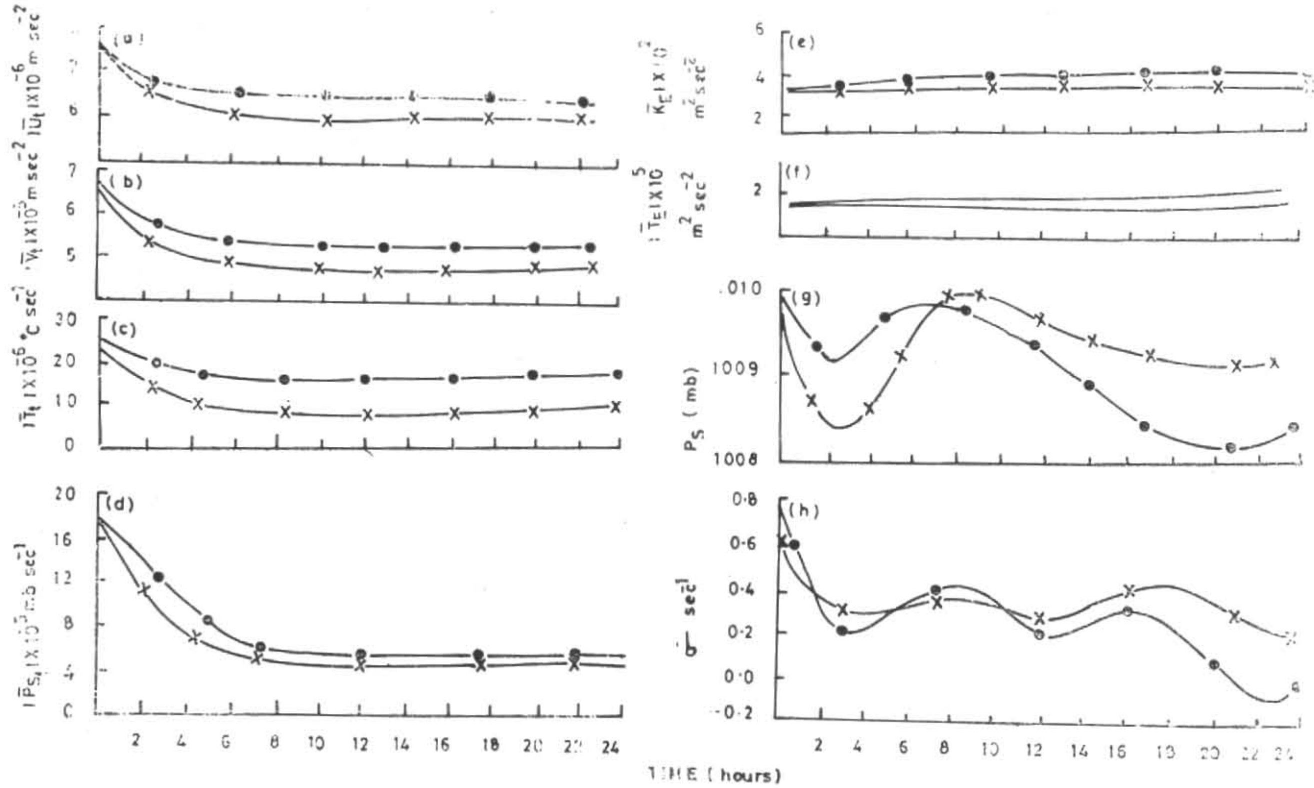

Fig. 2. Time-series during a one day forecasts obtained by split-explicit (×———×) and explicit (o———o) methods of time integration for, (a) $[\bar{u}_t]$, (b) $[\bar{v}_t]$, (c) $[\bar{T}_t]$, (d) $[\bar{p}_{st}]$, (e) $[\bar{K}_E]$, (f) $[\bar{T}_E]$, (g) p_s and (h) σ, with initialised initial data

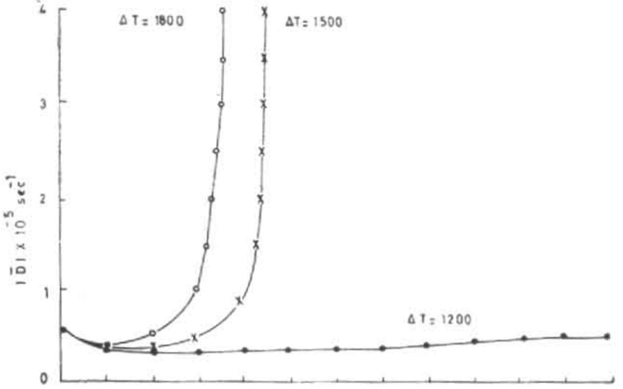

Fig. 3. Time series of $[\bar{D}]$ during a one day forecast obtained by split-explicit method of time integration with varying time steps ($\triangle t$)

4.2. *Properties of split-explicit time integration scheme*

In order to demonstrate the noise suppression characteristic of split-explicit method; experiments were conducted with initialised and uninitialised data of 22 May 1979 for both explicit & split-explicit methods, the results of these experiments are presented in Sec. 5.3.

