SESSION V

DIAGNOSTIC STUDIES—DYNAMICAL

CHAIRMAN : DR. J. KUETTNER

NCAR Boulder, USA

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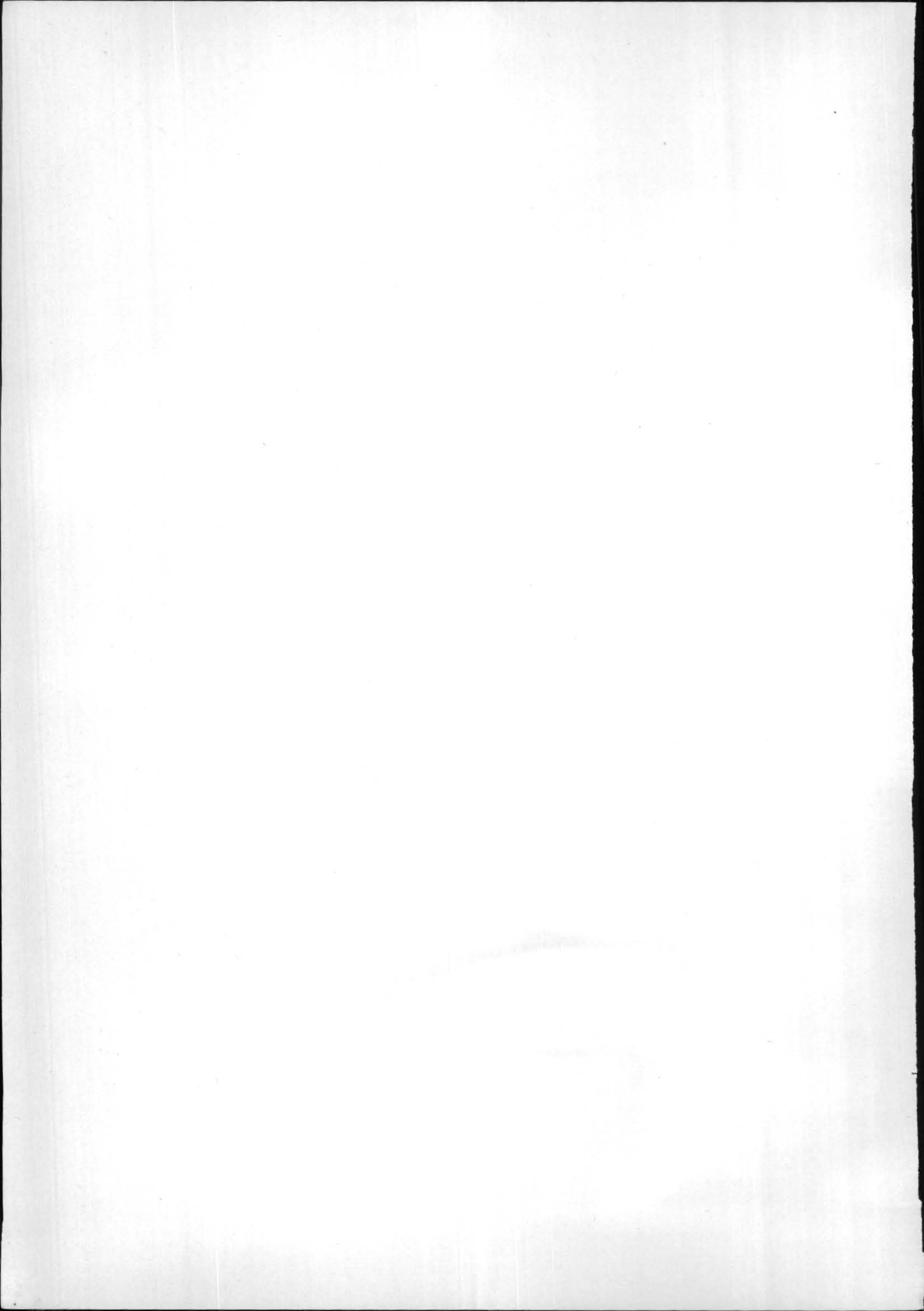
Lateral effects on monsoon systems*

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ABSTRACT. A theoretical study of the interaction of remote effects on the monsoon system is described. Such effects, in particular the cross-equatorial influence of the rapidly evolving southern hemisphere equinoctial circulation in the late northern hemisphere spring and again in the fall, have been speculated from observation.

To study such interaction, a primitive barotropic model is developed which contains a basic flow in the equatorial regions which is a function of both latitude and longitude. Such a flow contains the influence of the large-scale stationary eddies of low-latitudes which has the effect of producing a "critical latitude" that is also a function of both longitude and latitude. In this manner equatorward propagating energy from mid-latitude disturbances has the opportunity to propagate through the equatorial regions and thus modify the basic structure of the monsoon regime.

1. Introduction

Specifically addressing the problem of the influence of mid-latitude phenomena on equatorial motions, Charney (1969) and Bennett and Young (1971) defined the selectiveness of a basic field to lateral (*i.e.*, north-south) wave propagation. Waves with phase speeds somewhere equal to that of the basic flow at some latitude (the location being referred to as the "critical latitude" or in a more general sense as the "critical level") are absorbed or interact with the mean flow at that latitude. Only extremely large-scale waves which would propagate westward faster than the assumed westward basic flow were free to propagate across the equator. Such modes, argued Charney, possessed sufficiently small energy to be of significance and those mid-latitude modes of sufficiently high energy to invoke a significant response within the tropics would be successfully precluded by an efficient basic flow filter.

