

# A case study on the prediction of Indian summer monsoon of 1987 with the help of a global spectral model and sensitivity of the model to vertical resolution

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**सार** — 1987 की मानसून ऋतु के दौरान सक्रिय कालावधि के प्रत्येक दिन के लिये 240 घण्टों के लिये एक ग्लोबल स्पेक्ट्रल मॉडल समाकलित किया गया है। 24 घण्टों से 240 घण्टों तक के पूर्वानुमान खण्डों के लिये प्रत्येक सात प्रारम्भिक स्थितियों के लिये विश्लेषण और पूर्वानुमान परिवर्तनों के मध्य माध्य त्रुटि, मूल माध्य वर्ग त्रुटि, सहसम्बन्ध गुणांक और मूल माध्य वर्ग सदिश पवन त्रुटि का अभिकलन किया गया है। प्राप्त वर्षा से वर्षा पूर्वानुमानों का व्यक्तिनिष्ठ रूप में मूल्यांकन किया गया। दो उर्ध्वाधर वियोजनों के लिए उसी मॉडल से संवेदनशीलता पर एक प्रयोग भी किया गया है। यह देखा गया है कि निम्न वियोजन सीमित भौतिक मॉडल से उत्पन्न पूर्वानुमान उच्चतर वियोजन, उत्तम भौतिक मॉडल के 6 घण्टे के पूर्वानुमान का प्रयोग करते हुए उत्पन्न विश्लेषण के अधिक निकट है इसका तात्पर्य यह है कि उच्चतर उर्ध्वाधर वियोजन, परिवर्तन को उत्पन्न करता है जो कि और अधिक भौतिकी के जोड़ने से व्युत्पन्न से, विपरीत दिशा में है।

**ABSTRACT.** A Global spectral model has been integrated for 240 hours for each day of an active epoch during the monsoon season of 1987. The mean error, root mean square error, the correlation coefficient between analysis and forecast changes and root mean square vector wind error have been computed for each of the seven initial conditions for 24 hours to 240 hours forecast segments. The rainfall forecasts have also been evaluated subjectively against the realised rainfall values. An experiment on sensitivity has also been carried out with the same model for two vertical resolutions. It is seen that forecasts produced by the lower resolution limited physics model is closer to the analysis produced by using the 6-hr forecasts of a higher resolution, better physics model as first guess. This implies that higher vertical resolution produces changes which are in opposite direction to that produced by adding more physics.

## 1. Introduction

The results of a 23-day integration of the 12-layer 40-wave (rhomboidal truncation) global spectral model available at the New Delhi centre of India Meteorological Department, has been presented in an earlier paper (Basu 1990). In the present work result of 10-day integrations of the same model for seven initial conditions during a good monsoon epoch in August 1987 is presented.

During the period 11-17 August synoptic situation over India was characterised by an active monsoon trough at surface over the Gangetic plains with a few shallow low pressure areas either embedded in or lying close to the monsoon trough. In the lower troposphere a cyclonic circulation prevailed over the Bihar plateau and adjoining areas on most of the days during this period leading to southerlies over the head Bay of Bengal. Significant westerly component was present in the low level wind field along the west coast of India during most of the days of the period of this study. These westerlies along the west coast and the southerlies over the head Bay gave rise to large amounts of orographic rainfall over the Western Ghats and the

hills of northeast. Most of the remaining rainfall amounts during this period were in association with the low pressure areas or the cyclonic circulation over Bihar plateau. No organised system like monsoon depression or mid-tropospheric cyclone was present during this week.

Verification of the model forecast has been done following the procedure prescribed by the World Meteorological Organisation (WMO) in the *Manual on Global Data Processing System* (GDPS, attachment II.14, pp. 22-23). The mean error, root mean square error and correlation coefficient between the forecast and verification analysis of the surface pressure, geopotential heights and winds at standard pressure levels, for both the global domain and the Indian window (45°E-120°E, 15°S-55°N), have been computed and are presented in Tables 2 & 3 and Fig. 1. In the present study data used for both forecast run and verification are the 30-wave coefficients obtained as the output from the Global Data Assimilation System (GDAS) operational at NMC, Washington. This is the standard resolution at which analysis is archived at NMC.

