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The fractal geometry of winter monsoon clouds over the Indian region

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सार – लवजॉय के (1982) संबंधों $P \sim \sqrt{A^D}$ जहां, P, A और D क्रमश: परिसीमा क्षेत्र और मेघों के फ्रैक्टल आयाम है, का प्रयोग करते हुए, इन्सेट-1 बी से प्राप्त मेघचित्रों से 1987 तक के तीन वर्षों के दौरान भारतीय क्षेत्र पर शीतकालीन मानसून (अक्तूबर-दिसम्बर) मेघों के फ्रैक्टल आयाम सुनिश्चित किए गए ।

मेघों का अध्ययन लगभग 5×10^6 वर्ग कि० मी० के उपलब्ध अधिकतम क्षेत्रीय विस्तार तक सभी आकारों के मेघों की स्वत: समान फ्रैक्टल संरचना को स्पष्ट दर्शाता है । मेघ परिसीमा के फ्रैक्टल आयाम के मध्य मान को 1.30 के बराबर पाया गया है ।

ABSTRACT. The fractal dimension of winter monsoon (October-December) clouds over the Indian region during the 3 years 1985 to 1987 were determined from INSAT-1B cloud pictures using Lovejoy's (1982) relationship $P \sim \sqrt{A^D}$ where, P, A and D are respectively the perimeter, area and the fractal dimension of the cloud.

The cloud study clearly indicates self similar fractal structure to clouds of all sizes up to the available maximum areal extent of about 5×10^6 sq km. The mean value of the fractal dimension of cloud perimeter is found to be equal to 1.30.

1. Introduction

One of the recent approaches in improving our understanding of the cloud systems and the dynamical processes involved in them, is a geometrical analysis of the cloud horizontal pattern with the use of satellite or radar imageries.

Recent studies (Lovejoy 1981, Lovejoy and Schertzer 1986, Rys and Waldvogel 1986, Skoda 1987, Gifford 1989) provide conclusive evidence for the fractal geometry of clouds ranging in size from 0.16 km² to 1000 km². Fractal geometry is also exhibited by diverse chemical, physical and biological systems in nature (Sander 1987, Grebogi *et al.* 1987, Stanley and Meakin 1988, Malathi 1988, Mandelbrot 1989). The term "fractal" first coined by Mandelbrot (1981), indicates fractured or broken structure. Since a cloud is not a sphere, its fractal dimension is less than three. The perimeter of a cloud as measured from two dimensional satellite pictures will, therefore, have a fractal dimension less than two. Fractal geometry to cloud shape indicates basic self similarity, *i.e.*, repetition of single basic design in the internal structure of a wide range of cloud sizes and shapes. Since the self similar cloud structure is a signature of the turbulent updrafts and downdrafts inside the clouds a study of the fractal dimension of clouds will enable a fuller understanding of the turbulence scale buoyant energy generation proceses inside clouds and their role in the maintenance of cloud systems.

2. Fractal dimension of clouds

Following Mandelbrot's theory (Lovejoy and Schertzer 1986), the area-perimeter relation is used to investigate the geometry of satellite and radar-determined cloud and rain areas. The data are well fitted by a formula in which the perimeter (P) of a cloud area (A) is given approximately by the square-root of the area to the power D :

$$i.e., P \sim \sqrt{A^D}$$

where, D is interpreted as the fractal dimension of the perimeter. Lovejoy (1982) was, perhaps, the first to analyse meteorological data employing the fractal geometry theory and since then the significance of this approach to meteorology is beginning to be realised.

3. Fractal cloud geometry and deterministic chaos in the atmospheric boundary layer

The self similar fractal geometry is characteristic of the field of the universal period doubling route to chaos or deterministic chaos — a signature of nonlinearity and is found to occur in disparate physical, chemical and biological systems (Crutchfield *et al.* 1986). Lorenz (1963) showed that deterministic chaos is exhibited by the three coupled nonlinear ordinary differential equations for a heat convective system obtained by severe truncation of Navier-Stokes equations. Deterministic chaos governs the cloud dynamical processes and results in the observed fractal geometry to cloud shapes. Phenomenological observations of fractal (broken or fractured) structures in nature represent the two fundamental symmetries of nature, namely dilatation and translation and correspond respectively to change in unit of length or in the origin of the co-ordinate system. A self similar object is identified by its fractal dimension 'D' which in general is defined as :

$$d\ln M\,(R)/d\ln\,(R)$$

where, M (R) is the mass contained within a distance R from a typical point in the object. The basic physical mechanism of the observed self organised fractal geometry in nature is not yet identified (Kadanoff 1986).

