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An analytical study on the barotropic energy conversion in the lower tropospheric monsoonal flow

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ABSTRACT. The mean zonal flow profile along y, for July month at different levels in the lower troposphere and at different longitudes in the Indian region, confined between two latitudinal walls is considered. The analytical function for each zonal flow profile is obtained by expressing it as a cosine Fourier series. A barotropically neutral perturbation of different functional dependency along y, and the wave structure along x is specified initially. Using non-divergent barotropic model in a β-plane, the barotropic energy conversion tendency $(\partial c/\partial t)$ (K_E, K_Z) are evaluated analytically for different observed zonal flow profiles.

The study of the barotropic energy conversion tendency indicates the disturbance at levels 850 and 700 mb receives energy from zonal kinetic energy, the most preferred scale is around 6000 km. The barotropically growing wave moves from east to west with phase speed of about 8 m sec^{-1} and minimum local doubling time is about 4 days.

1. Introduction

The instability of monsoonal flow can be regarded as one of the possible mechanism for formation and growth of disturbances in monsoonal flow. One of the important disturbances of monsoon flow is a monsoon depression, they originate at head Bay of Bengal, travel north or northwest across India at average speed around 4 m sec^{-1}, remain confined in the troposphere, below 300 mb. Monsoon depressions are more intense in lower troposphere and have a horizontal scale of around 2500 km.

It is generally felt that the initial growth of a depression is due to the dynamic instability in particular barotropic instability, because the baroclinic instability is not operative for lack of sufficient vertical shear in zonal flow; the CISK (conditional instability of second kind) mechanism is responsible for the further growth.

The instability analysis is done by using perturbation technique. It is quite difficult to tackle the instability problem analytically by an eigen value method for more general flow, a flow having shear in meridional as well as in vertical direction. The barotropic instability has been studied by several authors, notably Kuo (1949), Eliasen (1954), Lipps (1962), Jacob and Wiin-Nielsen (1966) and Yani and Nitta (1968). The baroclinic instability of a zonal flow is investigated by Charney (1947), Eady (1949) and various other authors.

Instead of finding solution of a general initial value problem, insight into the instability characteristics of a flow can be gain by using a more restrictive treatment of so called initial tendency method, where initial tendency of perturbation flow and various energy exchange between basic current and disturbance are obtained. Generally, initial perturbation taken is barotropically and baroclinically neutral whose trough and ridge lines having no tilt in horizontal and vertical. By studies of the initial tendency of barotropic and baroclinic energy conversion the stability characteristics of the initial perturbation can be determined.

The advantages of initial tendency method for study of instability of a flow over eigen value approach are the followings: the amplitude of initial perturbation not necessarily be infinitesimal, the basic zonal flow having meridional as well as vertical shear can be tackled easily and analytical solution can be obtained in this case. This method is used by Kuo (1953), Fisher (1968) and others for barotropic instability problem, the baroclinic instability of the zonal flow is investigated by Wiin-Nielsen (1962) and combined instability problem by Lipps (1966), Fisher and Renner (1971), Lejenas (1973).

In this study we have used the initial tendency method to determine analytically the barotropic instability characteristics of July mean monsoon zonal flow in lower troposphere with the hope that it might be helpful in understanding the mechanism by which monsoon depressions grow.

2. The Fundamental equations

The flow is assumed to be barotropic, frictionless horizontal and non-divergent, the governing equation for such a flow is non-divergent barotropic vorticity equation. We have further assumed β-plane approximation. The flow is confined between two lateral rigid walls at $y=0$ and $y=D$, D is the channel width. The barotropic, non-divergent vorticity, equation can be written as

$$\frac{\partial}{\partial t} \nabla^2\psi + \mathbf{V}_{\psi} . \nabla\nabla^2\psi + \beta \frac{\partial \psi}{\partial x} = 0 \qquad (1)$$

where ψ is stream function. Eqn. (1) is linearised by usual method by assuming a stationary basic zonal flow $\bar{u}(y)$ which satisfies Eqn. (1) and superimposed on it is a infinitesimal perturbation $\psi'(x,y,t)$, neglecting quadratic terms in perturbation, finally we get the linear form of vorticity equation for perturbation. After dropping prime symbol from perturbation stream function, equation can be written in the form

$$\nabla^2 \frac{\partial\psi}{\partial t} = -\bar{u}\frac{\partial}{\partial x}\nabla^2\psi - \left(\beta - \frac{d^2\bar{u}}{dy^2}\right)\frac{\partial\psi}{\partial x} \qquad (2)$$