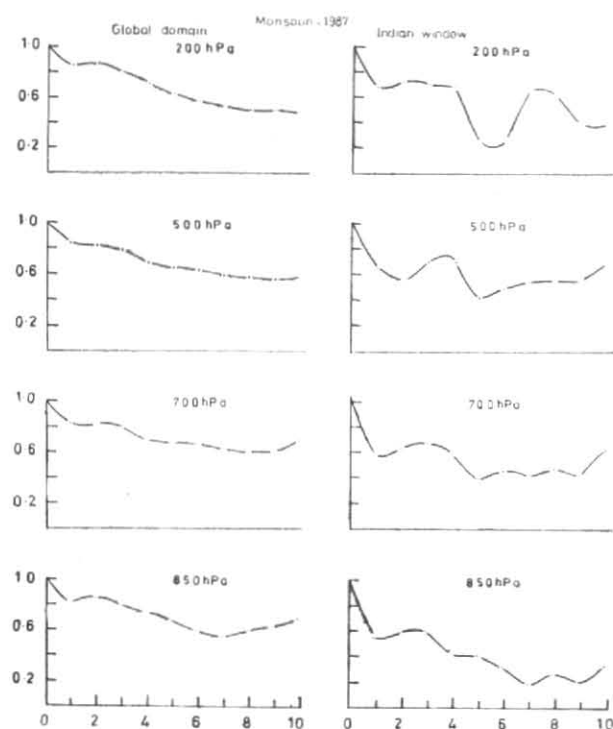


Fig. 1. Correlation coefficients between forecast and analysis changes in geopotential height (11-15 Aug)

In the sensitivity study the model was run at an upgraded vertical resolution of 18 layers for a 23-day period starting with the initial fields of 12 GMT of 15 December 1986 and the model outputs compared with the 12-layer version run reported in an earlier paper (Basu 1990). Sensitivity of model forecasts to vertical resolution has been studied by various authors (Kirkwood and Derome 1977, Lambert 1980, Derome 1981, Mechaso *et al.* 1982, Simmons and Strufing 1983, Anthes *et al.* 1989) in the past. Most of these work deal with the effect of raising the top of the model atmosphere by adding new layers on top of the existing ones. The data used in these cases are usually the outputs from a data assimilation system using the lower resolution model forecast as first guess. In the present study the top of the atmosphere is fixed for both the versions of the model and the additional layers are introduced only in the lower and middle troposphere (below 300 hPa). Comparing the sigma thickness (Table 1) of the two versions of the model it can be seen that close to ground the higher resolution model has more than three layers covering the same thickness of atmosphere covered by one layer in the earlier version. This finer resolution within the boundary layer is required as the change in resolution was done with the intention of introducing new packages dealing with surface physics and radiation. The data set used for verification was obtained from a GDAS system which used the forecast from the 18-layer full physics model as first guess.

The mean errors between forecast and verification temperature fields have been averaged over a 20-day period (4th day to 23rd day of forecast) for the global

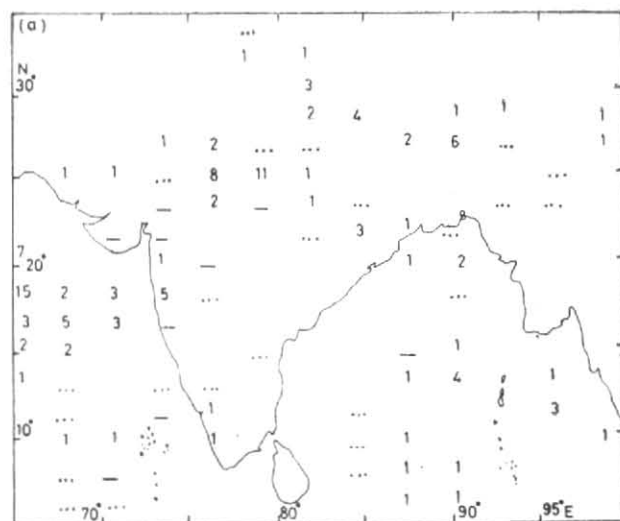


Fig. 2(a). Model (L12) predicted rainfall accumulated for the period 11-15 August 1987

domain, the northern hemispheric tropical belt (equator to  $31^{\circ}\text{N}$ ) and the Indian window have been computed for both the 12-layer and the 18-layer versions of the model.

