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Applicability of extreme value distribution for analysis of rainfall over India

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सार—देश के विभिन्न भागों में वार्षिक अधिकतम वर्षा के विश्लेषण के लिए चरम मान प्रकार I वितरण की व्यवहार्यता की जाँच की गई है। इसमें यह देखा गया है कि देश के पश्चिमी भागों को छोड़कर जहाँ पर कदाचित वर्षा में बहुत विभिन्नता के कारण व्यवहार्यता सीमित होती है, देश के अधिकांश भागों में सामान्यतः यह वितरण कार्य व्यवहार्य होता है।

ABSTRACT. Applicability of extreme value type I distribution, for the analysis of annual maximum rainfall over different parts of the country has been examined. It is observed that this distribution function is generally applicable in major parts of the country except in the western parts where it has limited applicability, perhaps due to large variation in rainfall.

1. Introduction

The extreme rainfall recorded over different regions of India is highly variable and could be associated with different meteorological situations. Hence it is felt that same extreme value distribution may not be applicable to all the regions. Rao & Krishnan (1958) examined the applicability of Gumbel's and Jenkinson's distribution for extreme rainfall in the Damodar catchment. They have indicated that for the computation of extreme rainfall probabilities it is necessary to consider daily rainfall data instead of extreme value series only. Raman and Mukherji (1964) applied the log-normal distribution for maximum 24-hr rainfall over five metropolitan cities in India. Goel and Kathuria (1984) studied the problem of out-liers by applying Gumbel Fisher & Tipper — type-II distribution to annual maximum rainfall series over Krishna basin. In case of out-liers, it has been suggested that Fisher & Tipper type II distribution may be preferred for evaluating the return periods of extremes rainfall events to Gumbel's distribution. Upadhyay *et al.* (1986) studied the problem of out-liers by using a mixture of two types of extreme value distributions.

In this paper, an attempt has been made to study the applicability of extreme value type I distribution to annual maxima series over different parts of the country.

2. Methodology

Extreme value type I (Gumbel's 1954) distribution has the cumulative probability $F(x)$ that x is not exceeded is

$$F(x) = e^{-e^{-\alpha(x-u)}} = e^{-e^{-y}} \tag{1}$$

where u and α are the location and scale parameters of the distribution respectively and $x = u + y/\alpha$ 1(a)

By inversion of Eqn. (1), the relationship may be written in terms of the return period $(T) = [1/\{1-F(x)\}]$.

$$y_T = -\ln.\ \ln[T/(T-1)] \tag{2}$$

$$= -\ln\{-\ln(1-1/T)\} \tag{3}$$

The parameters 'u' and 'α' have been estimated by the maximum likelihood procedure (WMO 1989). The steps for computation are given in appendix.

The rainfall estimates for a particular return period (T) are given by Eqn. 1(a) and (2) above. The standard error for the rainfall estimate is given by

$$\mathrm{SE}(x_T) = \frac{1}{\alpha N^{1/2}} (1.11 + 0.52\, y_T + 0.61 y_T^2)^{1/2} \tag{4}$$

In case of Gumbel's distribution the skewness is 1.14, (Natural Environment Research Council 1975). In Table 6.1 of W.M.O. operational Hydrology Report No. 33, selection of frequency distribution has been indicated by graphical and skewness tests, which have been used in this paper.

3. Data and analysis

Annual highest 24-hour rainfall series have been prepared by utilizing daily rainfall data for the period (1901-70) for 40 stations distributed all over the country and list of the stations is given in Table 1. These series were fitted by Gumbel's distribution and rainfall estimates for various return periods, *viz.*, 2, 5, ... , 1000 yr

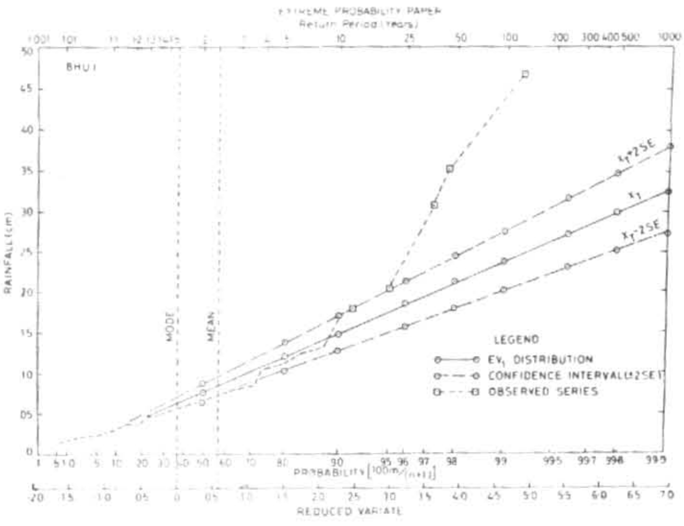

Fig. 1. Fitting of EV_1 distribution to the annual maximum rainfall series of station Bhuj

