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Numerical experiment with primitive equation barotropic model using quasi-Lagrangian advection scheme to predict the movement of monsoon depression and tropical cyclone

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ABSTRACT. A limited-area primitive equation barotropic model using quasi-Lagrangian advection scheme has been integrated upto 48 hours with wind as input to predict the movement of a monsoon depression and a tropical cyclone in the Indian region. The vector error in the forecast of the centre of the monsoon depression is 55 km in 24 hr and 220 km in 48 hr. The corresponding errors in the case of the tropical cyclone are 0 km and 80 km respectively. Verification statistics for the predicted u and v fields are presented and discussed.

1. Introduction

The problem of non-linear computational instability in numerical integration of prediction equations when finite difference technique is employed to evaluate the advective terms is quite well known. Spatial smoothing is sometimes applied to control such instability. Shuman (1962) developed a finite difference scheme which had considerable degree of built-in smoothing and selective damping. The scheme was successfully used by him later in a primitive equation barotropic model (Shuman and Vandermann 1966) and primitive equation multilevel model (Shuman and Hovermale 1968). An alternative approach to overcome the problem of non-layer instability has been to introduce a quasi-Lagrangian scheme in numer'cal integration. A systematic attempt on this concept was first made by Krishnamurti (1962). The method is analogous to that used in mechanics to determine the velocity of a particle moving with an initial velocity and under acceleration after a prescribed interval of time. However, to be able to apply this quasi-Lagrangian method, it is necessary to know as accurately as possible the location of the air parcel which after travelling under a non-conservative system of forces will arrive at a grid point after a given interval of time. The scheme has been successfully developed for numerical integrations of atmospheric motion by Krishnamurti (1962, 1969), Leith (1965), Oklands (1965), Mathur (1970), Krishnamurti *et al.* (1973) and others. As a result of these studies, it is now known that numerical integration of primitive equations by quasi-Lagrangian advection scheme is quite stable for short-range prediction of movement of tropical disturbances.

In the present study we have used an improved version of quasi-Lagrangian Advective scheme (Mathur 1970) in an primitive equation barotropic model to predict the movement of a monsoon depression and a tropical cyclone.

LIST OF SYMBOLS

f = coriolis parameters
g = acceleration due to gravity
h = height of free surface
i,I = x co-ordinate
j,J = y co-ordinate
m = map factor (secant of latitude)
t = time
u = wind speed towards east
v = wind speed towards north

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u^{-y} = mean of u between two consecutive grid points in the y-direction

v^{-y} = mean of v between two consecutive grid points in the y-direction

x = co-ordinate axis towards east

y = co-ordinate axis towards north

Δt = time step

ψ = stream function

$S_{ij}^{t_0+\Delta t}$ = value of any parameter S at time $t_0 + \Delta t$ at ij point

XP = distance along x-axis between origin of the particle P and the grid point

YP = distance along y-axis between origin of the particle P and the grid point

2. Basic equations and integrations scheme

In the case of a divergent barotropic model, the prediction equations in Cartesian co-ordinates on Mercator projection are :

$$A = \frac{Du}{Dt} = fv - mg\frac{\partial h}{\partial x} \quad (1)$$

$$B = \frac{Dv}{Dt} = -fu - mg\frac{\partial h}{\partial y} \quad (2)$$

$$C = \frac{Dh}{Dt} = -mh\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \quad (3)$$

where,

$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{dx} + v\frac{\partial}{\partial y}$ represents the Lagrangian advection operator.

The Lagrangian advective scheme is summarised as follows :

Initial values of A, B and C are calculated from the right hand side of (1), (2) and (3) respectively. For the time integration of the equations, the position of the particle P_{ij} is located at $t=t_0$ (Fig. 1) so that it may arrive at the grid point Q_{ij} at $t=t_0+\Delta t$.