## 2. Model topography

The spectral coefficients of the topography used in the model is formed as follows. At each point on the Gaussian grid of the model topographic height value is interpolated from the US Navy data set which has a resolution of 10 minutes of longitude/latitude. A fast Fourier transformation is done to convert these grid point values into the Fourier coefficients at each latitude which are then added up by a Gaussian quadrature to form the full spectral coefficients. These coefficients are passed through a Laplacian filter twice to get the smoothed topography actually used in the model. In the rhomboidal 40-wave resolution the model topography over the Indian region, as shown in Fig. 3, compares poorly with the actual topographic features of India and neighbourhood. The smoothed model topography spreads 4 to 5 degrees of latitude to the south of the Himalayas and smaller features like the hills of northeast India and the Burmese hills are not represented individually. Similarly, the Western Ghats are merged into a large elevated land covering the whole of the Peninsular India and parts of Rajasthan, Madhya Pradesh and Orissa. The low truncation number of the model leads to many dips over the sea areas and a large depression over the Gangetic West Bengal. The real topography has sharp slopes which gives rise to strong upward motion over a short distance in the windward side of the hills and often results in large amounts of

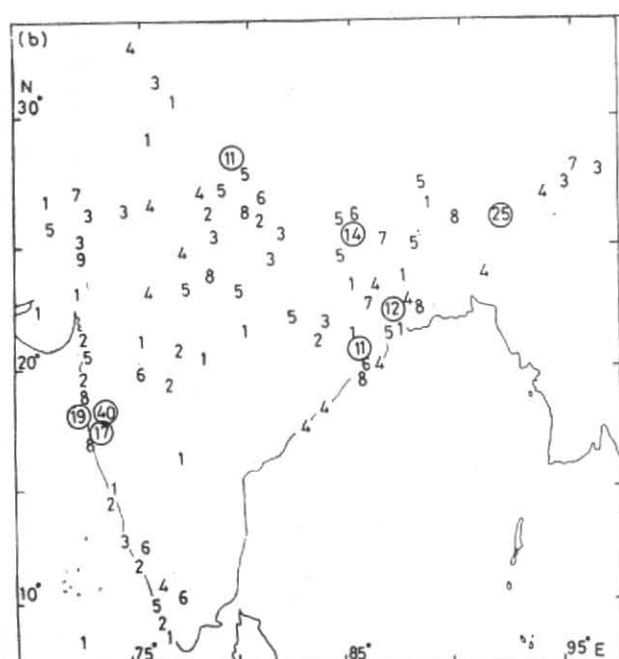


Fig. 2(b). Same as 2(a) except for observed rainfall

rainfall over a small area. The smoothed model topography leads to spreading of the upward motion field and the resulting rainfall over a much larger area with reduced intensity. Thus, the concentrated zones of heavy orographic rainfall cannot be simulated by the model unless the spectral resolution is increased sufficiently to ensure a more realistic representation of the topography.

### 3. Data

The data used to start the model integration and also for verification of forecasts are the optimum interpolation analysis done at NMC, Washington. This analysis is done over grid suitable to get unaliased rhomboidal 40-wave coefficients but is archived at the 30-wave resolution. This implies that though the mean field remains the same, the small scale features represented by wave numbers 31 to 40 are lost during archiving. In the present study, these 30-wave coefficients at constant pressure levels were augmented by zeroes to form the 40-wave coefficients required to start the model. The forecast integration generates higher wave numbers by non-linear interaction of lower ones. Thus, after the first time step wave numbers up to 91 ( $3J+1$ , where  $J$  is the highest wave number with non-zero coefficient) are created out of which only first 40 are retained as input for the next step. Thereafter, wave numbers up to 121 are created during the forecast integration at each time step. Thus, after a large number of time steps (time step for the present resolution is 12 minutes) the effect of zero coefficients for wave numbers between 31 and 40 will no longer have significant impact on the forecast fields.