5. Discussions

5.1. *Comparative behaviour of explicit and split-explicit method*

Though an explicit scheme like leapfrog scheme is considered inefficient due to the constraints of upper limit on the time-step, it yields to fairly satisfactory results with smaller time-steps within the range of CFL limits. In order to assess the characteristics of the split-explicit method as compared to explicit scheme, the model was integrated separately up to 48 hours with the two schemes with a small time step of 180 seconds, using initialised date of 22 May 1979. At each time step, mean absolute tendencies of u, v and T and mean absolute kinetic energy $[\overline{K}_E]$ and total energy $[\overline{T}_E]$ at each sigma surface and surface pressure tendency $[\overline{p}_{st}]$ were computed. In addition, surface pressure and vertical velocity (η) were also monitored for a few specified grid points.

Fig. 2. depict the time variation of above mentioned parameters for the explicit and split-explicit forecast runs of the model for the first day of the integration as no significant changes are observed beyond this period. It is seen from Fig. 2 that the two time integration schemes yield to nearly similar pattern of time evolution of above mentioned parameters. In both the cases, the tendencies of u, v, T and p_s stabilise after 5-7 hours of initial decrease and maximum decrease is shown in the case of p_s. Similarly, $[\overline{K}_E]$ and $[\overline{T}_E]$ values obtained from the two forecast runs are nearly identical. However, the magnitude of tendencies of u, v, T and p_s are lesser for the split-explicit run as opposed to the explicit forecast run of the model. In both the cases, p_s and η do not show large amplitude fluctuations. Thus, it can be inferred that both the split-explicit and explicit integration of the model display similar characteristics when we take a reasonably small time step.

It is seen from Fig. 3 the mean $[\overline{K}_E]$ and $[\overline{T}_E]$ remain almost constant (less than 10% changes). Similarly, the model is also found to conserve the total mass over the domain. p_s and η which are considered sensitive to initial data and approximations in the specification of lateral boundaries and discretization in space and time, do not show spurious oscillations and vary gradually during the period of integration. Though, the model incorporates various physical processes, in general, the mean absolute tendency of the variables over a very large domain should not vary sharply during the first day forecast, as in this time frame, the major contributing factor for the change of tendency is accredited to the large scale dynamics. The initial decrease in the tendency of the variables as observed may therefore, be due to initial mutual geostrophic balance of the mass and velocity fields.

It can be inferred that the LAM under consideration with all its approximations, largely satisfies the essential conserving properties of mass and energy during both explicit (short time step) and split-explicit time integration.

5.2. *Forecast performance*

The foremost requirement of a time integration scheme in a prediction model is to provide good forecast fields and, therefore, experiments were conducted to evaluate the forecasting performance of split-explicit time integration scheme. Forecasts for 48-hr were made from the initial date of 22 May 1979 (12 GMT) with explicit time integration scheme having time step of 180 sec $[E_1]$, split-explicit time integration scheme with time step of 180 sec $[E_2]$ and split-explicit scheme with time step of 900 sec $[E_3]$. Table 1 shows 24 hours and 48 hours root mean square [RMS] error in respect of zonal wind $[u]$, meridional wind $[v]$ and temperature $[T]$ for above three experiments. It brings out that the RMS error is of the same order for all the three experiments. The errors are least for experiment E_2, followed by E_3, and maximum for E_1. Superiority of split-explicit lies in the effective suppression of the growth of the gravity waves during time integration.

TABLE 1

Root mean square error — 22 May 1979

Levels (mb)	Variables	24 hr E_1	24 hr E_2	24 hr E_3	48 hr E_1	48 hr E_2	48 hr E_3
300	u	3.8	3.7	3.8	5.4	5.2	5.5
	v	4.5	4.2	4.4	7.0	6.7	7.0
	T	2.5	2.3	2.4	3.3	3.0	3.1
500	u	2.2	2.0	2.1	7.2	6.8	7.0
	v	2.8	2.9	2.8	2.4	2.4	2.3
	T	1.1	1.0	1.0	1.5	1.4	1.4
700	u	3.4	3.1	3.2	4.2	4.1	4.0
	v	2.5	2.3	2.2	3.3	3.0	3.2
	T	1.2	1.0	0.9	1.3	1.2	1.1
850	u	2.8	2.7	2.9	3.3	3.1	3.1
	v	2.3	2.2	2.2	2.6	2.6	2.7
	T	1.7	1.5	1.6	1.9	1.8	1.8

E_1 : Explicit scheme with 180 sec time step.