The boundary conditions

At $y = 0$ and $y = D$, the normal velocity to the wall, *i.e.*, the meridional velocity, v must vanish for all times, the following boundary condition follows immediately :

$$\psi_t = 0 \quad \text{at } y = 0 \quad \text{and} \quad y = D \qquad (3)$$

3. The barotropic energy conversion

The Eular equation of motion for barotropic and non-divergent equation in stream function ψ are

$$\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial y}\right) + J\left(\psi, \frac{\partial\psi}{\partial y}\right) + f\frac{\partial\psi}{\partial x} = \frac{\partial\phi}{\partial x} \qquad (4)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial x}\right) + J\left(\psi, \frac{\partial\psi}{\partial x}\right) - f\frac{\partial\psi}{\partial y} = -\frac{\partial\phi}{\partial y} \qquad (5)$$

where $\phi=gz$, is geopotential height. The equations governing zonal mean basic flow

$$\frac{\partial}{\partial t}\left(\frac{\partial\bar{\psi}}{\partial y}\right) = -\frac{\partial}{\partial y}\left(\overline{\frac{\partial\psi'}{\partial x}\frac{\partial\psi'}{\partial y}}\right) \qquad (6)$$

and perturbation

$$\frac{\partial}{\partial t}\left(\frac{\partial\psi'}{\partial y}\right) + f\frac{\partial\psi'}{\partial x} = \frac{\partial\phi'}{\partial x} + \frac{\partial\bar{\psi}}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial\psi'}{\partial y}\right) - \frac{\partial\psi'}{\partial x}\frac{\partial}{\partial y}\left(\frac{\partial\bar{\psi}}{\partial y}\right) \qquad (7)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\psi'}{\partial x}\right) - f\frac{\partial\psi'}{\partial y} = -\frac{\partial\phi'}{\partial y} + \frac{\partial\bar{\psi}}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial\psi'}{\partial x}\right) \qquad (7a)$$

A bar denotes the zonal mean and prime perturbation, in the further discussion the perturbation quantities will be represented without a prime. The zonal kinetic energy K_Z is defined as

$$K_Z = \frac{1}{2}\int_0^D \left(\frac{\partial\bar{\psi}}{\partial y}\right)^2 dy \qquad (8)$$

and perturbation or eddy kinetic energy as

$$K_E = \frac{1}{2}\int_0^D \left[\overline{\left(\frac{\partial\psi}{\partial x}\right)^2} + \overline{\left(\frac{\partial\psi}{\partial y}\right)^2}\right] dy \qquad (9)$$

The zonal kinetic energy equation is obtained by multiplying Eqn. (6) by $\frac{\partial\bar{\psi}}{\partial y}$, then integrating between $y = 0$ and D and using boundary conditions (3)

$$\frac{\partial K_Z}{\partial t} = C\,(K_E, K_Z) \qquad (10)$$

where,

$$C\,(K_E, K_Z) = \int_0^D \overline{\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial y}} \cdot \frac{\partial^2\bar{\psi}}{\partial y^2}\, dy \qquad (10a)$$

Similarly perturbation kinetic energy equation is obtained by multiplying Eqn. (7) by $\partial\psi/\partial y$, Eqn. (7a) by $\partial\psi/\partial x$, then adding the two and integrating between $y = 0$ and D.

$$\frac{\partial K_E}{\partial t} = -C(K_E, K_Z) \qquad (11)$$

$C\,(K_E, K_Z)$ represents the conversion from eddy kinetic energy into zonal mean kinetic energy;

this is barotropic process. When $C(K_E, K_Z) > 0$; the eddy kinetic energy is transferred to the zonal mean basic flow, thus the disturbance loses kinetic energy and in this case perturbation is stable; on the other hand when $C(K_E, K_Z) < 0$, disturbance receives kinetic energy from zonal flow and perturbation is unstable.

4. Instability criteria, growth rate and phase speed

The barotropic instability criteria of a flow can be expressed in term of initial barotropic energy conversion tendency; time derivative of the eddy kinetic, *i.e.*, $\partial K_E/\partial t$ at time δt ean be obtained with the help of Taylor expansion of $\partial K_E/\partial t$.

$$\left(\frac{\partial K_E}{\partial t}\right)_{t=\delta t} = \left(\frac{\partial K_E}{\partial t}\right)_{t=0} + \left(\frac{\partial}{\partial t}\frac{\partial K_E}{\partial t}\right)_{t=0} \delta t + \text{higher order terms} \quad (12)$$

We assumed that initial perturbation chosen is barotropically neutral one, this implies

$$C(K_E, K_Z) = 0;$$

using this condition and Eqn. (11) in Eqn. (12), it follows immediately

$$\left(\frac{\partial K_E}{\partial t}\right)_{t=\delta t} = -\left(\frac{\partial C(K_E, K_Z)}{\partial t}\right)_{t=0} \delta t + \ldots (13)$$