The verification statistics has been obtained at 40-wave resolution, again by padding up the 30-wave analysis by zeroes. This implies that the coefficients generated by the model between wave numbers 31 and 40 contributes only to forecast error during verification and

TABLE 1

Sigma thickness of model layers

Layer	1	2	3	4	5	6	7	8	9	>10
L12	.075	.125	.150	.150	.125	.075	.050	.050	.050	.050
L18	.010	.017	.025	.055	.073	.085	.093	.096	.096	.050

L12 — 12-layer model, L18 — 18-layer model

use of 40-wave coefficients or comparison at 30-wave resolution will improve the correlation coefficients. The conclusions arrived at about the sensitivity of the model to vertical resolution is, however, not significantly affected by the above considerations as results from both the runs are compared with same analysis.

It may further be noted that the GDAS system producing the analysis uses the 6-hourly forecast (after normal mode initialisation) produced by a 40-wave, 18-layer global spectral model, with radiation and elaborate surface physics parameterisation, as the first guess input for optimum interpolation method of data analysis. In data sparse regions the analysis is essentially the same as the first guess. Statistics for data reception at NMC for the month of October 1986 shows that on the average upper air data from 16 to 17 Indian stations reach the centre within a period of 12 hours of observation time. Since, the cut-off time for final analysis at NMC is 6-hr for 00 GMT and 8.5-hr for 12 GMT, it may be presumed that upper air data from 10 to 14 Indian stations are used for analysis. Out of these, data from some of the stations are rejected as they do not satisfy the quality control checks. Thus, the analysis over India and adjoining sea areas reflect mostly the 18-layer model climate.

### 4. Results

Results of verification presented below should be viewed with the following considerations in mind :

- (i) A 12-layer, 40-wave model of limited physics has been run using the 30-wave coefficients produced by a data assimilation system which uses the six-hour forecasts of a 18-layer model with detailed physics. Analysed fields used for verification are uninitialised,
- (ii) The Indian window has, within its domain, the largest plateau and the tallest mountain in the world, and
- (iii) In tropics, persistence is a much better estimate of realised fields than in extratropics.

A scrutiny of the root mean square errors (RMSE) of the forecast surface pressure and geopotential heights (Table 2) show that, for the global domain, these are better than persistence for 4 to 5 days. For the Indian window, however, RMSE of surface pressure and geopotential heights below 700 hPa are worse than those of persistence even for 24-hour forecasts. For levels between 700 hPa and 200 hPa of forecast geopotentials are some what better than those of persistence up to 72 hours. It may be noted here that during