If the arguments of Charney and Bennett and Young did apply to the real atmosphere the large scale and high amplitude disturbances propagating through the westerlies of the southern hemisphere (Webster and Curtin 1974, 1975), the growing quasi-stationary modes of the transitional seasons in either hemisphere and the intense winter disturbances occurring over eastern Asia and the western Pacific Ocean would appear to impart little impact on near-equatorial motions or phenomena in the adjacent hemisphere. In effect then each hemisphere would be effectively independent on time scales less than seasonal.

Theoretical predictions to the contrary, there is mounting observational evidence of considerable dynamical interaction between the hemispheres. For example, relationships between the upper tropospheric westerlies were discussed by Radok and Grant (1957) in the Australasian and Indian Ocean sectors. Considerable momentum flux by
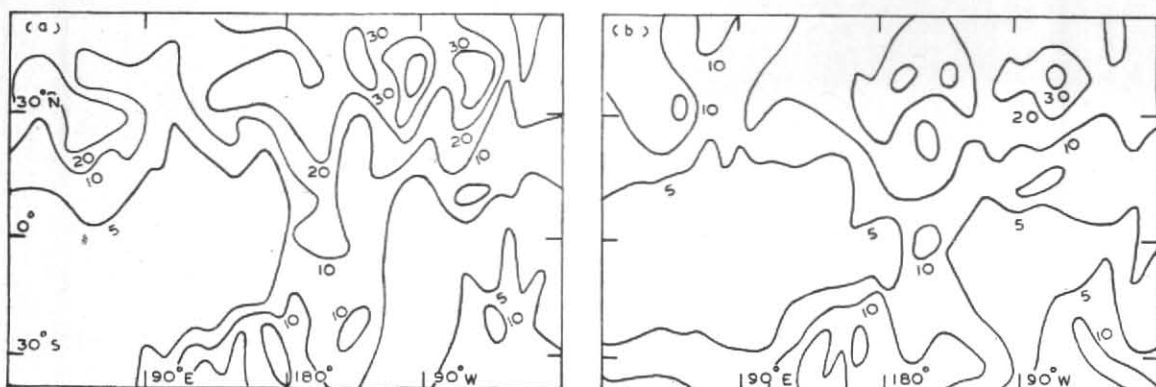
Fig. 1.   200 mb perturbation kinetic energy (m² s⁻²) for (a) January 1971 and (b) February 1971 (from Murakami and Unninayar 1977)

processes of subseasonal time scales was shown to exist by Tucker (1965) and later substantiated by Newell *et al.* (1972). The detailed mean monthly analysis of Sadler (1975), which show horizontal stream line analyses indicate large-scale systems with inter-hemispheric structure which show considerable variation from month to month. Webster and Curtin (1975) discuss the preference for large scale and slowly propagating mid-latitude upper tropospheric troughs to develop intense southeast to northwest tilt with latitude in the Pacific Ocean, an orientation essential for equatorial wave energy propagation. That such waves do indeed influence equatorial regions is suggested by Murakami and Unninayar (1977). Using the NMC data sets they compiled a number of statistics including distributions of perturbation kinetic energy. Examples at the 200-mb level for January and February 1971 are shown in Fig. 1. Three regions are immediately apparent for both months. Two are coincident with the mid-latitude storm tracks whereas the third, the weaker, centre straddles the equator. The third region resides over the equatorial Pacific Ocean. A similar region, although even weaker exists over the equatorial Atlantic Ocean.

The problem then reduces to reconciling the two apparently different indicators of inter-hemisphere interaction. In the following paragraphs we will argue that the differences between theory and observations resides not in errors in the theoretical arguments nor in misinterpretation of the observa-

tions, but rather the consideration of far too simple basic fields in the models of Charney (1969) and Bennett and Young (1971). Further, we will argue that if a more realistic basic field is allowed, the equatorial region may be considered to be significantly more porous to incident mid-latitude disturbances.

2. The basic fields

Charney (1969) and Bennett and Young (1971) assumed that the basic flow* of the equatorial regions was one dimensional with moderate easterlies predominating at the equator. However, in reality the basic flow is also a strong function of longitude (Webster 1972, 1973) such that on a seasonal time scale regions of easterlies and westerlies exist along the equator although the mean wind is westward. In other words the basic flow may be thought of as a principally easterly regime but with a strong superposition of wave numbers 1 and 2 (Webster 1972).