The first guess for the location of P_{ij} is

$$XP_{ij} = -u_{ij}\,\Delta t - \tfrac{1}{2}\,A_{ij}\,(\Delta t)^2 \quad (4)$$

$$YP_{ij} = -v_{ij}\,\Delta t - \tfrac{1}{2}B_{ij}\,(\Delta t)^2 \quad (5)$$

Once the location of P_{ij} is known, the values of u_{ij}, v_{ij}. A_{ij} and B_{ij} may be interpolated at P_{ij} using nine points Lagrangian interpolation scheme given by (6).

$$F\,P_{ij} = \sum_{\substack{i=I-1\\ j=J-1}}^{\substack{j=J+1\\ i=I+1}} W_{ij}\,F_{ij} \quad (6)$$

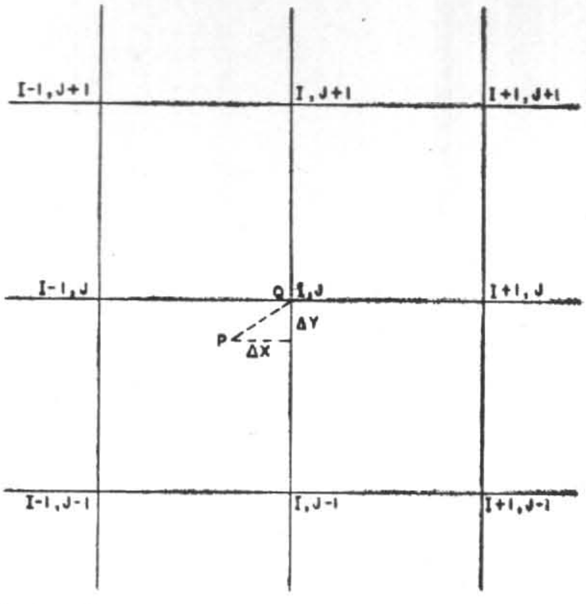

Fig. 1. Nine points for Lagrangian interpolation. A particle originally at P at time t_0 arrives at Q at time $t_0+\Delta t$

where F_{ij} is the quantity at grid-point Q_{ij} and W_{ij} is a weighting factor given by

$$W_{ij} = \prod_{\substack{k=I-1\\ k\neq i}}^{k=I+1} \frac{x-xk}{x_i-xk} \prod_{\substack{l=J-1\\ l\neq j}}^{l=J+1} \frac{y-yl}{y_j-yl} \quad (7)$$

A new guess for the location of P_{ij} is obtained by (8) and (9) using interpolated values uP_{ij}, vP_{ij}, AP_{ij} and BP_{ij}

$$XP_{ij} = -\,uP_{ij}\,\Delta t - \tfrac{1}{2}\,AP_{ij}\,(\Delta t)^2 \quad (8)$$

$$YP_{ij} = -\,vP_{ij}\,\Delta t - \tfrac{1}{2}\,BP_{ij}\,(\Delta t)^2 \quad (9)$$

Interpolation of u_{ij}, v_{ij}, h_{ij}, A_{ij}, B_{ij} and C_{ij} with new location of P_{ij} are made and forward difference in time is carried out using

$$u_{ij}^{t_0+\Delta t} = uP_{ij} + AP_{ij}\,\Delta t \quad (10)$$

$$v_{ij}^{t_0+\Delta t} = vP_{ij} + BP_{ij}\,\Delta t \quad (11)$$

$$h_{ij}^{t_0+\Delta t} = hP_{ij} + CP_{ij}\,\Delta t \quad (12)$$

Using the values of $u_{ij}^{t_0+\Delta t}$, $v_{ij}^{t_0+\Delta t}$, $h_{ij}^{t_0+\Delta t}$ given at left hand side of (10), (11), and (12) the values of $A_{ij}^{t_0+\Delta t}$, $B_{ij}^{t_0+\Delta t}$, $C_{ij}^{t_0+\Delta t}$ are evaluated by (1), (2) and (3) respectively, and new guess for location of point P_{ij} is made using the following relations :

$$XP_{ij} = -\tfrac{1}{2}\,[uP_{ij} + u_{ij}{}^{t_0 + \Delta t}]\,\Delta t \qquad (13)$$

$$YP_{ij} = -\tfrac{1}{2}\,[vP_{ij} + v_{ij}{}^{t_0 + \Delta t}]\,\Delta t \qquad (14)$$