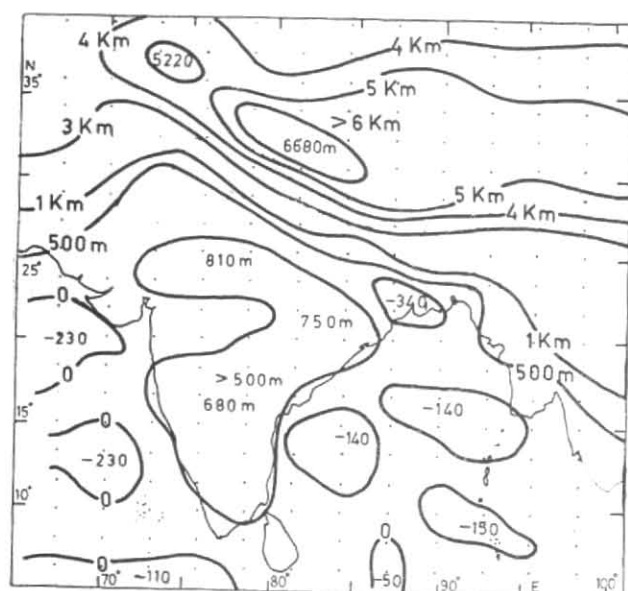
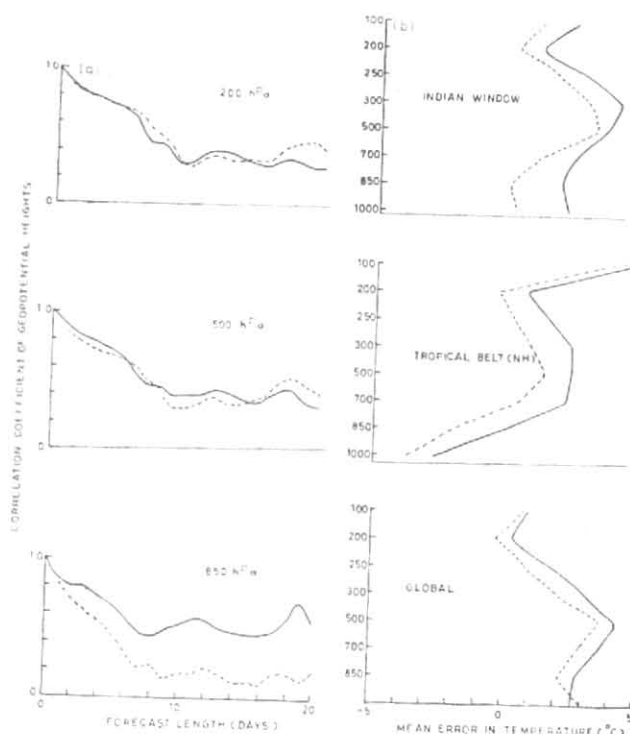


Fig. 3. Representation of topography in rhomboidal 40-wave resolution

monsoon RMSE values of forecast surface pressure and geopotentials are much smaller for the Indian window compared to those for the global domain. A comparison with the winter case shows that RMSE of surface pressure, geopotential and wind are better in the winter over the Indian region up to 500 hPa. The reason for this, is the presence of organised systems like monsoon depression in the lower and middle troposphere during monsoon. A small error in the location of these system leads to large RMSE values as these systems have strong pressure gradient associated with them.

Root mean square, vector wind errors (Table 3) also show a similar trend. For the global domain forecast winds have less RMSE than persistence even up to 10 days. For the Indian window forecast winds at levels below 700 hPa are worse than persistence even at 24 hours while at upper levels forecast winds are better than persistence upto 10 days. The rainfall forecast, Fig. 2(a) produced by the model have been subjectively compared with actual rainfall values, Fig. 2(b). It may be noted that the whole of India is covered by about 65 points of the model Gaussian grid on which physical processes are computed. Thus, each grid point represents an average over an area of about 50,000 square km. Considering the spatial variability of rainfall, an average over such a large area is not meaningful for the purpose of comparison with point observations.



Figs. 4(a & b). (a) Correlation coefficients between forecast and analysis changes in geopotential height (winter 1986) and (b) Mean error in temperature in degrees celsius (winter 1986). Solid lines are for L12 and dashed lines for L18

During the period under study significant rainfall amounts were predicted only over a few of the Gaussian grid points during any individual 24-hour period and over the whole of the 120-hour period about 15 grid points showed significant accumulated rainfall amounts. Due to the absence of any organised synoptic scale system, no definite spatial or temporal pattern could be discerned in either the observed or the forecast rainfall amounts. Since, conclusions derived from comparison of rainfall amounts during individual 24-hour periods are not much different, only the accumulated picture over the 120-hour period is presented in Figs. 2(a) and 2(b). A comparison of these two figures clearly shows the absence of concentrated heavy orographic rainfall areas in the predicted field though these are present in the realised one.