E_2 : Split-explicit scheme with 180 sec time step,

E_3 : Split-explicit scheme with 900 sec time step.

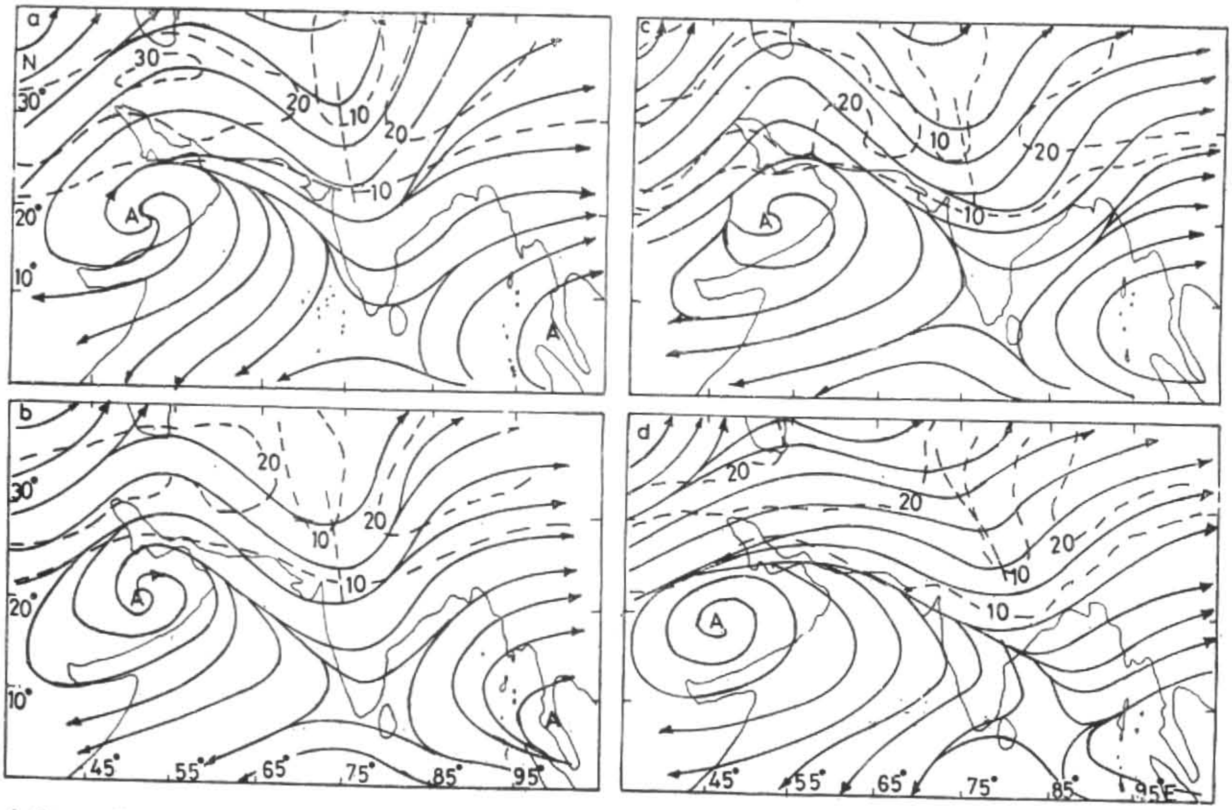

Fig. 4. Stream line and isotach analysis of 48 hours forecast wind field from the initial data of 22 May 1979 (12 GMT), (a) explicit scheme ($\triangle t$ = 180 sec), (b) split-explicit ($\triangle t$=180 sec), (c) split-explicit ($\triangle t$ =900 sec) and (d) corresponding verification-field

Figs. 4 (a-d) present 48-hr forecast streamline isotach analysis and corresponding verification field. A cut off low was observed over north Pakistan between 500 & 300 mb on 22 May 1979. It moved eastward and was over Jammu & Kashmir on 23 May 1979. It weakened rapidly and was seen as a feeble trough over Western Himalayas on 24 May 1979 (Fig. 4d). All three integration schemes have brought out general flow pattern and important synoptic features like weakening of cut off low into trough; locations of anticyclonic cells over Saudi Arabia and southeast Asia. However, the location of the trough is to the west of the actual position and forecast intensity is more marked. These discrepancies in general are common to all the three forecast runs and, therefore, can not be attributed to particular time integration scheme. Forecast wind field with explicit scheme (Fig. 4a) shows isotach maxima of 30 ms^{-1} over Iran, which is not seen in the split-explicit forecasts and verification fields.