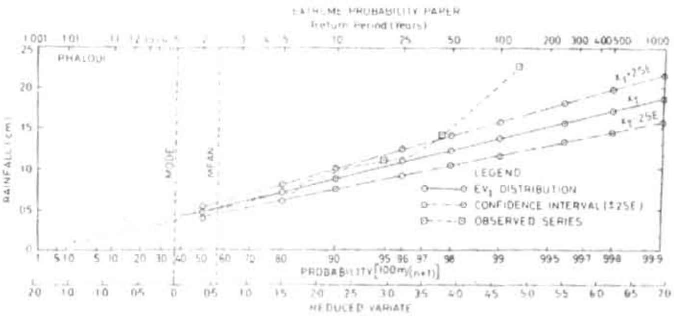

Fig. 2. Fitting of EV_1 distribution to the annual maximum rainfall series of station Phalodi

TABLE 1

Coefficient of skewness of annual extreme series

S. No.	Station (State/Sub-division)	No. of years of record	Skewness	S. No.	Station (State/Sub-division)	No. of years of record	Skewness
1	2	3	4	1	2	3	4
1.	Pasighat Arunachal Pradesh	50	0.75	21.	Pali (east Rajasthan)	70	0.56
2.	Gauhati (Assam)	61	1.21	22.	Phalodi (east Rajasthan)	69	2.30
3.	Sekoni (Assam)	58	1.39	23.	Kota (east Rajasthan)	69	0.85
4.	Deemapur (Nagaland)	57	0.88	24.	Sagar (M.P.)	70	1.08
5.	Berhampore (West Bengal)	70	1.28	25.	Shahpur (M. P.)	66	1.27
6.	Sambalpur (Orissa)	69	0.92	26.	Raipur (M.P.)	70	1.67
7.	Deogarh (Bihar)	62	1.30	27.	Janakpur (M.P.)	30	0.79
8.	Darbhanga (Bihar)	68	1.00	28.	Bhuj (Gujarat)	70	2.86
9.	Kundra (west U.P.)	69	0.91	29.	Tharad (Gujarat)	69	1.60
10.	Okhimath (west U.P.)	68	1.28	30.	Dahanu (Maharashtra)	69	1.13
11.	Hydergarh (east U.P.)	68	0.65	31.	Brahmpuri (Maharashtra)	69	0.67
12.	Panipat (Haryana)	68	1.06	32.	Nanded (Marathwada)	70	1.69
13.	Dasuyia (Punjab)	70	1.66	33.	Chintalapudi (A.P.)	69	1.02
14.	Hamirpur (H.P.)	69	1.60	34.	Borlan (A.P.)	46	1.15
15.	Kangra (H.P.)	62	0.59	35.	Kodanur (A.P.)	45	0.81
16.	Kothai (H.P.)	70	1.27	36.	Madurai (Tamil Nadu)	70	1.28
17.	Baramula (J & K)	66	0.64	37.	Mangalore (Karnataka)	70	1.64
18.	Shri Ranbir Singh Pura (J&K)	65	1.08	38.	Virajpet (Karnataka)	70	1.35
19.	Barmer (west Rajasthan)	69	1.93	39.	Bijapore (Karnataka)	70	1.28
20.	Bikaner (west Rajasthan)	69	1.00	40.	Adur (Kerala)	70	0.74

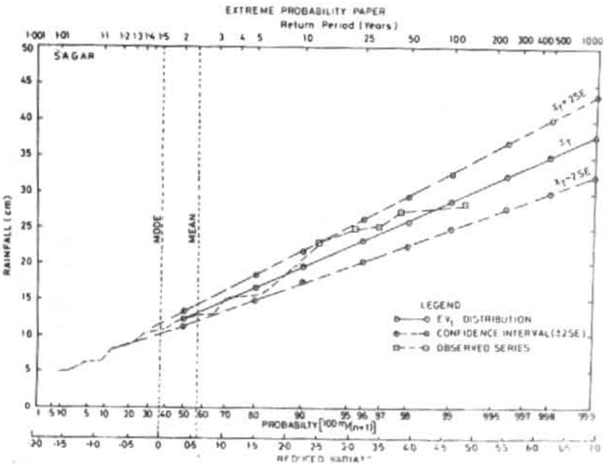

Fig. 3. Fitting of EV_1 distribution to the annual maximum rainfall series of station Sagar