In most of the situations first term on Taylor's expansion (13) dominate compared to the higher order terms up to $\delta t \sim 1$ day; this assumption is justified on the ground that the results of numerical integration of basic equation by Brown (1969), and Fisher and Renner (1971) confirm it. We get the following criteria for instability of zonal flow from Eqn. (13)

$$\left[\frac{\partial C}{\partial t}(K_E, K_Z)\right]_{t=0} < 0 \quad \text{barotropically unstable}$$

$$\left[\frac{\partial C}{\partial t}(K_E, K_Z)\right]_{t=0} > 0 \quad \text{barotropically stable}$$

We further assume that wave grow exponentially as in the case in a eigen value formulation of the instability problem; since under the assumption the eddy kinetic energy also grow exponentially, the growth rate of the wave is equal to

$$\left[\frac{1}{K_E}\frac{\partial K_E}{\partial t}\right]_{t=\delta t} = -\left[\frac{1}{K_E}\frac{\partial C}{\partial t}(K_E, K_Z)\right]_{t=0}$$

provided in the series (13) only first term is retained on the right hand side.

$$-\frac{1}{\left[\frac{1}{K_E}\frac{\partial C}{\partial t}(K_E, K_Z)\right]_{t=0}}$$

represent e-folding time, the time taken by the wave to grow in amplitude by e times to its initial value.

To arrive at an equation for phase velocity, however, within the framework of the initial tendency method, we proceeded as follows : Let C_r is the real part and C_i the imaginary part of phase velocity along x axis of a wave. C_r represents the velocity of propagation of wave's phase and C_i the amplification or decaying of the wave depending upon the sign. The partial differential equation governing propagation and amplification of a single wave.

$$\frac{\partial\psi}{\partial t} + C_r\frac{\partial\psi}{\partial x} - K\psi = 0 \quad (14)$$

where $C_r > 0$ and $C_i > 0$. On differenciating Eqn. (14) with respect to x and substituting from Eqn. (14) for $\partial\psi/\partial x$, we get the equation

$$\frac{\partial^2\psi}{\partial t\,\partial x} + C_r\frac{\partial^2\psi}{\partial x^2} + \frac{K}{C_r}\left(\frac{\partial\psi}{\partial t} - K\psi\right) = 0 \quad (15)$$

If we assume the following form of a amplifying wave :

$$\psi = \epsilon^{Kt}\sin k\,(x - C_r t)$$

and substituting in first two terms of Eqn. (15), we get

$$\frac{\partial^2\psi}{\partial t\,\partial x} + C_r\frac{\partial^2\psi}{\partial x^2} = k\,K\epsilon^{Kt}\cos k\,(x - C_r t)$$

For large scale wave and small growth rate, the first two terms nearly balance each other to a good degree of approximation,

$$\frac{\partial^2\psi}{\partial t\,\partial x} + C_r\frac{\partial^2\psi}{\partial x^2} \approx 0$$

Thus for larger wavelength and small growth rate first two terms of Eqn. (15) approximately govern the propagation of wave, thus the phase velocity along x direction is given by

$$C_r = -\frac{\dfrac{\partial^2\psi}{\partial t\,\partial x}}{\dfrac{\partial^2\psi}{\partial x^2}} \quad (16)$$

The phase velocity as given by Eqn. (16) depend upon y, the mean velocity for the channel, is given by the following equation

$$C_r = -\frac{\int_0^D \frac{\partial^2\psi}{\partial t\, \partial x}\, dy}{\int_0^D \frac{\partial^2\psi}{\delta x^2}\, dy} \tag{16a}$$

From Eqn. (11) one get easily

$$\frac{\partial\, C\,(K_E, Kz)}{\partial t} = \int_0^D \left[\frac{\partial^2\psi}{\partial t\, \partial x}\, \frac{\partial\, \psi}{\partial\, y} + \frac{\partial\, \psi}{\partial\, x}\, \frac{\partial^2\psi}{\partial t\, \partial y}\right] \times \times\, \frac{\partial^2\bar{\psi}}{\partial y^2}\, dy \tag{17}$$

5. Analytical barotropic tendency for three different initial perturbation

We have obtained, in this section, the analytical expression for the barotropic energy tendency for three different structures of initial wave, in two cases the wave chosen have symmetric structure and in third case initial perturbation chosen is asymmetric around the centre of channel. The basic zonal flow $\bar{u}\,(y)$ is expanded in terms of cosine Fourier series, only first four harmonics are retained.

$$\bar{u} = C_0 + C_1 \cos ly + C_2 \cos 2\, ly + C_3 \cos\, 3\, ly + + C_4 \cos 4\, ly \tag{18}$$

where $l = \pi/D$, D is the width of channel.