The interpolation of u_{ij}, v_{ij}, h_{ij}, A_{ij}, B_{ij} and C_{ij} with new location of P_{ij} are made and implicit difference in time is carried out using

$$u_{ij}{}^{t_0 + \Delta t} = uP_{ij} + \tfrac{1}{2}\,[AP_{ij} + A_{ij}{}^{t_0 + \Delta t}]\,\Delta t \qquad (15)$$

$$v_{ij}{}^{t_0 + \Delta t} = vP_{ij} + \tfrac{1}{2}\,[BP_{ij} + B_{ij}{}^{t_0 + \Delta t}]\,\Delta t \qquad (16)$$

$$h_{ij}{}^{t_0 + \Delta t} = hP_{ij} + \tfrac{1}{2}\,[CP_{ij} + C_{ij}{}^{t_0 + \Delta t}]\,\Delta t \qquad (17)$$

Some tests were carried out to determine the time step required for convergence of the integration scheme. The convergence criterion here indicates that the difference between the absolute values computed from first guess [using relations (10), (11) and (12)] and corresponding absolute values from second guess [using relations (15), (16) and (17)] should be minimum. This criterion is carried out for different time step by checking whether $(\,|\,Q_1\,| - |\,Q_2\,|\,) < \varepsilon$, where ε is sufficiently small number (*i.e.*, 10^{-3} for h field and 10^{-2} for u and v field), may be either u, v or h and suffix 1 and 2 denote first and second guess. Fig. 2 gives the variation of the number of grid points which do not show convergence of variables u, v and h with different time step. It may be seen from Fig. 2 that the convergence is achieved for all the variables at all grid points with time step of seven minutes. Thus, in the present experiment a time step of seven minutes for the particular choice of grid length and the integration scheme has been found suitable.

3. Boundary conditions

For the purpose of forecasting on the boundaries cyclic community is assumed on the eastern and the western boundaries. On the northern and southern boundaries ($y=y_B$), boundary conditions are specified as follows :

$$u(x, y_B, t) = \frac{\oint u(x,\, y_B,\, t=0)\, dx}{\oint dx}$$

$$v(x, y_B, t) = 0$$

$$h(x, y_B, t) = \frac{\oint h(x,\, y_B,\, t=0)\, dx}{\oint dx}$$

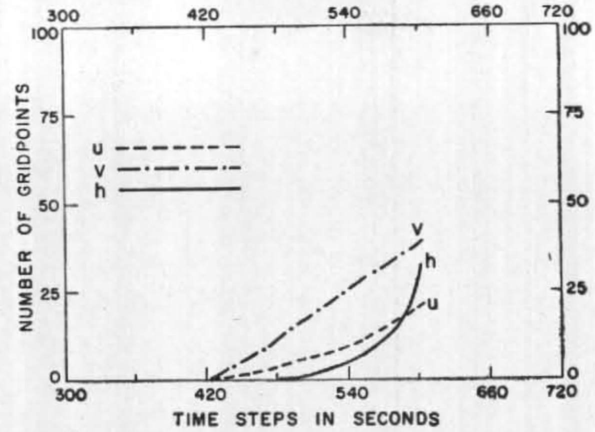

Fig. 2. The total number of non-converged grid points for different magnitudes of time step

The boundary conditions specified above do not change with time. The values of A, B and C are also kept zero for all the time on northern and southern boundaries.

