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Some effects of asymmetry of pressure distribution in a cyclone field

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सार — जब चक्रवात क्षेत्र में समदाब रेखाएं संकेन्द्र में वृत्ताकार या वर्तुल होती हैं तो भ्रमिल की सममान रेखा भी स्पष्टतः वृत्ताकार होती हैं। दाब प्रोफाइल में एक असममिति के कारण भ्रमिल प्रोफाइल में भी असममिति आने की संभावना हो सकती है। सामान्य दाब मॉडल का प्रयोग करते हुए इस पहलू का अध्ययन किया जाता है। यह दर्शाया गया है कि अपने बाहरी क्षेत में चक्रवातीय तूफान के भ्रमिल प्रोफाइल की असममिति, अनुकूल (सदृश) दाब प्रोफाइल से कहीं कम होती है। इसलिए इससे यह निष्कर्ष निकाला गया है, कि सुस्पष्ट असममित समदाब रेखाओं से चक्रवातीय तूफान असममिति के वर्षा वितरण और रेडॉर/उपग्रह बिम्बों में वही डिग्री व्यक्त नहीं कर सकता।

ABSTRACT. When the isobars in a cyclone field are concentric circular, the isopleths of vorticity are also obviously circular. An asymmetry in the pressure profile can be justifiably expected to cause an asymmetry in the vorticity profile also. This aspect has been studied by making use of a general pressure model. It has been shown that the asymmetry of vorticity profile of a cyclonic storm in its outer region is far less than that of the corresponding pressure profile. There upon it has been concluded that a cyclonic storm with markedly asymmetric isobars may not manifest the same degree of asymmetry in its rainfall distribution and in radar/satellite imageries.

1. Introduction

For a stationary cyclone with concentric circular isobars, the pressure profile in the cyclone field can be expressed as :

$$\frac{P_r - P_o}{P_N - P_o} = \Psi \left(\frac{r}{R}\right) \tag{1}$$

where, P_o, P_r and P_N are respectively the central, radial and peripheral pressures, r the radial distance and R the radius of maximum cyclostrophic wind (RMW). A number of forms of $\Psi(r/R)$ could be found in the literature. Bretschneider (1982) has suggested a general form of the pressure profile as :

$$\Psi \left(\frac{r}{R}\right) = 1 - \left[1 + a\left(\frac{r}{R}\right)^2\right]^{-b} \tag{2}$$

Here $a=1/b$. If $a=2$, $b=1/2$ it leads to Fujita model and if $a=b=1$, to the Bret-X model (*loc. cit*).

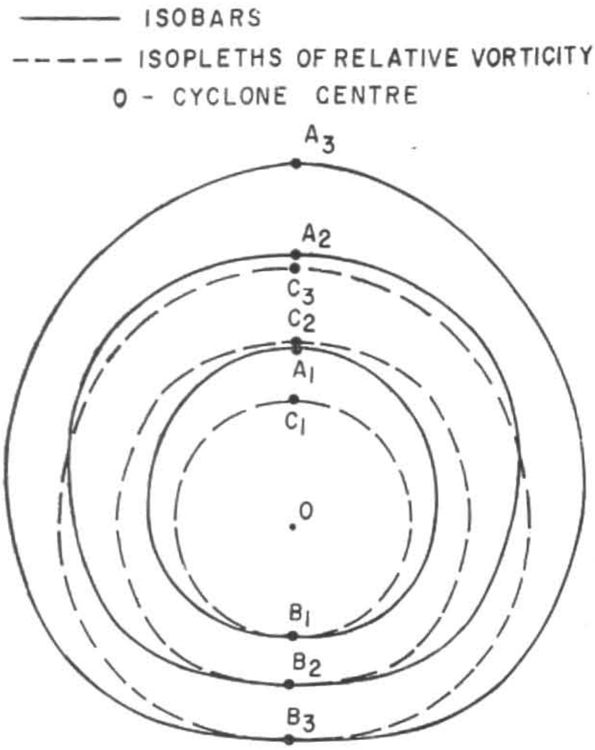
The suitability of a theoretical model for a given cyclone can be determined by comparing the theoretically obtained pressure values with the observed values at various time intervals. If the correlation between the two is consistently high, then the model could be considered adequate.

When the isobars in a cyclone field are concentric circular, the isotachs and isopleths of relative vorticity are also obviously circular. An asymmetry in the pressure field can be justifiably expected to induce an asymmetry in the vorticity field also. The object of this note is to make use of the above model to study the effect of any asymmetry in the pressure profile (in the cyclone field) in the profile of relative vorticity. We first discuss the adequacy of this model in generating the observed pattern of relative vorticity in a cyclone field.