Case I : The perturbation at initial time is a wave along x direction, its form is so chosen to satisfy the boundary conditions. It is symmetric around the centre of channel. The form of wave is identical to the neutral Rossby wave and along y the sign of vorticity remain unchanged

$$\psi_0 = A \sin ly \sin kx \tag{19}$$

where $k = 2\pi/l$, is wave number and l represents wavelength along x, A is the amplitude of the wave. We will not fix the value of amplitude A, because growth rate, phase velocity and the criteria of the instability do not depend upon amplitude. Substituting Eqns. (18) and (19) in Eqn. (2), then solving the resulting Poisson's for $\partial\psi/\partial t$, we get :

$$\left(\frac{\partial\psi}{\partial t}\right)_{t=0} = -Ak \cos kx \Bigg[\Big\{\left(k^2 + l^2\right) C_0 - -\beta - \frac{C_2}{2}\left(k^2 - 3l^2\right)\Big\}\frac{\sin ly}{(k^2+l^2)} + + \tfrac{1}{2}\Big\{C_1 k^2 - C_3\,(k^2 - 8l^2)\Big\}\frac{\sin 2ly}{(k^2+4l^2)} + + \tfrac{1}{2}\Big\{C_2\,(k^2 - 3l^2) - C_4\,(k^2 - 15l^2)\Big\} \times \times\, \frac{\sin 3\, ly}{(k^2 + 9l^2)} + + \frac{C_3}{2}\left(k^2 - 8l^2\right)\frac{\sin 4\, ly}{(k^2+16\, l^2)} + + \frac{C_4}{2}\left(k^2 - 15\, l^2\right)\frac{\sin 5\, ly}{(k^2 + 25\, l^2)}\Bigg] \tag{20}$$

$$\frac{1}{K_E}\left[\frac{\partial\, C\,(K_E, Kz)}{\partial\, t}\right]_{t=0} = \frac{k^2\, l^2}{(k^2+ l^2)}\Bigg[\frac{3\, C_1^2\, k^2}{2\,(k^2 + 4\, l^2)} + + 4C_2^2\, \frac{(k^2 - 3l^2)}{(k^2+9l^2)} + 9\, C_3^2\, \frac{(k^2 - 8\, l^2)}{(k^2+4\, l^2)} \times \times\, \frac{(k^2 + 6\, l^2)}{(k^2 + 16\, l^2)} - - 3\, C_1 C_3 \frac{(k^2 - 4\, l^2)}{(k^2+4\, l^2)} - 8 C_2 C_4 \frac{(k^2 - 9l^2)}{(k^2+9l^2)} + + 16\, C_4^2\, \frac{(k^2 - 15\, l^2)\ (k^2+13\, l^2)}{(k^2+9\, l^2)\ \ (k^2+25\, l^2)}\Bigg] \tag{21}$$

The phase velocity equation is easily obtained from Eqns. (16a), (20) and (19).

$$C_r = \Bigg[\frac{1}{(k^2+l^2)}\Big\{\left(k^2+l^2\right)C_0 - \beta - \frac{C_2}{2}\left(k^2 - 3l^2\right)\Big\} + \frac{1}{6\,(k^2+9\, l^2)}\Big\{C_2\,(k^2 - 3\, l^2) - C_4\,(k^2 - 15\, l^2)\Big\} + \frac{C_4}{2}\, \frac{(k^2 - 15\, l^2)}{(k^2 + 25\, l^2)}\Bigg] \tag{22}$$

The contribution of C_0-component of basic flow and β to ψ_t, as seen from Eqn. (20), has same latitude dependence on ψ_0; therefore, it is implied that the interaction of β and C_0 with the initial perturbation ψ_0 leads to the propagtion of the perturbation without change of slopes of troughs and ridges. The zonal average momentum transfer tendency across the channel due to β and C_0 vanishes; hence these interactions do not lead to the energy exchange between basic flow and the perturbation, this is the physical reason for not appearing of β and C_0 terms in growth rate Eqn. (21).

To understand the role of each terms of Eqn. (21) in the growth of the initial wave, we have to know the distribution of the momentum transfer across the channel associated with each components of the basic flow. The tendency of momentum transfer $\partial\,(\overline{uv})/\partial t$ and its divergence $\partial^2\,(\overline{uv})/\partial y\,\partial t$ can easily be found with the help of Eqns. (19) and (20). Since the initial momentum transfer vanishes, therefore $\partial^2\,(\overline{uv})/\partial y\,\partial t$ is representative of the distribution of $\partial\,(\overline{uv})/\partial y$ atleast for a short time. The interaction of a component of the basic flow with the initial wave leads to the transfer of momentum $(\overline{uv})$ with certain distribution of $\partial\,(\overline{uv})/\partial y$. If $\partial\,(\overline{uv})/\partial y$ for a given component of the basic flow, has a component whose latitudinal variation is same and also in phase with the component of the basic flow, in this situation the momentum is always transported horizontally away from the jet axis, and kinetic energy is transferred from the basic flow to the wave. When the component of $\partial\,(\overline{uv})/\partial y$ has the same latitudinal variation but phase is opposite to that of basic flow component, in this situation reverse process takes place, momentum is transferred toward jet axis and basic flow component gain energy from the initial wave.