4. Initialization

The basic input to the model is the wind. The stream function (ψ) is obtained from the observed wind by solving the equation :

$$\nabla^2\psi = m^2\left[\frac{\partial}{\partial x}\left(\frac{v}{m}\right) - \frac{\partial}{\partial y}\left(\frac{u}{m}\right)\right].$$

From the computed ψ field, u and v are obtained by using the relations,

$$u = -\,m\,\frac{\partial\psi}{\partial y}, \qquad v = m\,\frac{\partial\psi}{\partial x}$$

The u and v values thus obtained are used as input to the balance equation to obtain the height field, h ;

$$\nabla^2 h = -\left(\frac{1}{g}\right)\left[\frac{\partial}{\partial x}\left(u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} - \frac{f}{m}\,v\right) + \frac{\partial}{\partial y}\left(u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} + \frac{f}{m}\,u\right)\right]$$

5. Finite difference scheme

Simple centred difference scheme is adopted for space derivatives. The time difference scheme has been discussed in section 2. Along the short trajectories, forward and centred implicit iterative time difference schemes have been adopted.

6. Data

The basic input in the model being the wind, the wind field is manually analysed for streamlines and

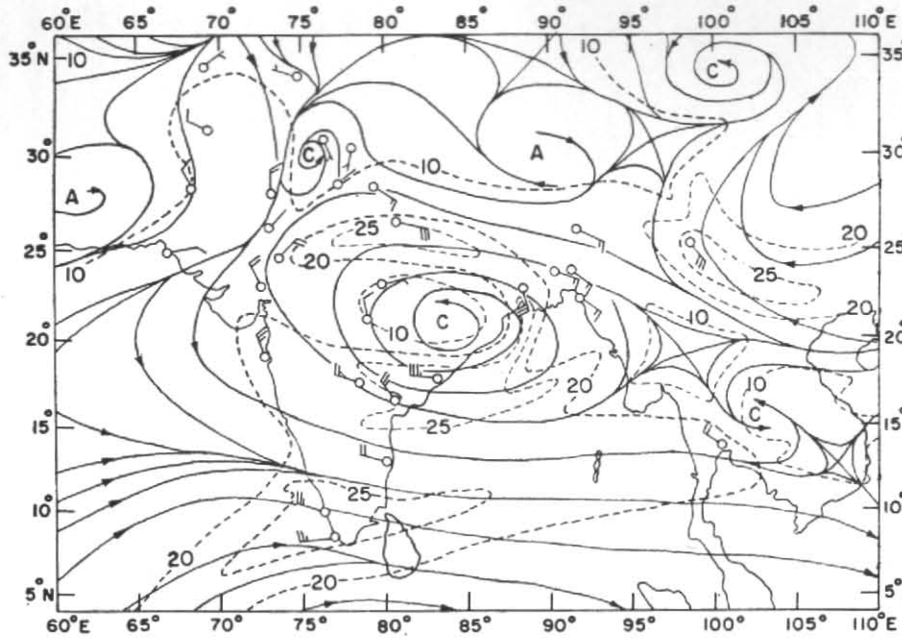

Fig. 3. Streamline-isotach analysis at 700 mb for the case of monsoon depression of 4 August 1968 at 00 GMT. Station winds are also shown.

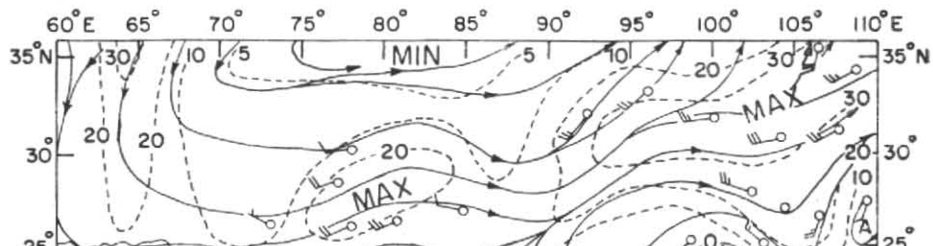

Fig. 4. Streamline-isotach analysis at 500 mb for the case of tropical cyclone of 14 October 1971 at 00 GMT. Station winds are also shown.