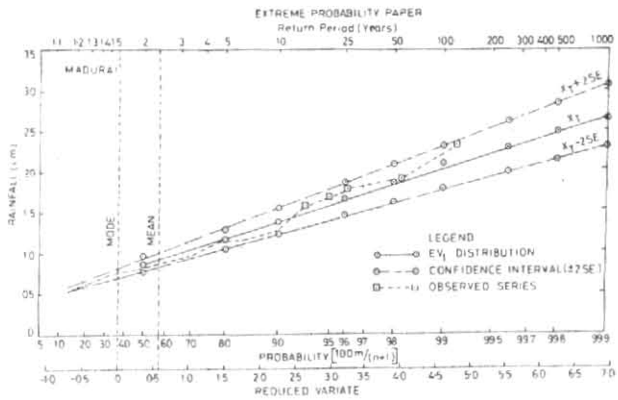

Fig. 4. Fitting of EV_1 distribution to the annual maximum rainfall series of station Madurai

TABLE 2

Statistical estimates of annual extreme series

S. No.	Station	Mean	Standard deviation	Maximum recorded	Return periods alongwith their standard errors [SE (x_T)]							
					x_5	S.E.	x_{50}	S.E.	x_{100}	S.E.	x_{1000}	S.E.
1	2	3	4	5	6	7	8	9	10	11	12	13
1.	Pasighat	213.5	82.50	467.9	284.2	18.52	458.9	36.17	509.7	41.57	677.5	59.68
2.	Gauhati	102.5	36.20	232.9	127.7	6.33	193.7	12.37	212.9	14.21	276.3	20.41
3.	Sekoni	99.3	29.00	198.4	117.5	4.82	166.4	9.41	180.7	10.82	227.7	15.53
4.	Deemapur	97.7	33.80	197.1	119.7	5.93	179.5	11.59	196.8	13.32	254.2	19.12
5.	Berhampore	108.9	44.32	286.3	137.5	6.89	214.5	13.47	236.8	15.48	310.8	22.22
6.	Sambalpur	148.7	56.38	312.4	187.2	9.23	289.5	18.04	319.3	20.73	417.6	29.76
7.	Deogarh	100.5	46.63	264.2	102.4	7.07	176.6	13.80	198.1	15.86	269.4	22.76
8.	Darbhanga	127.5	46.70	266.7	158.5	7.55	241.5	14.74	265.6	16.94	345.3	24.31
9·	Okhimath	104.9	35.00	208.3	125.1	5.13	181.6	10.02	198.0	11.52	252.3	16.54
10.	Kundra	114.1	58.39	315.0	159.3	10.38	274.3	20.27	307.7	23.29	418.2	33.44
11.	Hydergarh	112.5	40.45	233.2	77.1	5.97	142.7	11.66	161.8	13.40	224.9	19.23
12.	Panipat	96.5	45.10	254.0	127.1	7.38	208.3	14.42	231.9	16.57	310.0	23.79
13.	Dasuyia	95.1	36.30	245.9	118.8	5.61	181.4	10.95	199.5	12.58	259.7	18.07
14.	Hamirpur	116.3	50.00	351.0	148.7	7.73	234.4	15.10	259.3	17.35	341.6	24.91
15.	Kangra	156.6	44.03	305.3	191.5	8.51	280.9	16.62	305.9	19.10	392.7	27.41
16.	Kothai	72.3	27.65	182.9	90.3	4.33	138.6	8.45	152.6	9.71	199.0	13.95
17.	Baramula	58.97	17.70	107.4	72.1	3.16	106.5	6.18	116.5	7.11	149.5	10.21
18.	Sri Ranbir Singh Pura	103.5	34.30	207.0	128.4	5.88	191.3	11.48	209.7	13.2	270.4	18.9
19.	Barmer	66.95	46.90	285.8	91.5	6.28	161.1	12.28	181.4	14.1	248.3	20.26
20·	Bikaner	61.96	32.50	165.6	83.8	5.21	141.9	10.18	158.9	11.70	214.8	16.79
21.	Pali	85.35	37.09	206.6	115.22	6.81	191.2	13.30	213.3	15.28	286.3	21.94
22.	Phalodi	55.05	35.44	225.0	71.9	4.57	122.6	8.93	137.4	10.27	186.1	14.74
23.	Kota	103.0	42.00	249.2	134.4	7.32	215.5	14.30	239.1	15.4	317.0	23.59
24.	Sagar	135.2	59.64	381.5	167.3	8.43	261.4	16.47	288.8	18.93	379.2	27.18
25.	Shahpur	111.6	42.20	289.6	87.0	6.29	155.2	12.29	175.0	14.13	240.6	20.28
26.	Raipur	129.1	51.60	370.3	95.0	7.44	178.0	14.53	202.1	16.69	281.9	23.96
27.	Janakpur	98.0	25.67	165.1	78.0	5.89	121.1	11.51	133.6	13.23	174.9	18.99
28.	Bhuj	88.3	74.94	467.0	119.4	8.26	211.5	16.12	238.3	18.53	326.8	26.60
29.	Tharad	116.7	78.96	370.3	158.7	10.95	280.1	21.39	315.4	24.59	432.0	35.30
30.	Dahanu	203.6	77.50	481.0	257.4	12.78	399.0	24.96	440.2	28.63	576.2	41.17
31.	Brahmpuri	148.9	61.25	323.6	104.7	8.89	203.3	17.36	231.9	19.95	326.5	28.64
32.	Nanded	93.4	38.01	254.0	115.1	5.38	175.2	10.51	192.6	12.08	250.4	17.35
33.	Chintalapudi	86.6	26.36	160.8	103.4	4.10	148.9	8.01	162.1	9.12	205.8	13.23
34.	Borlan	114.9	47.67	243.8	144.2	9.01	225.8	17.60	249.5	20.22	327.8	29.04
35.	Kodanur	79.4	53.90	298.2	60.4	10.16	143.9	19.85	169.2	22.81	248.5	32.75
36.	Madurai	90.2	34.86	229.0	116.5	6.00	183.6	11.73	203.1	13.48	267.5	19.36
37.	Mangalore	159.0	41.55	360.9	189.0	6.91	266.1	13.50	288.6	15.51	362.6	22.27
38.	Virajpet	156.3	60.35	366.5	193.8	9.12	295.6	17.80	325.2	20.46	422.9	29.37
39.	Bijapur	68.7	26.68	181.1	88.2	4.48	138.2	8.76	152.8	10.07	200.9	14.45
40.	Adur	113.9	34.00	223.5	138.1	5.73	202.0	11.18	220.6	15.07	282.0	18.45