The distribution of $\partial\,(\overline{uv})/\partial y$ for C_1 has a component which is in all situations opposite in phase with C_1 for all k; hence the C_1 interaction with the initial wave results in damping it. The interaction of other components C_2, C_3, C_4 with ψ_0 leads to the damping of short wave and amplification of long wave. Eqn. (21) for growth-rate of the wave also contains terms which depend upon the products C_1C_3 and C_2C_4. If products C_1C_3 and C_2C_4 are negative in that case the contribution of these terms are qualitatively same as that of other square terms in Eqn. (21). The terms of this type arise, because the distribution of $\partial\,(\overline{uv})/\partial y$ due to the interaction of ψ_0 with a component of the basic flow also contains a component whose latitudinal variation is same as that of another component.

Case II : Here we consider an initial wave which is symmetric in y around the centre of channel; the vorticity field associated with the wave is also symmetric, it has one sign except near the boundaries where sign is opposite. The form of the *initial* wave is different from that of neutral Rossby wave. In Case II the initial ψ chosen is of the form :

$$\psi_0 = A\,(1 - \cos 2\,ly)\sin kx \tag{23}$$

The $\frac{\partial\psi}{\partial t}$ for this case is given by

$$\left(\frac{\partial\psi}{\partial t}\right)_{t=0} = Ak\cos kx\Bigg[\Big(F_0+F_1\cos ly+F_2\cos 2ly+ + F_3\cos 3\,ly + F_4\cos 4\,ly + F_5\cos 5\,ly + +F_6\cos 6\,ly\Big)+\left\{\left(\frac{1-e^{kD}}{e^{kD}-e^{-kD}}\right)e^{-ky} - \left(\frac{1-e^{-kD}}{e^{kD}-e^{-kD}}\right)\times e^{ky}\right\}\Big(F_0+F_2+F_4+F_6\Big)+\left\{\left(\frac{1+e^{kD}}{e^{-kD}-e^{kD}}\right)e^{-ky} -\left(\frac{1+e^{-kD}}{e^{-kD}-e^{kD}}\right)e^{ky}\right\}(F_1+F_3+F_5)\Bigg] \tag{24}$$

where,

$$F_0 = \frac{\beta}{k^2} - \left(C_0 - \frac{C_2}{2}\right);$$

$$F_1 = \frac{(k^2-5l^2)\,(C_3-C_1)}{2\,(k^2+l^2)}\;;$$

$$F_2 = C_0 - \frac{\beta}{(k^2+4l^2)} + \frac{(4\,l^2-k^2)}{(k^2+4\,l^2)}\,C_2 + \frac{C_4}{2}\,\frac{(k^2-12\,l^2)}{(k^2+4\,l^2)}\;;$$

$$F_3 = \frac{(C_1/2)\,(k^2+3\,l^2) - C_3\,(k^2-9\,l^2)}{(k^2+9\,l^2)}$$

$$F_4 = \frac{\frac{C_2}{2}\,k^2 - C_4\,(k^2-16\,l^2)}{(k^2+16\,l^2)}\;;$$

$$F_5 = \frac{C_3\,(k^2-5\,l^2)}{2\,(k^2+25\,l^2)}\;;$$

$$F_6 = \frac{C_4\,(k^2-12\,l^2)}{2\,(k^2+36\,l^2)}$$

Substituting in Eqn. (17) with the help of Eqns. (23) and (24), we can write the expression for the growth rate in the form

$$\left[\frac{1}{K_E}\,\frac{\partial C\,(K_E,\,K_Z)}{\partial t}\right]_{t=0} = -\,\frac{k^2\,l^2}{(3\,k^2+4\,l^2)}\times \{5\,F_1C_1 + 8\,F_2C_2 + 18\,F_3C_3 + 32\,F_4C_4 - 5\,C_1F_3 - -12\,C_2F_4 - 21\,C_3F_5 - 32\,C_4F_6 + 8\,C_2F_0\} + -\frac{4\pi k^3\,l}{(3\,k^2+4l^2)}\Bigg[\left(\frac{1-e^{kD}}{e^{kD}-e^{-kD}}\right)\times \times\Bigg\{\frac{C_1\,(e^{-kD}+1)\,(23\,\pi^2-D^2k^2)}{(D^2k^2+\pi^2)\,(D^2k^2+9\pi^2)} + +\frac{2\,C_2\,(e^{-kD}-1)\,(D^4k^4-24\,\pi^2D^2k^2-64\,\pi^4)}{D^2k^2\,(D^2k^2+4\,\pi^2)\,(D^2k^2+16\,\pi^4)} + +\frac{3\,C_3\,(e^{-kD}+1)\,(-D^4k^4+22\,\pi^2D^2k^2+151\pi^4)}{(D^2k^2+\pi^2)\,(D^2k^2+9\,\pi^2)\,(D^2k^2+25\,\pi^2)} +$$