Fig. 2 shows the 200 mb zonal wind field from Newell *et al.* (1972) for the two seasons, December to February and March to May. Obvious in both charts is the variability of the mean near-equatorial flow. In particular, the regions of equatorial westerlies coincide rather well with the regions of maximum near-equatorial perturbation kinetic energy; the mid-pacific Ocean and the mid-Atlantic Ocean. It is the aim of this paper to consider the influence of propagating mid-latitude disturbances

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*The basic flow may be defined to be that component of the total atmospheric state which possesses a time scale significantly longer than the characteristic time scale of the propagating mid-latitude disturbance.
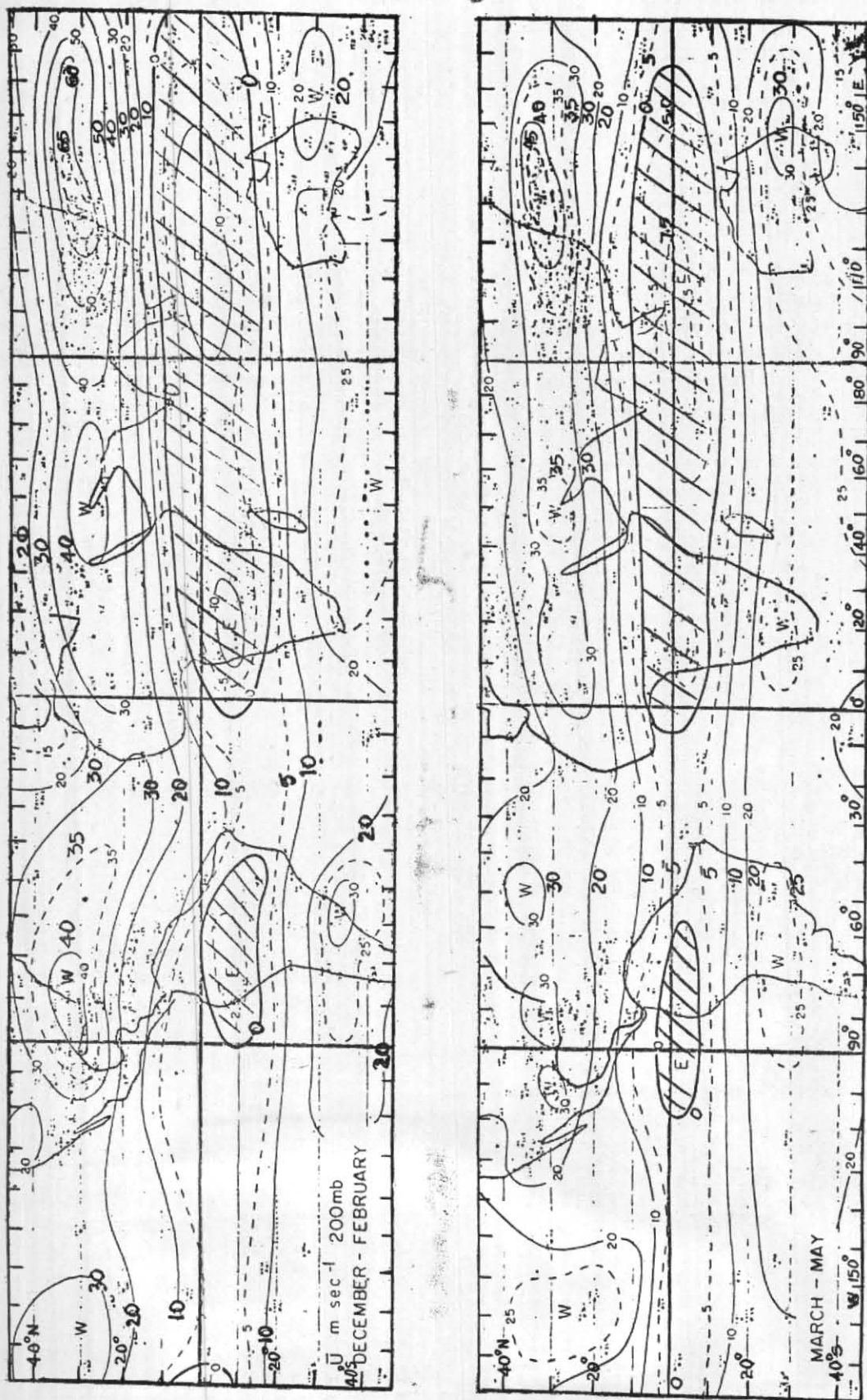
Fig. 2. 200 mb zonal wind field (ms⁻¹) for December-February and March-May (from Newell et al. 1972)

on the tropical atmosphere defined by a basic flow which is a strong function of longitude and latitude. In particular, we are interested in whether such a flow possesses different transmission characteristics to the one dimensional flow considered by Charney and Bennett and Young.

3. The model

For an initial study, a free-surface barotropic primitive equation model is developed. A Mercator projection is used to represent the equatorial channel confined by rigid boundaries at 50°N and 50°S. The governing equations are :

$$u_t - fv + m(uu_x + vu_y) = -gmH_x - au \quad (1)$$
$$v_t + fu + m(uv_x + vv_y) = -gmH_y - av \quad (2)$$
$$H_t + m[(uH)_x + (vH)_y] - Hvm_y = F \quad (3)$$

u and v are the depth averaged horizontal velocity components, H is the total depth of the column, a is a dissipative coefficient, F some forcing coefficient and m is the Mercator map factor.