It is seen from above discussions that split-explicit time integration scheme with larger time step gives results which are not inferior to explicit scheme having smaller time step and are equally good as compared to split-explicit scheme having smaller time step. Thus, on the grounds of both the time economy and operating performance, split-explicit scheme with larger time step is preferred over other two schemes.

5.3. *Noise suppression characteristic of the split-explicit method*

The split-explicit time integration technique employs much smaller time steps to treat the fast moving gravity modes. Unlike the split-implicit scheme, an split-explicit time differencing scheme does not suffer from severe truncation errors (Gadd 1978). In view of the above, the split-explicit time differencing scheme suppresses and does not permit generation of gravity wave noise. In order to illustrate this characteristic, the model was integrated with the uninitialized data of 22 May 1979 with split-explicit technique and with the explicit method respectively. In the case of the split-explicit forecast run of the model, a time step of 900 seconds was used while for the explicit case, a smaller time step of 180 seconds was used.

The model could be successfully integrated even with the uninitialized data with split-explicit time difference scheme up to a period of 48 hours. On the other hand, the model integrations failed after 16 hours of integration by the conventional explicit scheme. This result is based on just one example and have been taken with caution, because initialisation is any way important.

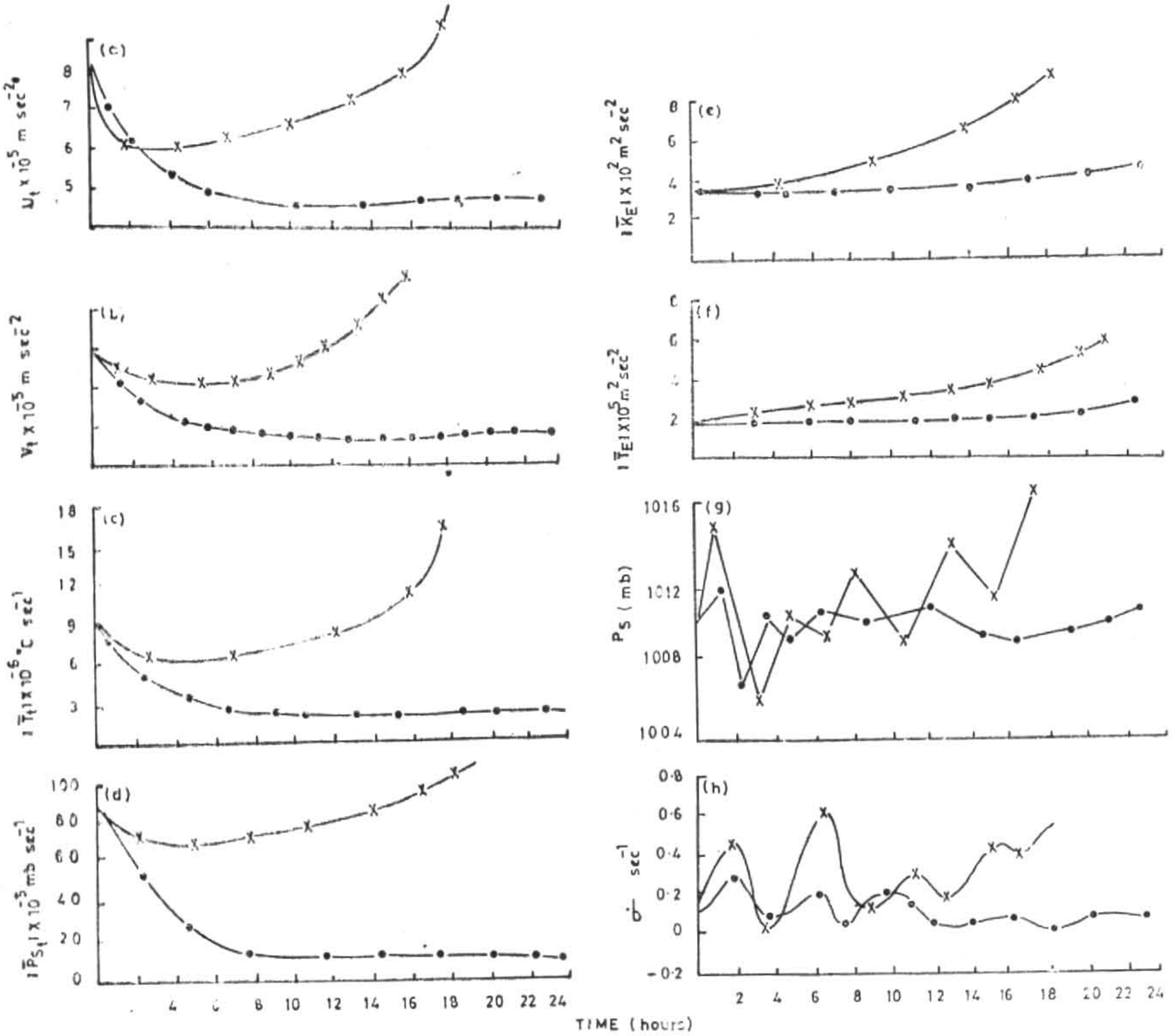

Fig. 5. Time-series during one day forecasts obtained by split-explicit (o———o) and explicit (×———×) methods of time integration with uninitialised initial data, (a) $[\overline{u_t}]$, (b) $[\overline{v_t}]$, (c) $[\overline{T_t}]$, (d) $[\overline{p_{st}}]$, (e) $[\overline{K_E}]$, (f) $[\overline{T_E}]$, (g) p_s and (h) $\dot{\sigma}$

Fig. 5 depict the changes in various parameters (as in Fig. 3) during the integrations of the model by the above mentioned two time difference schemes. It can be seen that $[\overline{u_t}]$, $[\overline{v_t}]$, $[\overline{T_t}]$ and $[\overline{p_{st}}]$ grow rapidly after one to two hours during the explicit run. Similarly $[\overline{K_E}]$ and $[\overline{T_E}]$ show a progressive rapid rise after a few hours of the model run. On the other hand, $[\overline{u_t}]$, $[\overline{v_t}]$, $[\overline{T_t}]$ and $[\overline{p_{st}}]$ show a initial rapid fall during the first 6-8 hours of model integration by the split-explicit method and become almost constant thereafter. A similar nature of decrease in these parameters are noticed during the split-explicit run with uninitialised data as against the similar forecast run with initialised