and the corresponding standard errors [SE (x_T)] computed by using the equations above are provided in Table 2. The values of the skewness for these series are given in Table 1.

The different return period estimates (x_T) were plotted on probability paper for EV_1 distribution along with confidence limits ($\pm$2SE). If the observed values are lying within the confidence limits, the fitness of the distribution is considered as adequate (WMO 1989). The observed extreme rainfall data for all the 40 stations were also plotted on the probability paper by using Gringorton's (1963) plotting positions in which the return period (T) corresponding to observed values with m^{th} rank in descending order series ($m=1$ for the highest value and N is number of observations) is given by the empirical expression :

$$T = \frac{N + 0.12}{m - 0.44} \tag{5}$$

From these diagrams it is observed that Gumbel istribution fits adequately as the rainfall values are ying between the confidence intervals for most of the stations except for some stations in west Rajasthan and Saurashtra & Kutch. A few representative diagrams for the stations—Bhuj, Phalodi, Sagar and Madurai are given in Figs. 1 and 4. Gumbel distribution fits satisfactorily to the observed series for the stations given in Figs. 3 and 4. In case of Bhuj and Phalodi, the fit is unsatisfactory as some of the observed points are lying much outside the confidence intervals. Such outlying observations may occur due to their belonging to a different population. For such cases other distribution functions may be attempted. It may also be seen from Table 1 that for the stations where departure of coefficient of skewness from 1.14 (coefficient of skewness for Gumbel distribution) is large, the Gumbel distribution has limited applicability.

4. Conclusion

The above study suggest that extreme value Type I (Gumbel) distribution fits adequately the annual maximum (daily) rainfall series over most of the country except for west Rajasthan and Saurashtra & Kutch.

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Appendix

(*i*) Arrange the data in ascending order.

(*ii*) Calculate the sextile means.

(*iii*) Calculate the mean ($\bar{x}$) and standard deviation (σ) of the sextile means.

(*iv*) Calculate $\hat{\alpha} = 0.8333\, x$

$$\hat{u} = x - 0.4833\, \sigma$$

These values are used as initial estimate for the maximum likelihood solution.

(*v*) Tabulate $x_i = (x_i - \hat{u})/\hat{\alpha}$.

(*vi*) Compute

$$P = N - \sum_{i=1}^{N} e^{-x_i}$$

$$R = N - \sum_{i=1}^{N} x_i + \sum_{i=1}^{N} x_i\, e^{-x}$$

(*vii*) Compute corrections $\delta\alpha$, δu to the values $\hat{\alpha}$, $\hat{u}$ given by

$$\delta\alpha = (0.26P - 0.608R)\, \hat{\alpha}/N,$$

$$\delta u = (1.11P - 0.26R)\, \hat{\alpha}/N.$$

(*viii*) The corrected (new) estimates become

$$\hat{\alpha} = \hat{\alpha} + \delta\alpha$$

$$\hat{u} = \hat{u} + \delta u$$

Repeat the above procedures from step (*v*) to step (*viii*) till corrections $\delta\alpha$ and δu become small.