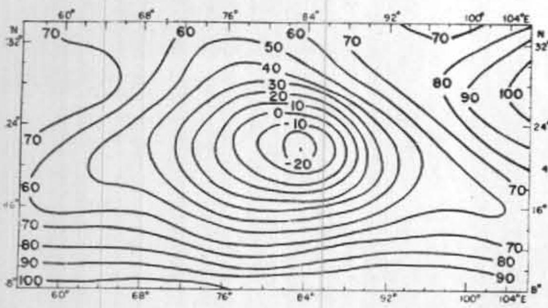

Fig. 5. Initial map (00 GMT, 4 Aug 1968)

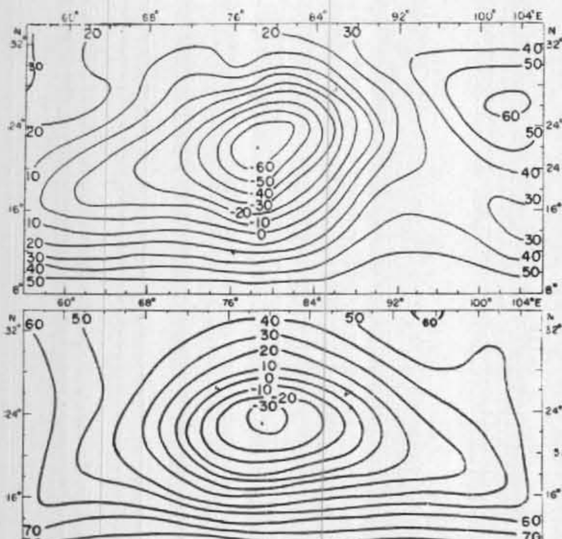

Fig. 6. Forecast stream-function charts at 700 mb for 24 hr and corresponding verification charts. Initial map is shown in Fig. 5 (x gives the centre of depression)

isotachs at 700 mb for the case of monsoon depression and at 500 mb for the case of tropical cyclone. The selection of 700 mb level for monsoon depression is based on the observation that the circulation of monsoon depression hardly extends above 300 mb. In the case of the tropical cyclone, the system extends to a great depth of the atmosphere, *i.e.,* nearly upto the tropopause. For the present study, a typical case of monsoon depression at 00 GMT, 4 Aug 1968, and a typical case of tropical cyclone at 00 GMT, 14 Oct 1971, have been selected. Figs. 3 and 4 present the streamline-isotach analysis in respect of these two situations. The domain of integration extends in the case of monsoon

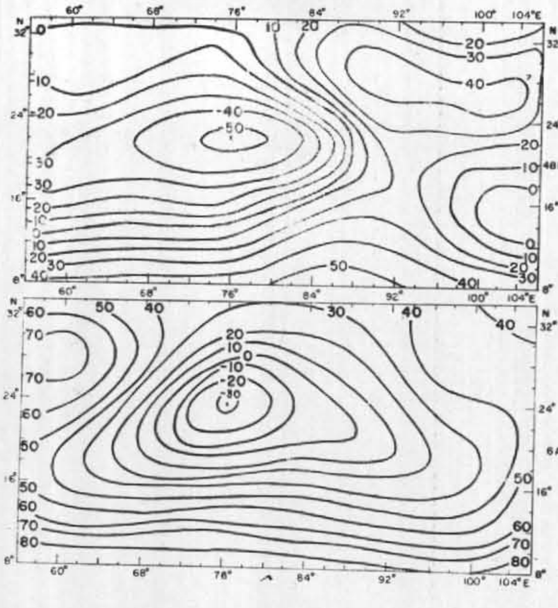

Fig. 7· Forecast stream-function charts at 700 mb for 48 hr and corresponding verification charts. Initial map is shown in Fig. 5 (x gives the centre of the depression)