For the experiment on sensitivity verification statistics for surface pressure, geopotential heights and winds at standard pressure levels, for the globe as well as the Indian window have been prepared for the 18-layer model version. Correlation coefficients between forecast and analysis height fields are presented in Fig. 4(a). In Fig. 4(b) mean error between forecast and verification fields, averaged over a 20-day period, are presented for the global domain, the northern hemispheric tropical belt and the Indian window.

It is seen that the root mean square error (RMSE) of forecasts for surface pressure and geopotential



TABLE 2

Model verification for 11-17 August 1987 (pressure and geopotential height)  
 Root mean square error (RMSE) for surface pressure and geopotential heights at standard millibaric levels. Quantities  
 in bracket are corresponding values for persistence

Pressure (hPa)	24-hr	48-hr	72-hr	96-hr	120-hr	144-hr	168-hr	192-hr	216-hr	240-hr
<b>Global</b>										
Surface	4.8 (6.0)	6.0 (7.5)	7.1 (8.7)	8.2 (9.2)	9.3 (9.6)	9.4 (9.8)	10.0 (10.1)	9.9 (10.1)	9.5 (10.0)	9.7 (10.5)
<b>Height (gpm)</b>										
1000	37.6 (48.6)	48.1 (60.1)	57.3 (70.5)	66.4 (74.3)	75.1 (77.7)	76.8 (79.8)	81.1 (82.2)	81.0 (82.0)	78.0 (81.6)	80.3 (85.3)
850	28.7 (45.1)	38.5 (55.7)	46.4 (66.2)	54.3 (70.5)	62.0 (73.9)	64.9 (75.9)	68.8 (77.6)	68.5 (78.1)	67.8 (78.7)	70.5 (82.5)
700	26.3 (45.1)	38.1 (57.5)	47.3 (69.3)	57.5 (74.9)	64.7 (78.8)	70.6 (80.7)	75.9 (82.0)	78.4 (83.3)	81.7 (84.7)	86.3 (88.3)
500	31.2 (55.5)	50.5 (72.0)	67.3 (87.4)	81.5 (95.3)	94.1 (100.7)	106.4 (102.6)	115.8 (104.1)	124.1 (107.7)	132.1 (110.5)	141.7 (114.3)
300	42.8 (75.1)	72.4 (98.7)	99.0 (119.1)	121.3 (130.5)	142.3 (139.2)	162.3 (141.7)	178.5 (143.3)	193.1 (148.1)	205.6 (151.1)	221.2 (155.0)
250	43.6 (75.3)	74.2 (100.6)	101.9 (122.2)	125.8 (134.2)	148.7 (143.7)	170.4 (146.4)	188.5 (147.4)	204.5 (151.5)	218.3 (154.8)	235.1 (159.8)
200	42.7 (69.8)	72.3 (95.3)	98.7 (116.5)	123.2 (128.7)	146.9 (138.5)	169.3 (140.6)	188.9 (140.4)	205.7 (143.6)	220.9 (148.1)	238.5 (155.3)
100	46.4 (55.3)	70.5 (76.5)	92.9 (93.0)	115.2 (104.6)	137.3 (115.7)	161.1 (117.4)	181.2 (114.7)	197.4 (116.3)	211.0 (123.9)	225.5 (141.6)
<b>Indian window</b>										
Surface	4.2 (2.4)	5.0 (3.3)	6.3 (3.6)	7.6 (3.3)	8.7 (3.4)	9.5 (3.4)	9.3 (3.7)	9.2 (3.9)	9.5 (4.1)	10.5 (4.3)
<b>Height (gpm)</b>										
1000	36.8 (20.7)	44.1 (29.1)	55.4 (31.6)	66.1 (30.8)	75.4 (29.5)	83.0 (30.0)	81.4 (32.3)	79.8 (33.8)	83.1 (35.6)	90.8 (37.0)
850	24.4 (17.0)	28.1 (24.2)	35.2 (27.2)	43.2 (27.1)	50.7 (26.4)	57.0 (27.7)	54.7 (30.7)	53.0 (32.7)	56.1 (34.5)	62.7 (36.2)
700	18.4 (16.7)	21.6 (23.6)	25.6 (26.9)	30.6 (27.2)	36.9 (26.7)	41.0 (28.8)	38.5 (32.1)	36.1 (34.2)	38.5 (35.5)	41.1 (36.0)
500	17.3 (21.9)	22.9 (29.6)	28.3 (33.3)	34.4 (33.5)	41.3 (33.0)	44.8 (36.5)	42.8 (40.1)	38.5 (42.2)	40.0 (42.3)	40.8 (43.7)
300	24.5 (33.6)	32.7 (44.9)	42.8 (48.3)	54.5 (45.8)	63.9 (46.4)	68.5 (51.7)	65.9 (55.3)	58.2 (56.9)	58.8 (57.0)	58.6 (59.1)
250	26.7 (36.4)	36.0 (48.8)	48.0 (51.4)	61.5 (48.9)	72.0 (49.5)	77.4 (55.6)	74.3 (59.4)	66.5 (61.9)	67.8 (62.9)	68.1 (65.5)
200	28.9 (38.8)	37.3 (49.9)	48.7 (51.0)	63.7 (48.0)	75.5 (48.4)	81.8 (55.80)	78.7 (59.9)	71.7 (63.0)	73.3 (65.0)	75.7 (68.2)
100	38.8 (33.7)	43.2 (39.8)	46.9 (40.6)	61.0 (40.0)	69.8 (41.7)	75.3 (47.1)	77.6 (49.6)	74.2 (53.0)	73.7 (55.5)	72.9 (57.1)