$$+ \frac{4\,C_4\,(e^{-kD}-1)\,(D^4k^4-16\pi^2D^2k^2-272\,\pi^4)}{(D^2k^2+4\pi^2)\,(D^2k^2+16\,\pi^2)\,(D^2k^2+36\,\pi^2)}\Bigg\}$$

$$+\left(\frac{1-e^{-kD}}{e^{kD}-e^{-kD}}\right)\Bigg\{\frac{C_1\,(e^{kD}+1)\,(-D^2k^2+23\,\pi^2)}{(D^2k^2+\pi^2)\,(D^2k^2+9\,\pi^2)}$$

$$+ \frac{2\,C_2\,(e^{kD}-1)\,(D^4k^4-24\,\pi^2D^2k^2-64\,\pi^4)}{D^2k^2\,(D^2k^2+4\,\pi^2)\,(D^2k^2+16\,\pi^2)}$$

$$+ \frac{3\,C_3\,(e^{kD}+1)\,(-D^4k^4+22\,\pi^2D^2k^2+151\,\pi^4)}{(D^2k^2+\pi^2)\,(D^2k^2+9\,\pi^2)\,(D^2k^2+25\,\pi^2)}$$

$$+ \frac{4\,C_4\,(e^{kD}-1)\,(D^4k^4-16\,\pi^2D^2k^2-272\,\pi^4)}{(D^2k^2+4\,\pi^2)\,(D^2k^2+16\,\pi^2)\,(D^2k^2+36\pi^2)}\Bigg\}$$

$$\times\Big(F_0+F_2+F_4+F_6\Big)+\left(\frac{1+e^{kD}}{e^{-kD}-e^{kD}}\right)\times$$

$$\times\Bigg\{\frac{C_1\,(e^{-kD}+1)\,(-D^2k^2+23\,\pi^2)}{(D^2k^2+\pi^2)\,(D^2k^2+9\,\pi^2)}+$$

$$+ \frac{2\,C_2\,(e^{-kD}-1)\,(D^4k^4-24\,\pi^2D^2k^2-64\,\pi^4)}{D^2k^2\,(D^2k^2+4\pi^2)\,(D^2k^2+16\,\pi^2)}$$

$$+ \frac{3\,C_3\,(e^{-kD}+1)\,(-D^4k^4+22\,\pi^2D^2k^2+151\,\pi^4)}{(D^2k^2+\pi^2)\,(D^2k^2+9\,\pi^2)\,(D^2k^2+25\,\pi^2)}$$

$$+ \frac{4\,C_4\,(e^{-kD}-1)\,(D^4k^4-16\,\pi^2D^2k^2-272\,\pi^4)}{(D^2k^2+4\,\pi^2)\,(D^2k^2+16\,\pi^2)\,(D^2k^2+36\,\pi^2)}\Bigg\}$$

$$+\left(\frac{1+e^{-kD}}{e^{-kD}-e^{kD}}\right)\Bigg\{\frac{C_1\,(e^{kD}+1)\,(-D^2k^2+23\,\pi^2)}{(D^2k^2+\pi^2)\,(D^2k^2+9\,\pi^2)}+$$

$$+ \frac{2\,C_2\,(e^{kD}-1)\,(D^4k^4-24\,\pi^2D^2k^2-64\,\pi^4)}{D^2k^2\,(D^2k^2+4\,\pi^2)\,(D^2k^2+16\,\pi^2)}+$$

$$+ \frac{3\,C_3\,(e^{kD}+1)\,(-D^4k^4+22\,\pi^2D^2k^2+151\,\pi^4)}{(D^2k^2+\pi^2)\,(D^2k^2+9\,\pi^2)\,(D^2k^2+25\,\pi^2)}$$

$$+ \frac{4\,C_4\,(e^{kD}-1)\,(D^4k^4-16\,\pi^2D^2k^2-272\,\pi^4)}{(D^2k^2+4\,\pi^2)\,(D^2k^2+16\,\pi^2)\,(D^2k^2+36\,\pi^2)}\Bigg\}\times$$

$$\times(F_1+F_3+F_5)\Bigg] \qquad (25)$$

The phase velocity in this case is given by,

$$C_r = -F_0 + \frac{2\,(e^{kD/2}-e^{-kD/2})}{kD\,(e^{kD/2}+e^{-kD/2})}(F_0+F_2+F_4+F_6) \qquad (26)$$