4. Scope of the present study

Utilising the horizontal projections of cloud areas as seen by the satellite picture of INSAT-1B the present study aims to find out the fractal properties of the cloud patterns over the Indian seas mainly to understand the tropical cloud systems during the months of October, November and December which is normally referred in India as the post monsoon or northeast monsoon season.

Post monsoon season is normally associated with widespread cloud activity over the Indian seas due to the formation and passage of weather systems ranging from trough of low pressure to low pressure areas, depressions, cyclones and hurricanes affecting the Indian coasts. This is also the period of maximum cyclone activity over the Indian seas, the yearly average being 3 to 4 over Bay of Bengal and 1 to 2 over Arabian Sea.

5. Data

Satellite cloud cover data over the Indian region obtained from INSAT-1B for winter monsoon (Oct-Dec) 1985 to 1987 was used for the study. The cloud area and perimeter were measured using a grid size length of 110 km. A total of 757 cloud cases ranging in areal size from 0.5 to 397 unit squares were used in the study (one unit square = 110 × 110 km²). The area was measured correct to 0.5 unit square.

The data analysis was done as follows. Since the cloud area was measured correct to 0.5 unit square, the cloud perimeter values were grouped and the mean perimeter values P were obtained for each of the cloud area (A) values 0.5, 1.0, 1.5 ....397 unit squares. As shown in the following, there is a high significant correlation between log A and log P and, therefore, grouping the data into narrow class intervals for analysis purposes will not introduce significant errors.

The mean values of log A, log P, their respective standard deviation σ_A and σ_P and the covariance between log A and log P were computed. The correlation coefficient between log A and log P was evaluated.

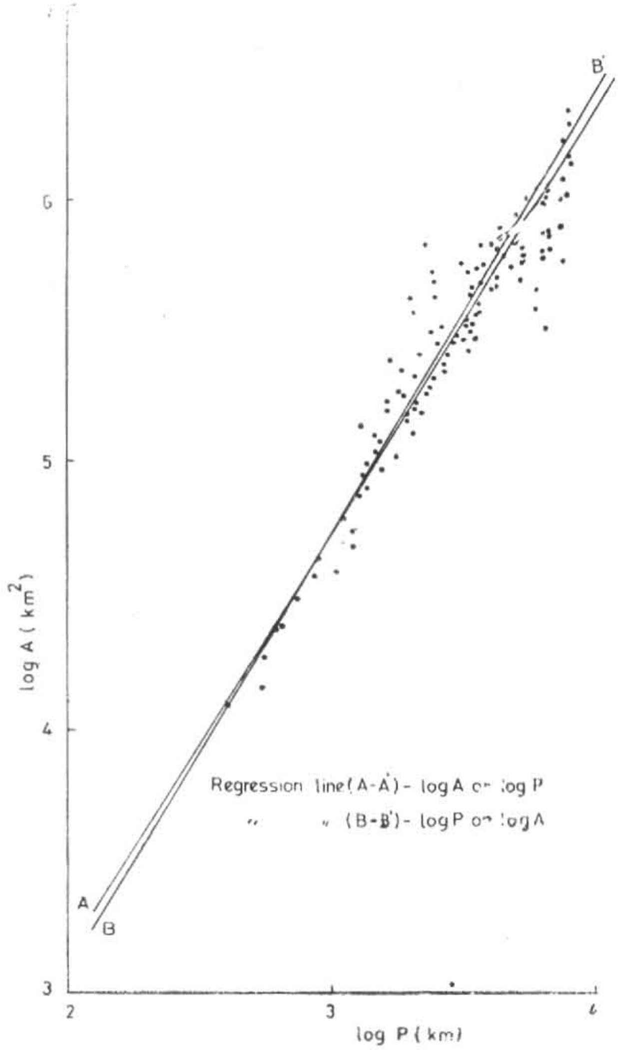


Fig. 1. Scatter diagram of log A *versus* log P

The correlation coefficient is found to be equal to 0.98 which is highly significant. Further the regression lines between log A and log P were evaluated using standard statistical methods. The regression equation of log A and log P is given by:

$$\log A = 1.501 \log P + 0.138$$

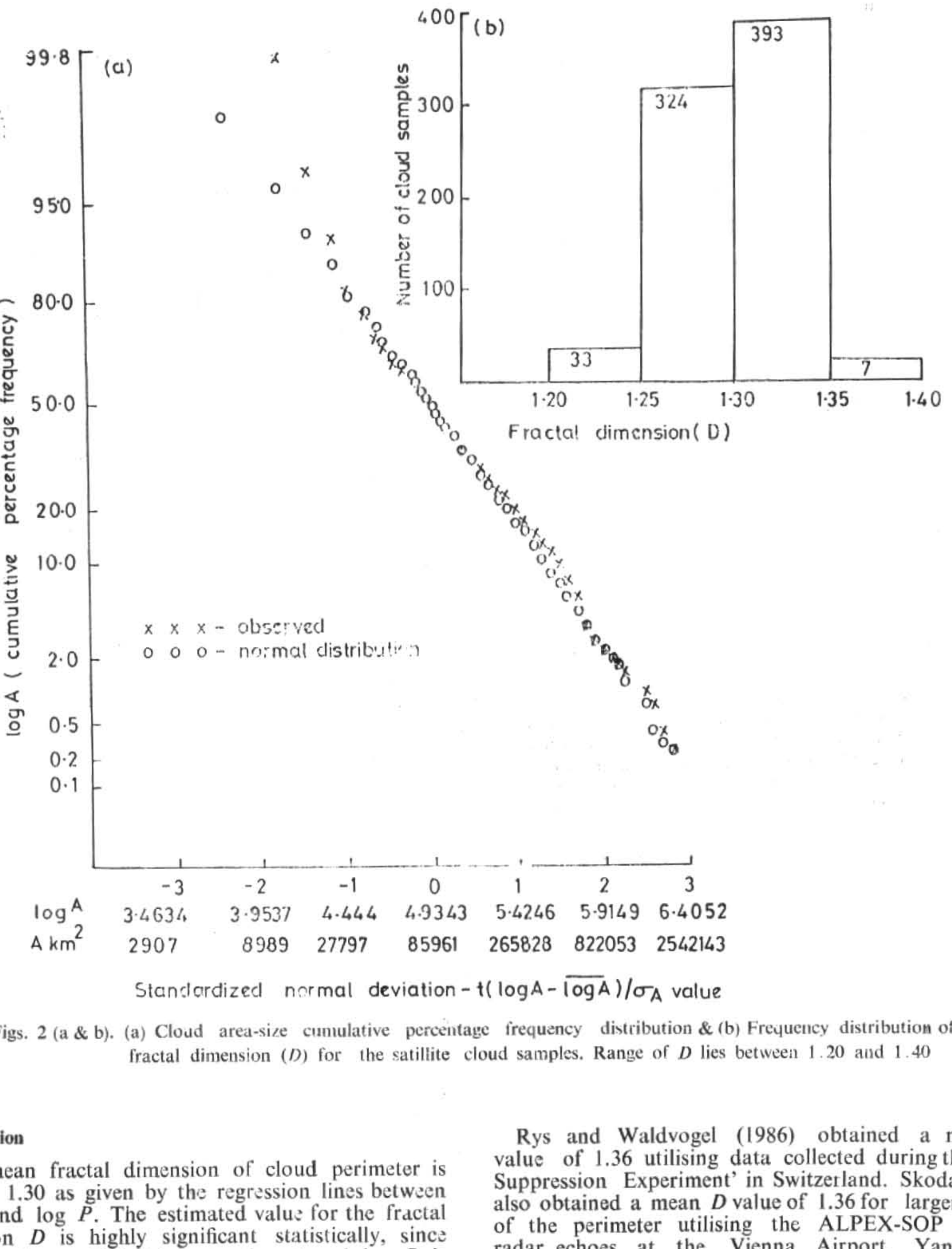
The regression equation of log P on log A is given as

$$\log P = 0.64 \log A + 0.038$$

The fractal dimension D of cloud perimeter as given by the above two regression equations are respectively 1.28 and 1.33 the mean value for D being equal to 1.30.