data (Fig. 3). It is, therefore, attributable to the initial balancing of mass and motion field by the model itself during its integration. The mean kinetic energy and mean total energy remain almost constant by the split-explicit time integration method. Fig. 5 also depict time evolution of surface pressure (p_s) and R (at σ=0.5) at a specified location (30°N, 65° E) during the integrations of the model by explicit and split-explicit time difference methods. The time series of p_s and $\dot{\sigma}$ by explicit method shows considerable oscillations (as much as 10 mb). On the other hand p_s and $\dot{\sigma}$ fluctuations during the model integration by the split-explicit technique are smaller and quite realistic.

From the above discussions, it is apparent that during the model integration by explicit method with uninitialised input data, gravity wave noise grows and contaminates the meteorological fields to the extent of blowing up the model integration after a few hours. On the other hand, the split-explicit time integration with uninitialised input data leads to mutual balance of mass and motion fields. It also effectively suppresses growtn of high amplitude gravity wave oscillations during the integration of the model. The main concept behind the split-explicit scheme is to apply a time averaging of the high frequency modes during their integration with smaller time steps. Thus, the split-explicit technique of time integration has an inherent mechanism of smoothing high frequency modes and hence to some extent operates like a dynamic initialisation.

6. Conclusions

(*i*) The split-explicit and explicit time integration schemes lead to nearly same results with a small time step and initialised input data set. Further from the results it can be inferred that the LAM displays conserving properties of mass and energy during both explicit and split-explicit time integration.

(*ii*) The results of the study with initialised input data show that the split-explicit time integration is nearly five times more economical than the explicit method in terms of time step.

(*iii*) An explicit time integration with uninitialised input fields leads to failure of integrations after a few hours even with a small time step. On the other hand, a split-explicit method with uninitialised input data and with large time step is found to effectively suppress gravity wave noise for model integration up to 48 hours. Thus, the split-explicit time integration technique has an inherent mechanism of smoothing high frequency waves.

(*iv*) Split-explicit schemes as used in limited area model simulates general flow pattern and important synoptic feature very well up to 48 hours and its root mean square errors for wind and temperature are less compared to explicit scheme.

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