Case III : In the third case we have chosen initial perturbation of the form

$$\psi_0 = A \sin 2\,ly \sin kx \qquad (27)$$

which is asymmetric around the centre of the channel, the vorticity field is also asymmetric around the centre of channel, this case is not interesting for July mean flow because the perturbation is always stable. For the sake of completeness we have given here the expression for $\left(\frac{\partial \psi}{\partial t}\right)_{t=0}$ and $\left[\frac{1}{K_E}\frac{\partial C}{\partial t}(K_E, Kz)\right]_{t=0}$ they are respectively

$$\left(\frac{\partial \psi}{\partial t}\right)_{t=0} = -Ak\cos kx\,(F_1'\sin ly + F_2'\sin 2\,ly + F_3'\sin 3ly + F_4'\sin 4ly + F_5'\sin 5\,ly + F_6'\sin 6\,ly)$$

$$\left[\frac{1}{K_E}\,\frac{\partial C}{\partial t}\,(K_E,\; Kz)\right]_{t=0} = \frac{k^2\,l^2}{2\,(k^2+4l^2)}\times$$

$$\Bigg[\frac{8\,C_1^2\,(k^2+3\,l^2)\,(k^2+4\,l^2)}{(k^2+l^2)\,(k^2+9\,l^2)}+\frac{12\,C_2^2\,k^2}{(k^2+16\,l^2)}$$

$$+\frac{18\,C_3^2\,(k^2-5\,l^2)\,(k^2-3\,l^2)}{(k^2+l^2)\,(k^2+25\,l^2)}-\frac{24\,C_2C_3l^2}{(k^2+l^2)}$$

$$+\frac{32\,C_4^2\,(k^2-12\,l^2)}{(k^2+36\,l^2)}\Bigg]$$

where,

$$F_1' = -\frac{1}{2\,(k^2+l^2)}\Big\{C_1\;(k^2+3l^2)+ + C_3\,(k^2-5\,l^2)\Big\};$$

$$F_2' = C_0 - \frac{\beta}{(k^2+4\,l^2)} - \frac{C_4}{2}\,\frac{(k^2-12\,l^2)}{(k^2+4\,l^2)};$$

$$F_3' = \frac{C_1\,(k^2+3\,l^2)}{2\,(k^2+9l^2)}\,;\; F_4' = \frac{C_2\,k^2}{2\,(k^2+16\,l^2)};$$

$$F_5' = \frac{C_3\,(k^2-5\,l^2)}{2\,(k^2+25\,l^2)}\,;\; F_6' = \frac{C_4\,(k^2-12\,l^2)}{2\,(k^2+36\,l^2)}$$

6. Results

The data set consist of the monthly mean zonal wind in the latitudinal belt between 0° and 30°N, grid interval of 5°, for longitudes from 80°E to 120°E at the interval of 5° and at levels 850 mb, 700 mb and 500 mb for the month of July 1963. Fig. 1 (a, b, c) shows the meridional distribution of zonal wind at longitude 90°E, 95°E and 100°E for 850 mb, 700 mb and 500 mb respectively. At 850 mb the flow is mostly westerly in the latitudinal belt, and wind maxima of strength around 11 m sec^{-1} is situated just south of the centre of the belt, these features of the flow is exhibited at all longitudes under consideration. The zonal wind at 700 mb exhibits qualitatively same features except that the value of wind maxima is around 9 m sec^{-1} just lies south of the centre of the belt. The zonal wind at 500 mb is westerly upto 20°N and easterly