In anticipation of the method of solution we group the nonlinear terms so that (1)-(3) become :

$$u_t - fv + I(u) = -gmH_x - au \quad (4)$$
$$v_t + fu + I(v) = -gmH_y - av \quad (5)$$
$$H_t + I(H) = F \quad (6)$$

where, $I(u) = m(uu_x + vv_y)$
$$I(v) = m(uv_x + vv_y) \quad (7)$$
and $I(H) = m[(uH)_x + (vH)_y] - Hvm_y$

Two basic methods are available for the solution of the nonlinear set (4)-(6). These are the grid-point method and the spectral method. Because of the wave-wave interaction aspects of the problem, the latter method is most useful. A fully spectral representation of each variable (*i.e.*, spherical harmonics) is difficult because of the latitudinal truncation of the equatorial channel. To overcome this a "semi-spectral" resolution is adapted in which only eigen-functions in longitude are sought and full variability in latitude retained. That is, we set;

$$u(x,y,t) = \sum_n u_n(y,t) \cos nx + \bar{u}_n(y,t) \sin nx,$$

$$v(x,y,t) = \sum_n v_n(y,t) \cos nx + \bar{v}_n(y,t) \sin nx.$$

$$H(x,y,t) = \sum_n h_n(y,t) \cos nx + \bar{h}_n(y,t) \sin nx$$

and

$$F(x,y,t,) = \sum_n F_n(y,t) \cos nx + \bar{F}_n(y,t) \sin nx$$

$$(8)$$

Substitution of (8) into (7) raises another serious problem. The nonlinear terms, which represent the wave-wave interactions, produce triple sums which require extreme amounts of computer time to calculate. To avoid this we utilize a modification of the Fourier-grid transform method (the "flip-flop" scheme) in which all linear arithmetic is computed in spectral-space whilst nonlinear arithmetic is carried out in grid-space. The efficiency of this techniques is shown by Bourke (1972). Access between the two domains is accomplished by fast Fourier transform techniques. The grid spacing is so chosen to exactly represent the Fourier components. In summary we express u, v and $H(x, y, t)$ in grid space by inverse-transforming (8). The nonlinear terms (7) are computed in grid space and then Fourier-transformed so that;

$$I[u(x,y,t)] = \sum_n a_n(y,t) \cos nx + \bar{a}_n(y,t) \sin nx,$$

$$I[v(x,y,t)] = \sum_n b_n(y,t) \cos nx + \bar{b}_n(y,t) \sin nx$$

and

$$I[H(x,y,t)] = \sum_n c_n(y,t) \cos nx + \bar{c}_n(y,t) \sin nx.$$

$$(9)$$

$a_n, \bar{a}_n, b_n, \ldots\ldots$ and etc may then be considered known quantities and treated as linear terms in (4)-(6). With (8) and (9) in (4)-(6), (6) the following "linear" equations emerge in the Fourier coefficients of (8):

$$(u_n)_t - fv_n + a_n = -gmn\bar{h}_n - au_n \quad (10)$$
$$(\bar{u}_n)_t - f\bar{v}_n + \bar{a}_n = gmnh_n - a\bar{u}_n \quad (11)$$
$$(v_n)_t + fu_n + b_n = -gm(\bar{h}_n)_y - av_n \quad (12)$$
$$(\bar{v}_n)_t + f\bar{u}_n + \bar{b}_n = -gm(\bar{h}_n)_y - a\bar{v}_n \quad (13)$$
$$(h_n)_t + mH_0(v_n)_y + mnH_0\bar{u}_n - H_0\bar{v}_nm_y + \bar{c}_n = \bar{P}_n \quad (14)$$
$$(\bar{h}_n)_t + mH_0(\bar{v}_n)_y - mnH_0u_n - H\bar{v}_nm_y + \bar{c}_n = \bar{P}_n \quad (15)$$

A set of such equations exist for each wave number $n=0$ to N.

(10)-(11) are represented in a staggered space-difference form in y. Boundary conditions are that the zonal flow must be geostrophic at the boundaries and that the meridional flow must vanish. A semi-implicit time differencing scheme is used following Holton (1976) where we relate a quantity X at time step J to X at $J-1$ and $J+1$ by :

$$X^* = [X^{J-1} + 2X^J + X^{J+1}]/4 ,$$

so that $X_t = (X^{J+1} - X^{J-1})/\triangle t$
$$= [2X^* - (X^J + X^{J-1})]/\triangle t ,$$

so that equations (10)-15) become sets of algebraic equations at each of $K+1$ latitudinal grid points with unknowns u^*, $\bar{u}^*$, v^* and etc. Elimination in (10)-(15) produces two equations sets in h^* and $\bar{h}^*$ which altogether produce $2(N+1)(K+1)$ sets of simultaneous, although linear, equations which are solved by an efficient matrix inversion technique. The numerical scheme is extremely stable allowing time steps in excess of one hour.

4. Three experiments

As an initial attempt to study the cross-equatorial propagation of waves generated in mid-latitudes through a nonhomogenous media, three simple experiments were performed. The experiments may be summarized as follows :
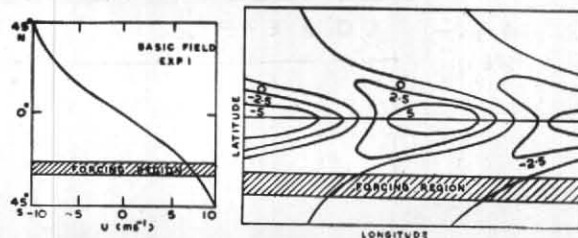