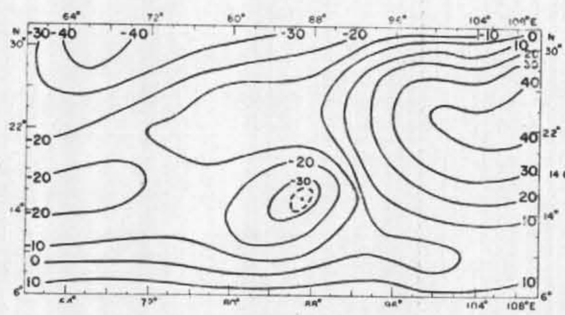

Fig. 8. Initial map (00 GMT, 14 Oct 1971)

depression from 6°N to 36°N and from 56°E to 106°E and in the case of tropical cyclone from 4°N to 34°N and from 60°E to 110°E.

A two-degree latitude-longitude grid is used in all the cases. A time step of 7 minutes is found suitable for this particular choice of grid distance. In the case of monsoon depression, the mean free surface is taken as 1500 gpm corresponding to 700 mb and in the case of tropical cyclone, the mean free surface is taken as 2500 gpm corresponding to 500 mb.

7. Forecast and verification

The forecast results for both the typical cases are discussed separately.

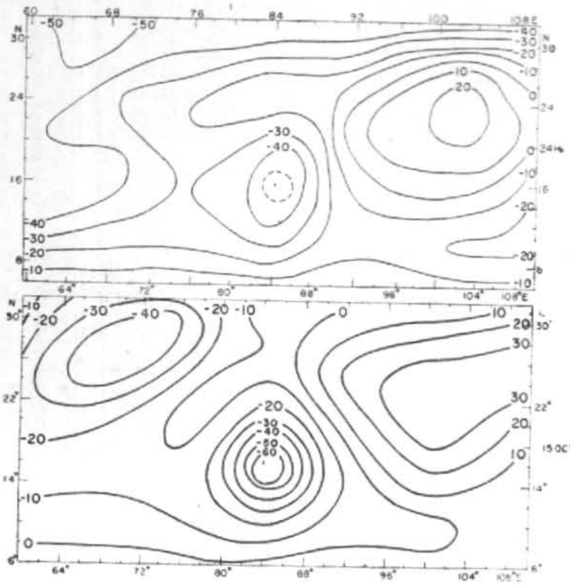

Fig. 9. Forecast stream-function charts at 500 mb for 24 hr and corresponding varification charts. Initial map is shown in Fig. 8 (× gives the centre of the tropical cyclone)

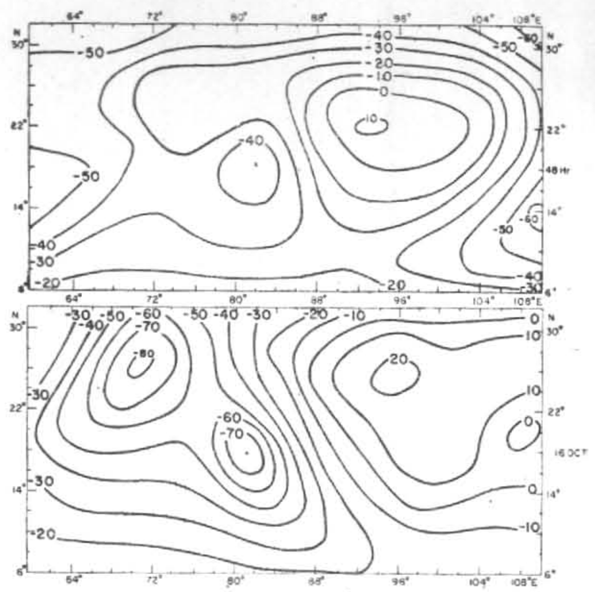

Fig. 10. Forecast stream-function charts at 500 mb for 48 hr and corresponding verification charts. Initial map is shown in Fig. 8 (× gives the centre of the tropical storm)