are significantly better than that of persistence for a forecast period up to 5 days for the global domain. For the Indian window corresponding values of forecast lengths are less by two days.

The RMSE of forecast wind field in winter is better than that of persistence for about 6 days (except at lower troposphere over Indian window) for both the global domain and the Indian window. The difference between forecast and persistence is more significant at upper tropospheric levels than that at lower and middle troposphere. At 100 hPa, the small value of persistence may be due to lack of observation at that level.

A scrutiny of the correlation coefficients for the global domain shows that for the 12-layer model these are better than 0.70 upto a forecast length of 5 days. For the 18-layer model the correlation coefficients for geopotential height is much less than the 12-layer

version at the 1000 hPa level but equals or betters the latter at middle (500 hPa) and upper (250 hPa) tropospheric levels. This may be due to the fact that the 18-layer version has three layers close to the surface but no physical package to account for the surface processes.

The mean temperature errors, averaged over the 20-day period shows that the 12-layer version has smaller mean error than the 18-layer version except at 1000 hPa for the northern hemispheric tropical belt.

##### 5. Conclusions

The average of seven 120-hr forecasts during an active monsoon period of 1987 shows that correlation between forecast and analysis changes decreases sharply from winter to monsoon period as far as India and neighbourhood is concerned. Further, the model in

TABLE 3

Model verification for 11-17 August 1987 (wind)  
 Root mean square error (RMSE) for winds at standard millibaric levels. Quantities in bracket are corresponding values for persistence