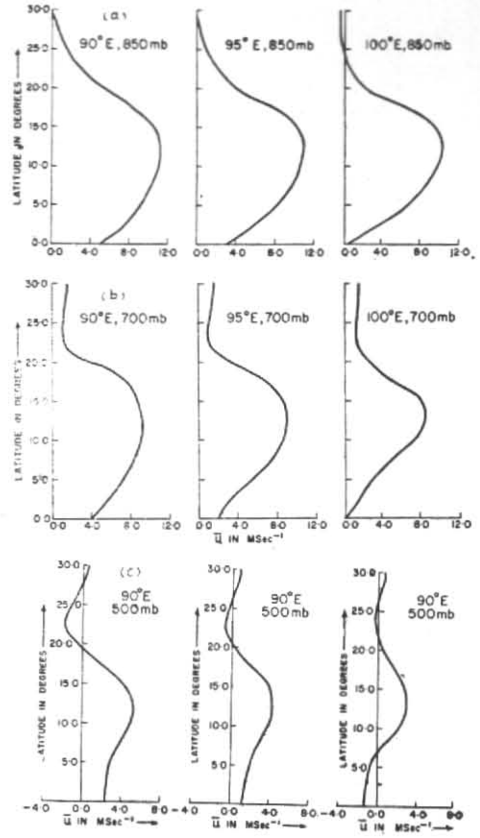

Fig. 1. The observed meridional distribution of monthly mean zonal flow for July

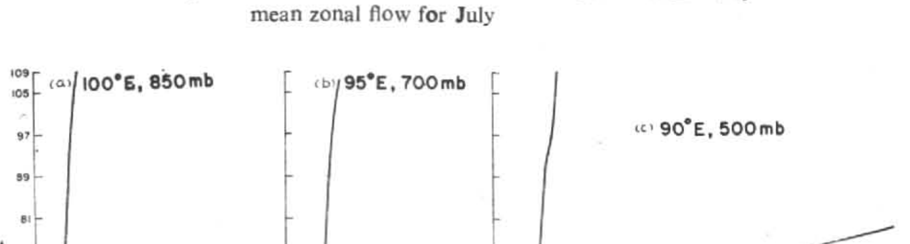

Fig. 2. Doubling time of unstable wave as a function of wavelength flow

beyond it, westerly maxima of around 4·5 m/sec just lies south of the centre of the belt.

The data set at each pressure levels and longitudes is subjected to the cosine Fourier series analysis along meridional direction, the coefficients C_0, C_1, C_2, C_3 and C_4 of the series are evaluated. These coefficients for the various levels and longitudee are given in Table 1.

Here for various computation we have taken $D=3.30\times10^5$ m and $\beta=2.151\times10^{-11}$ m/sec at

TABLE 1

First five coefficients of the cosine Fourier series

Along		Coefficient of cosine Fourier series				
Level (mb)	Long.	1st	2nd	3rd	4th	5th
850 mb	90°E	6·77	4·06	—3·47	—0·88	0·03
850 mb	95°E	6·18	3·65	—3·68	—1·45	0·08
850 mb	100°E	4·87	3·43	—4·08	—1·99	0·26
700 mb	90°E	5·45	3·17	—2·76	—1·21	0·54
700 mb	95°E	4·82	2·24	—3·21	—1·65	0·51
700 mb	100°E	3·80	1·16	—3·26	—1·60	0·66
500 mb	90°E	1·96	2·41	—1·31	—1·48	0 91
500 mb	95°E	1·85	1·14	—1·28	—0·92	0·41
500 mb	100°E	0·68	—0·25	—1·63	—0·78	0·57

15°N, centre of the channel. The growth rate and phase speed of the initial wave for all the three cases is computed by using the respective expression given in the last section. Fig. 2 (a, b, c) gives doubling time for unstable wave of different wave length for the flow at 850 mb; 100°E, 700 mb; 95°E and 500 mb; 90°E respectively. From computations it turns out that asymmetric initial wave of case III is stable at all pressure levels and longitudes under consideration, and for initial wave of case II, either it is stable or it's growth rate is quite small compared to the initial wave of case I, as can be seen from Fig. 2(c) for one situation. Therefore, here we will discuss only the results of the case I.

It can be seen that locally, the flow at 850 mb is most unstable compared to flow at 700 mb and 500 mb, this occurs at 100°E where doubling time for the initial growing wave is about 4.2 days, doubling time for the wave increase rather rapidly for the flows to the east and west of 100°E. The unstable wave attains maximum growth rate at longitudes 100°E and 90°E for the flow at level 700 mb and 500 mb respectively. On average (90°E, 120°E) the 700 mb zonal flow is slightly more unstable compared to 850 mb. The preferred scale of most unstable wave is little more than 6000 km at 850 mb and little less than 6000 km at 700 mb, and phase speeds on average is —8·5 m sec^{-1} at 850 mb and —7·5 m sec^{-1} at 700 mb. From Table 1 it can be seen that the sign of the product terms C_1C_3 and C_2C_4 is negative in all cases; this situation is favourable for growth of the initial wave, around of 40 per cent of the total growth rate is contributed by these two terms in all cases.

The curves for the doubling time are quite flat around its minima in all cases, this indicates that there is no sharp selection of most preferred scale of the disturbance; this may be due to our approximation in retaining only one term in the Taylor's expansion (13) for the barotropic energy conversion; improvement in this regard can be brought about by considering more terms in the series (13).

7. Conclusion

The flow at 850 mb and 750 mb and between longitudes 80°E and 120°E is unstable, the wavelength of the most unstable wave is around 6000 km; doubling time about 8 days and wave propagate from east to west with average phase speed of 8 m sec^{-1}. Locally, at few longitudes the flow is more unstable with doubling time between 4 to 5 days. Thus the basic flow can support the disturbance by barotropic process in lower troposphere.

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