Fig. 3. Exp. 1   Fig. 4. Exp. 2 and 3
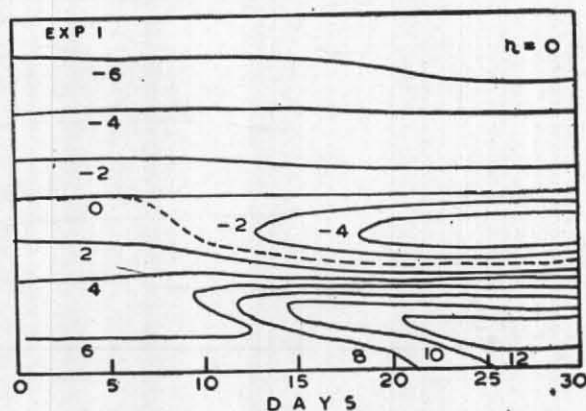
Basic zonal wind field (Forcing region is hatched)

Fig. 5. Evolution of the zonal wind ($n=0$) for Experiment 1 as a function of time and latitude. Heavy line is the zero contour, negative values denote easterlies and positive values westerlies.

Experiment	Basic Flow	Forcing
1	$u = A \tanh y/a$	$h = C \cos x \exp [(y_0-y)^2/c]$
2	$h = H_0 + B \cos x \exp (-y^2/b)$	$h = C \cos x \exp [(y_0-y)^2/c]$
3	$h = H_0 + B \cos x \exp (-y^2/b)$	$h = C \sin x \exp [(y_0-y)^2/c]$

In each case the forcing was allowed to grow slowly for four days at which time it approached its maximum value. Experiment 1 is summarized in Fig. 3 which shows a basic zonal field which is independent of x. Experiments 2 and 3 are shown in Fig. 4 and represent a basic field which is a function of x and y, or more precisely, and exponentially equatorially located wave number 1 field with forcing occurring at $y=y_0$. Experiment 3 is the same except that the forcing is out-of-phase with the basic flow.

5. Results

All experiments were run for a thirty-day period with the forcing at the same southern hemisphere latitude ($y=y_0$) and with no dissipation. The results are presented for the $n=0$ and 1 modes in

Figs. 5 to 7 in the form of contours of basic zonal flow for $n=0$ and as amplitude-phase vector diagrams for $n=1$. Each field is shown as a function of time (abscissa) and latitude (ordinate).

The major feature of Experiment 1 is the rapid southward propagation of the zero contour (the critical level for a steady wave) toward the latitude of maximum forcing (indicated by an arrow). This occurs concurrently with the intensification of the westerlies to the south of the critical level and of the easterlies to the north. Similar features are observed for both Experiments 2 and 3, both of which do not initially possess a $n=0$ field. The westerly and easterly maximum rapidly develop in such a manner as to establish a critical latitude between the forcing latitude and the fluid to the north. The $n=1$ field for both Experiments 2 and 3
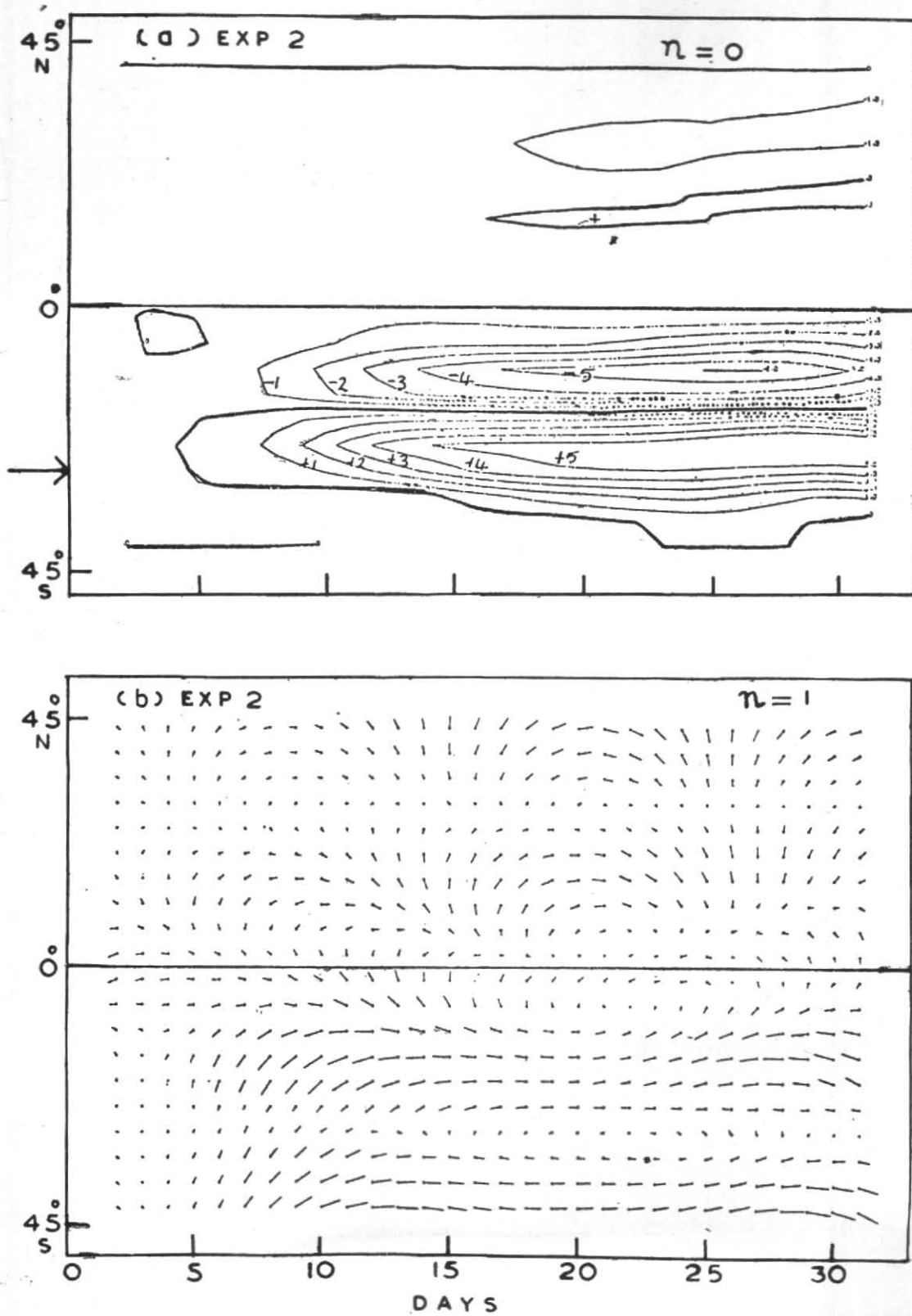
Fig. 6. (a). Same as Fig. 5 but for Experiment 2