The scatter diagram of log A *versus* log P and the regression lines are shown in Fig. 1.

The cumulative frequency distribution of cloud areas (log A) as observed and as given by the corresponding normal distribution value is plotted on log probability paper in Fig. 2(a). Fig. 2(b) gives the frequency distribution at select class intervals of 'D' for the cloud data sample under study by means of histograms.



Figs. 2 (a & b). (a) Cloud area-size cumulative percentage frequency distribution & (b) Frequency distribution of fractal dimension (D) for the satillite cloud samples. Range of D lies between 1.20 and 1.40

6. Discussion

The mean fractal dimension of cloud perimeter is equal to 1.30 as given by the regression lines between log A and log P. The estimated value for the fractal dimension D is highly significant statistically, since the correlation coefficient between log A and log P is very nearly equal to one.

The estimated 'D' values compare well with the values reported by earlier studies. Lovejoy (1982) obtained the 'D' value to be equal to 1.35 with a correlation coefficient of 0.994 utilising area perimeter relationship for radar rain data from the tropical Atlantic as well as for data from infrared geostationary satellite data over the Indian Ocean.

Rys and Waldvogel (1986) obtained a mean 'D value of 1.36 utilising data collected during the 'Hail Suppression Experiment' in Switzerland. Skoda (1987) also obtained a mean D value of 1.36 for larger values of the perimeter utilising the ALPEX-SOP weather radar echoes at the Vienna Airport. Yano and Takeuchi (1987) reported a higher value of D of 1.5 utilising the IR satellite imagery of clouds over inter-tropical convergence zone.

The recent study by Jain (1989) on the fractal dimension of post monsoon clouds around Madras utilising 1986 radar data also shows a mean D value of 1.30.

The standard statistical chi-square test (Spiegel 1961) was applied to test the "goodness of fit" of the observed cumulative area size distribution to the log-normal distribution. The cumulative area size distribution is found to follow the log-normal distribution with "goodness of fit" being highly significant at less than 0.5 per cent level. Earlier studies using radar data gave similar results for convective to mesoscale cloud ensemble over the region around Madras (13° 04'N, 80°17'E) for cloud sizes up to a few hundred sq km only (Raghavan *et al.* 1983).

The frequency distribution as seen in the histogram clearly indicates that 98% of the sample has fractal dimension equal to 1.30 ± 0.1.

7. Conclusion

Although this field has advanced at a great rate in recent years, there is a wealth of challenging fundamental questions, that are yet to be adequately dealt with, *e.g.*, whether fractal boundaries can have different dimensions in different regions and the related physical mechanism.

However, the above study clearly establishes the fractal nature of cloud perimeter in the post monsoon season (or winter monsoon) over Indian seas. The fractal dimension of clouds as computed using the area-perimeter relation gives a quantitative measure of the non-Euclidean (irregular) geometrical shape of the cloud. The recently identified constant value for the fractal dimension D for a wide range of cloud sizes indicates identical geometrical shape to clouds of all sizes as also inferred intuitively from the commonly observed typical cloud shape consisting of billows upon billows. Therefore, the cloud geometry is independent of the horizontal length scale in the size range for which the fractal dimension is a constant. The present study indicates that there is no preferred or characteristic horizontal length scale for the cloud samples in the size range up to about 10^6 km^2 since the observed cloud fractal dimension is a constant in this size range.

The mean value of the fractal dimension of winter monsoon cloud over the Indian region is equal to 1.30 for sizes ranging from the meso-scale to synoptic scale and is in agreement with the values reported elsewhere.

Also the 'D' values of the entire sample is found to be 1.30 ± 0.1 and in 98% of the cases the value is within the range 1.30 ± 0.05.

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