2. Distribution of relative vorticity associated with the general cyclone model

We assume that the cyclone is stationary and the forces acting on an air parcel are the pressure gradient, coriolis and the centrifugal forces only and that these are in gradient balance. Firstly, we assume that the isobars are concentric circular. We denote $P_N - P_o$ by $\triangle p$ which is the pressure defect of the storm and $P_N - P_r$, the pressure drop at radial distance r by $g(r)$. Thus from Eqns. (1) and (2) we have :

$$g(r) = \frac{\triangle p}{\left[1 + \frac{1}{b}\left(\frac{r}{R}\right)^2\right]^b} \tag{3}$$



$$\frac{OA_1}{OB_1} = \frac{OA_2}{OB_2} = \frac{OA_3}{OB_3} = \frac{5}{3} = 1 \cdot 67$$

$$\frac{OC_1}{OB_1} = \frac{OC_2}{OB_2} = \frac{OC_3}{OB_3} = \left(\frac{5}{3}\right)^{1/3} = 1 \cdot 19$$

Fig. 1. Shape of isopleths of vorticity in the outer region of cyclonic storm with asymmetric isobars

Once b is fixed, R completely determines the rate of decrease of $g\ (r)$ from the centre, the larger the R the smaller the rate of decrease. The value of R varies from cyclone to cyclone with an approximate range of 20-60 km.

Let P be the pressure, V the wind speed, ζ the vorticity and R_s the radius of curvature of streamlines and f the coriolis parameter. We have from Holton (1972) and Hess (1959);

$$\frac{V^2}{R_s} + fV = -\frac{1}{\rho}\ \frac{\partial P}{\partial n}$$

$$\zeta = -\frac{\partial V}{\partial n} + \frac{V}{R_s} \tag{4}$$

As the isobars are circular and the streamlines are counter clockwise circuited, $-\frac{\partial}{\partial n} = \frac{\partial}{\partial r}$ and $R_s = r$.

Thus Eqn. (4) becomes :

$$\frac{V^2}{r} + fV = \frac{1}{\rho}\ \frac{\partial P}{\partial r} = -\frac{1}{\rho}\ \frac{\partial g}{\partial r}$$

$$\zeta = \frac{\partial V}{\partial r} + \frac{V}{r} \tag{5}$$

where, $g\ (r) = P_N - P_r$, P_r being the radial pressure. It can be easily verified from Eqn. (5) that :

$$\zeta\ (r) = -f + \frac{f^2 r - \frac{3}{\rho}\ \frac{\partial g}{\partial r} - \frac{r}{\rho}\ \frac{\partial^2 g}{\partial r^2}}{\left(f^2 r^2 - \frac{4r}{\rho}\ \frac{\partial g}{\partial r}\right)^{\frac{1}{2}}} \tag{6}$$

Now, in the outer region where $r > 3R$ we have approximately from Eqn. (3)

$$g\ (r) = \triangle p\, b^b \left(\frac{R}{r}\right)^{2b} \tag{7}$$

Differentiating Eqn. (7) twice and substituting in Eqn. (6) we get

$$\zeta\ (r) = -f + \frac{f^2 + \frac{4}{\rho}\ \triangle p\, b^{b+1}(1-b)\ \frac{R^{2b}}{r^{2b+2}}}{\left(f^2 + \frac{8 \triangle p}{\rho}\ b^{b+1}\ \frac{R^{2b}}{r^{2b+2}}\right)^{\frac{1}{2}}} \tag{8}$$

It is obvious from the above expression that if $b \geqslant 1$, $\zeta(r) < 0$ for all values of r. According to Gray (1981), the relative vorticity in a cyclone field is negative generally beyond 4-5° from the centre. Thus values of $b \geqslant 1$, do not give realistic profiles of vorticity. The Bret-X model for which $b=1$, comes under this category. For the Fujita model, for which $b=\frac{1}{2}$, Eqn. (8) reduces to :

$$\zeta(r) = -f + \frac{f^2 + \frac{\triangle p}{\sqrt{2}\rho}\frac{R}{r^3}}{\left(f^2 + 2\sqrt{2}\frac{\triangle p}{\rho}\frac{R}{r^3}\right)^{\frac{1}{2}}} \tag{9}$$

It can be shown from Eqn. (9) that the region of positive ζ extends up to a distance of $6R$ to $7R$ from the centre. The Fujita model thus generates the observed relative vorticity profile of a cyclone. We will, therefore, use the Fujita model and the vorticity profile based thereon in the ensuing analysis.

3. Cyclone with asymmetric isobars

The pressure profile given by Eqns. (1) and (2) are meant only for cyclones with concentric isobars. Cyclones with asymmetric isobars are not uncommon. Such a feature can be easily incorporated in Eqn. (2) by taking R the RMW as a variable which varies with θ [*i.e.*, $R = R(\theta)$] where θ is the polar angle of a point in the cyclone field with the centre of the cyclone as the initial point.