(b). Variation of the amplitude and phase for Experiment 3 of the $n=1$ mode as a function of
time and latitude. Anti-clockwise rotation of the vector with time denotes westward
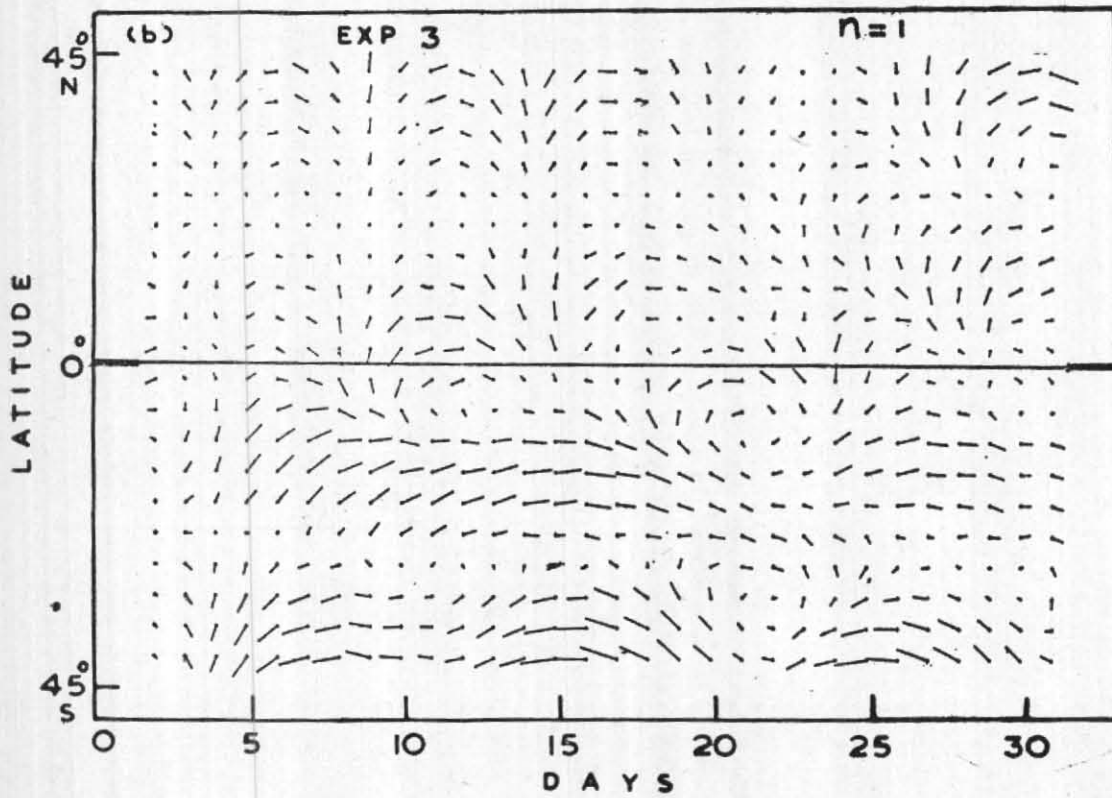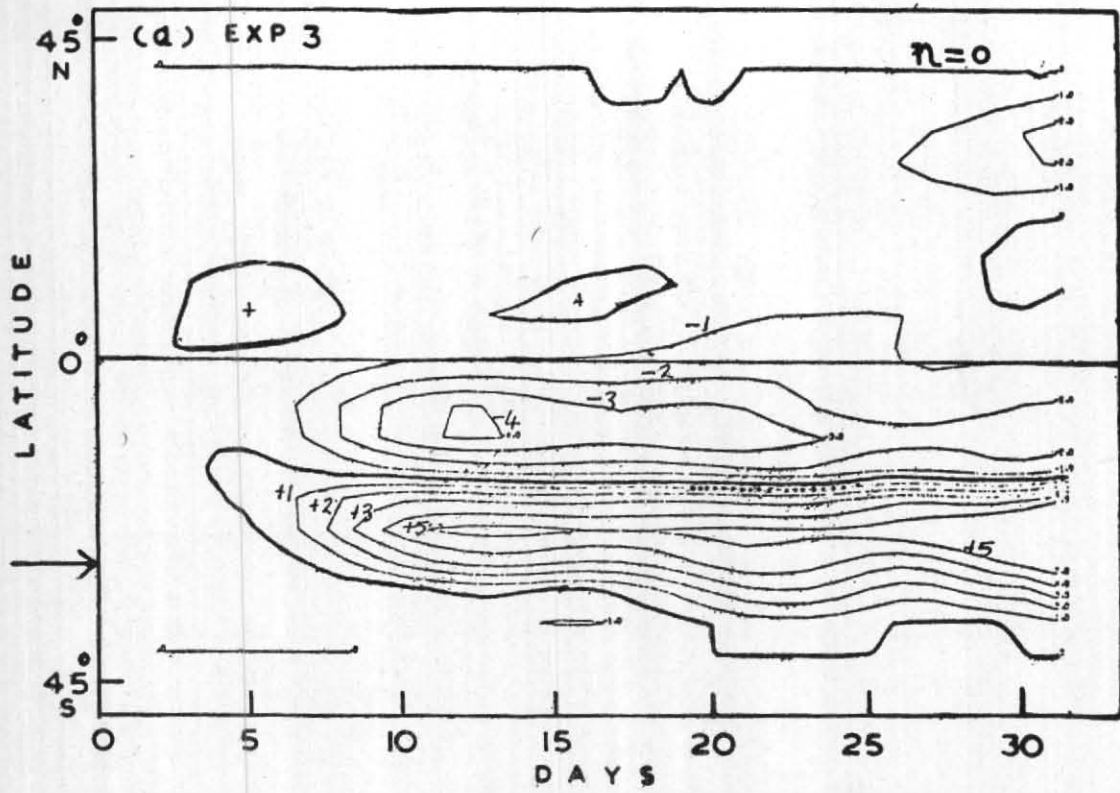propagation