TABLE 1

Forecast and observed position of the centre of the monsoon depression

Date	4 Aug 1968 Initial time	5 Aug 1968 24 hours	6 Aug 1968 48 hours
Observed position at 700 mb	22°N, 84°E	23°N, 79°E	24°N, 76°E
Forecast position at 700 mb	—	22·5°N, 79°E	22°N, 76°E
Vector error (km)	—	55	220

TABLE 2

Forecast and observed position of the centre of the tropical cyclone

Date	14 Oct 1971 Initial time	15 Oct 1971 24 hours	16 Oct 1971 48 hours
Observed position at 500 mb	16°N, 88°E	16°N, 84°E	17·5°N, 81·5°E
Forecast position at 500 mb	—	16°N, 84°E	18°N, 82°E
Vector error (km)	—	0	80

TABLE 3

Verification of forecasts of the u and v field (m sec^{-1})

Period of forecast	u-component		v-component	
	Root mean square error	Correlation coefficient	Root mean square error	Correlation coefficient
	Initial chart, 0000 GMT 4 August 1968			
24 hours	3·5	·86	2·7	·64
48 hours	4·3	·75	4·5	·11
	Initial chart, 0000 GMT, 14 October 1971			
24 hours	4·0	·62	3·5	·50
48 hours	6·3	·31	6·0	·14

7.1. *Case of monsoon depression*

Fig. 5 shows the initial stream-function field at 700 mb on 00GMT on 4 August 1968. Figs. 6 and 7 show the forecast and corresponding verification charts respectively for the case of the monsoon depression. Table 1 gives the positional error of the forecast.

7.2. *Case of tropical cyclone*

Fig. 8 shows the initial stream-function field at 500 mb at 00GMT on 14 October 1971. Figs. 9 and 10 show the forecast and corresponding verification charts for 24 hr and 48 hr respectively for the case of tropical cyclone. Table 2 gives the positional error of the forecast.

It may be seen from the above results that the forecast for 24 hr and 48 hr are reasonably good and error in the position of the centre of the monsoon depression is 55 km in 24 hour and 220 km in 48 hours. The corresponding error in the case of tropical cyclone is zero km in 24 hours and 80 km in 48 hours.

Table 3 gives the verification of the forecast computed separately for the u and v components. It gives the root mean square errors and correlation coefficients between actual and forecast for the whole forecast domain. It may be seen from Table 3 that with the increase errors their correlation coefficients decrease with period of forecast for both the typical situations. The v-forecast field corresponding to 48 hours for both the cases are poorly correlated with actual fields. Though errors in forecast centres are not very significant but there is a general tendency in forecast flow field to become westerly. It is suspected that this might perhaps be due to damping inherent in the barotropic system. When a smoother was applied prior to 24 hours forecast output, the 48 hours forecast field showed extreme damping effect, and flow pattern in general became westerly. This has further strengthened the view that damping might be the main reason for poor prediction of v-field.

8. General remarks and conclusion

Results obtained for both the cases as presented in Tables 1 to 3 to suggest that foreasts upto 48 hours are encouraging but that beyond this period (not presented here) deteriorates rapidly. Several factors may be responsible for the limited success with the scheme tried. Further work is clearly needed in respect of initialization, boundary conditions and time integration scheme for improvement in accuracy and increase in the range of useful forecasts.

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DISCUSSION

(Paper presented by Kshudiram Saha)

M. SHANKAR RAO : How Lagrangian and Eulerian methods compare ?

AUTHOR : Results obtained (vector error of position) with the two schemes for the two cases were as follows :

	24-hr F/C	48-hr F/C	72-hr F/C
	Quasi-Lagrangian Scheme		
Monsoon depression	55 km	220 km	—
Tropical cyclone	0 km	80 km	—
	Eulerian Scheme		
Monsoon depression	56 km	110 km	245 km
Tropical cyclone	0 km	110 km	693 km