Pressure (hPa)	Wind (m/s)									
	24-hr	48-hr	72-hr	96-hr	120-hr	144-hr	168-hr	192-hr	216-hr	240-hr
<b>Global</b>										
1000	7.6 (9.5)	8.8 (10.5)	9.6 (11.1)	10.2 (11.1)	10.7 (11.3)	11.0 (11.5)	11.0 (12.0)	11.0 (11.7)	11.1 (11.5)	10.9 (11.5)
850	7.5 (10.3)	8.6 (11.3)	9.7 (11.4)	10.3 (12.1)	10.7 (12.4)	11.1 (12.5)	11.2 (12.8)	11.2 (12.8)	11.1 (12.3)	11.0 (12.7)
700	7.0 (10.4)	8.4 (11.3)	9.3 (12.2)	10.0 (12.6)	10.6 (13.0)	11.1 (12.9)	11.4 (13.5)	11.3 (13.0)	11.1 (13.0)	11.1 (13.1)
500	8.1 (12.6)	9.9 (14.2)	11.4 (15.5)	12.2 (15.9)	12.9 (16.6)	13.4 (16.7)	13.6 (17.2)	14.5 (17.1)	14.2 (17.2)	14.2 (17.1)
300	11.5 (17.4)	13.9 (20.7)	15.8 (21.9)	16.9 (22.8)	18.1 (23.6)	19.4 (23.7)	20.4 (24.3)	20.3 (24.1)	20.0 (24.3)	20.3 (24.9)
250	11.3 (17.0)	13.5 (20.3)	15.5 (22.1)	16.8 (23.1)	18.2 (23.9)	19.5 (23.9)	20.6 (24.5)	20.5 (24.6)	20.5 (24.9)	20.7 (25.5)
200	10.7 (15.2)	12.2 (18.3)	13.7 (20.3)	15.1 (20.9)	16.4 (21.7)	17.6 (21.6)	18.8 (22.5)	18.7 (22.5)	18.9 (22.6)	19.3 (23.1)
100	8.5 (10.2)	9.2 (11.9)	10.3 (13.1)	10.6 (13.4)	11.5 (14.3)	12.3 (14.5)	13.2 (14.5)	13.7 (14.7)	14.1 (15.2)	14.3 (16.2)
<b>Indian window</b>										
1000	6.3 (5.3)	6.6 (6.3)	6.9 (6.6)	7.3 (6.4)	7.6 (6.0)	7.9 (5.8)	8.5 (6.1)	8.4 (6.5)	8.6 (7.0)	8.9 (6.9)
850	6.0 (5.5)	6.9 (7.0)	7.6 (7.6)	8.4 (7.4)	8.7 (6.7)	9.0 (6.4)	10.0 (6.8)	9.8 (7.4)	10.1 (8.2)	10.3 (7.7)
700	5.5 (5.6)	6.3 (7.4)	7.2 (8.1)	7.9 (8.0)	8.5 (7.4)	8.9 (7.3)	9.4 (7.7)	9.4 (8.1)	9.3 (8.5)	9.3 (8.3)
500	5.1 (6.4)	6.4 (8.5)	7.1 (9.4)	8.0 (9.3)	8.7 (8.7)	9.0 (8.8)	9.1 (9.2)	8.9 (9.5)	9.0 (9.5)	8.7 (9.3)
300	7.0 (8.9)	8.7 (11.5)	9.9 (12.4)	10.8 (11.7)	11.5 (11.4)	12.0 (12.3)	11.7 (12.9)	11.7 (12.9)	11.0 (12.6)	10.7 (12.6)
250	7.9 (9.7)	9.4 (12.5)	10.7 (13.4)	11.7 (12.8)	12.5 (12.3)	13.3 (13.2)	13.0 (14.0)	12.3 (14.2)	12.4 (14.1)	12.5 (14.2)
200	8.3 (10.1)	9.9 (12.9)	11.1 (13.8)	12.5 (13.1)	13.3 (12.5)	14.1 (13.5)	14.2 (14.5)	13.7 (14.8)	13.8 (14.7)	14.0 (14.4)
100	8.2 (8.2)	9.1 (9.5)	9.7 (10.1)	10.3 (10.3)	11.1 (9.7)	11.6 (9.5)	12.7 (9.8)	13.7 (10.4)	13.7 (11.6)	13.9 (12.3)

its present resolution and in absence of adequate physical packages is not suitable for direct prediction of rainfall, particularly, during monsoon. The model, however, has some skill in prediction of geopotential height and wind especially in the middle and upper troposphere. A comparison of RMSE of 12-layer and 18-layer versions of the model show that the model version with lower vertical resolution yields better result. A plausible explanation of this is that the changes introduced by greater vertical resolution is in a direction opposite to that due to the physical processes like radiation, boundary layer physics and shallow convection. A model forecast using these physical packages was used by the data assimilation system at NMC.

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