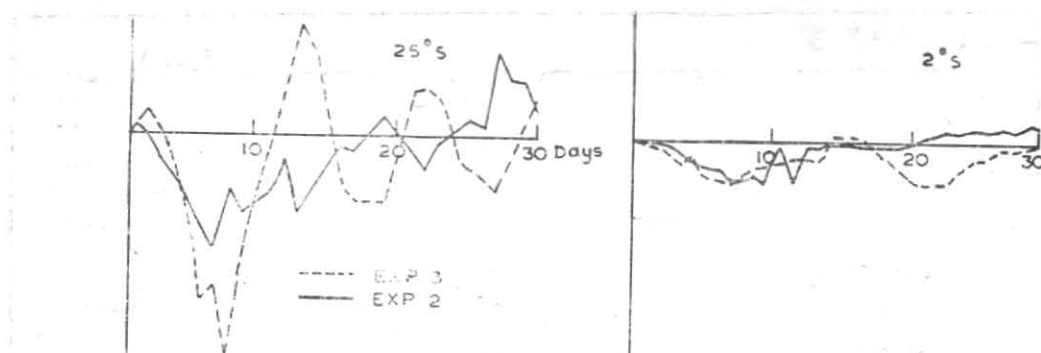Fig. 7 (a) and (b).   Same as Fig. 6 except for Experiment 3

Fig. 8.   Comparison of the variation of the momentum flux at 25°S and at the equator
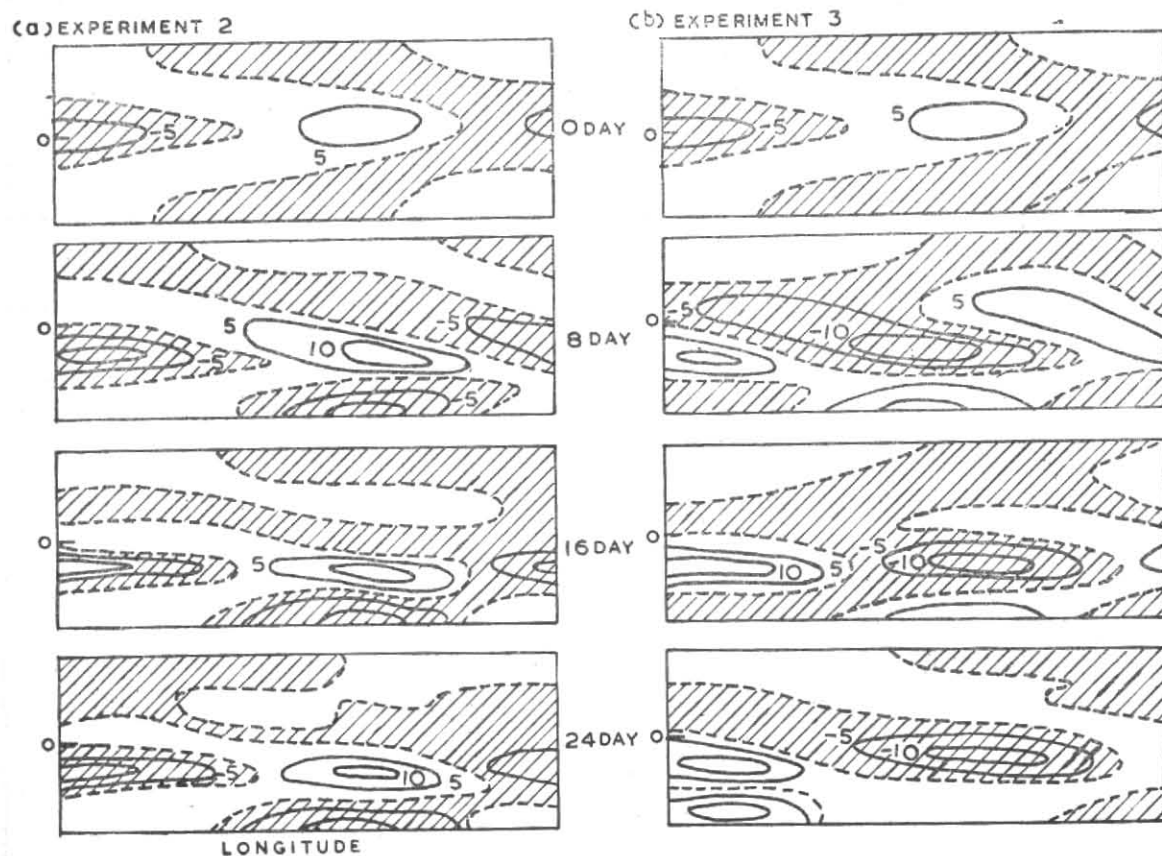for Experiments 2 (solid line) and 3 (dashed line)