We have from Eqn. (3) with $b = 1/2$

$$g(r,\theta) = \frac{\triangle p}{\sqrt{1 + 2[(r/R(\theta)]^2}} \tag{10}$$

It is evident from Eqn. (10) that the lines of constant g and so the isobars are given by the equation

$$r = \lambda R(\theta) \tag{11}$$

where, λ is a positive scalar. We may define a coefficient of asymmetry C for a closed curve given by Eqn. (11) with the initial point as a central point as :

$$C_g = 100\left(\frac{\text{Max } r}{\text{Min } r} - 1\right) \tag{12}$$

4. Vorticity profile in the outer region of a cyclone with asymmetric isobars

Taking RMW as a function of angle θ, we have the relation Eqn. (9) redefined as :

$$\zeta(r,\theta) = -f + \frac{f^2 + \frac{\triangle p}{\sqrt{2}\rho}\frac{R(\theta)}{r^3}}{\sqrt{f^2 + 2\sqrt{2}\frac{\triangle p}{\rho}\frac{R(\theta)}{r^3}}} \tag{13}$$

From Eqn. (13) it is evident that isopleths of ζ in the outer region of the cyclone are given by the equation $r^3 = \mu^3 R(\theta)$, *i.e.*,

$$r = \mu[R(\theta)]^{\frac{1}{3}} \tag{14}$$

where, μ is a positive scalar. For the family of closed curves defined by Eqn. (14) the coefficient of asymmetry may be defined as :

$$C_\zeta = 100\left[\left(\frac{\text{Max } r}{\text{Min } r}\right)^{\frac{1}{3}} - 1\right] \tag{15}$$

To illustrate the degree of asymmetry of isobars *vis-a-vis* the isopleths of vorticity, we compute the coefficient of asymmetry C_g in Eqn. (12) and C_ζ in Eqn. (15) by taking an arbitrary ratio of Max r/Min r as 5/3. It is seen that the value of C_g is 67% as against 19% for C_ζ thus indicating that asymmetry in the pressure field is much more than that in the vorticity field. Even when the isobars are highly asymmetric, the isopleths of relative vorticity are more or less circular. The two patterns are depicted in Fig 1. The rainfall distribution, clouding as revealed by radar and satellite imageries correspond better with the vorticity pattern than the pressure pattern in a storm field. We may not, therefore, expect the same degree of asymmetry as obtained in the pressure field in the rain/cloud distributions.

5. Derivation from the wind profile

It is interesting to see that the results obtained in Sec. 4, which are based on the vorticity profile derived from a pressure profile, could also be obtained from a widely used wind profile. Riehl (1963), Hughes (1952) and Gray (1982) have noted that the radial profiles of tangential wind V of a cyclonic storm can be approximated by the equation :

$$V r^x = \text{Constant} \tag{16}$$

where, x is an exponent. Here, $x = -1$ has been shown to be valid inside the RMW and $x = 0.5$ outside thereof. From Eqn. (16) we easily have

$$V = V_m\left(\frac{R}{r}\right)^x \tag{17}$$

where, V_m is the maximum wind,

and hence from Eqn. (5) :

$$\zeta = \frac{V_m R^x (1-x)}{r^{x+1}} \tag{18}$$

Now the isopleths of wind speed, *viz.*, isotachs are obviously given by the family of curves $r = \lambda R$ where λ is a positive scalar. The isopleths of vorticity are given by

$$r^{x+1} = \mu R^x,\ i.e.,\ r = \mu_1 R^{\frac{x}{x+1}} = \mu_1 R^{1/3} \text{ for } x = 0.5,$$

where μ_1 is a positive scalar. If we define the coefficient of asymmetry in a similar way as in the previous sections, we get a coefficient of asymmetry of 67% for wind speed profile against a 19% asymmetry for vorticity. This shows that the profile of wind speed *vis-a-vis* vorticity also leads to similar results.

6. Discussion and concluding remarks

The results we have derived are valid in the outer region of the cyclone which we have taken as the region where $r > 3R$. The error in ζ due to the assumption Eqn. (7) decreases sharply with increasing r. The form of $R(\theta)$ as a function of θ needs, however, to be determined in order to define the asymmetry of pressure and vorticity fields. In this study we have not considered frictional forces and the force arising due to the translation of a cyclone. Basu and Ghosh (1987) and Shapiro (1983) have discussed the effect of the motion of the cyclone in inducing asymmetry in the isotach profile even if pressure profile were to be asymmetric. By incorporating these forces in the dynamic equations governing the cyclone flow and carrying out a similar analysis we may be able to derive, perhaps, more realistic results.

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