Fig. 9.   Longitude-latitude plots of the total zonal velocity component at days 0, 8, 16 and 24 for
(a) Experiment 2 and (b) Experiment 3

indicates an initial cross equatorial wave propaga-
tion during the first few days of the experiment
at which time a transient is set up in the
northern hemisphere, the nature of which is indi-
cated by the rotating amplitude-phase vectors.
To the north and the south of the forcing lati-
tude, but still within the southern hemisphere, a

large amplitude standing wave (i.e., constant
phase) with a 180° phase difference across the
forcing latitude is set up. This is consistent with
the momentum fluxes shown in Fig. 8.

It is interesting to note that the amplitude of
the northern hemisphere wave $n=1$ does not change

after its initial excitation and *after* the setting up of the critical latitudes in the $n=0$ field as is evident in Figs. 6(a) and 6(b). This is best seen by viewing longitude-latitude plots of the *total* zonal velocity component (*i.e.*, u summed from $n=0$ to N) at specific times which are shown in Fig. 9 for Experiments 2 and 3. It will suffice to point out that the total field quickly forms a band of easterlies that appears to act as an effective barrier against further lateral wave propagation.

The only difference between Experiments 2 and 3 is that the steady forcing is in-phase with the basic flow (*see* Fig. 4) in the former case but out-of-phase in the latter case. Comparisons of the evolving fields, for both $n=0$ and $n=1$, show significant differences, especially in the $n=0$ easterlies to the north of the critical latitude as well as in the form of the $n=1$ field. It would seem that the initial relative phases of mid-latitude wave and the equatorial basic flow is of importance especially during the initial propagation period. The gaining of an understanding of the differences between the two responses would appear to be an important problem.

Fig. 8 shows the equatorial momentum flux for Experiments 2 and 3. Like the velocity fields the momentum fluxes differ considerably. The "in-phase" case suggests a relatively slow extraction of westerly momentum from low latitudes which slowly diminishes with time. This appears consistent with the relatively steady $n=1$ field north of the critical latitude. On the other hand, the "out-of-phase" case has assumed an oscillatory type of behaviour with a period between 15 and 20 days. This also seems consistent with the $n=1$ field (Fig. 7b) which undergoes considerable variations both to the north of the $u=0$ line and to the south. It would appear that the differences between Experiments 1 and 2 are more than merely those that would be apparent from linear addition of the two initial fields and their subsequent advection.

The evolution of these two cases as a function of longitude and latitude at days 0, 8, 16 and 24 is shown in Figs. 9(a) and 9(b). Starting with the same initial field (day 0), large differences quickly emerge in the total fields.

6. Conclusions

An extremely simple model has been developed with the aim of examining the mechanisms by which energy may propagate from one hemisphere to the other through a complicated two-dimensional basic flow. Three initial experiments were attempted and in each case wave-wave interactions produced a zonal field which acted to form a critical latitude to the equatorward of the forcing region. During its establishment, wave propagation across the equator promotes the establishment of transients in the hitherto undisturbed hemisphere. Momentum fluxes are consistent with this picture.

It is obviously too early to understand either the relevance of cross-equatorial influences in the atmosphere or understand the character of the model wave interactions and propagations. At this stage, the most that can be said is that the existence of a longitudinally dependent structure in the basic zonal field is *porous* to wave energy propagation at least until the nonlinear interactions form a critical latitude to prevent further equatorial penetration. This result is decidely different to the cases discussed by Charney (1968) and Bennett and Young (1971) when the basic flow was constrained to vary only in latitude.

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DISCUSSION

G.C. Asnani : In your first basic flow, the condition of inertial stability is not satisfied in the
neighbourhood and south of the equator. Would you not like to remove this instability?

Author : I do not believe that inertial instability is present in the basic flow.