

## Design wave height and wave period of sea waves at Digha

S. K. DASKAVIRAJ and S. K. SARKAR

River Research Institute, Haringhata Central Laboratory, Mohanpur, Nadia

(Received 8 June 1976)

**ABSTRACT.** The most probable height and period of the waves which will occur in any given length of time have been determined from systematic instrumental records of sea waves at Digha. In determining the design wave height and wave period the method as suggested by Draper (1963) has been used. Cumulative distribution of the highest and the significant wave heights and the distribution of wave periods in percentage of occurrence have also been presented in this paper.

### 1. Introduction

For scientific investigation of the problem of coastal erosion at Digha in West Bengal, it was decided to record the waves and tides for a considerable period. This is because reliable values of long term characteristics of waves and tides should be used by the engineer in his design equations before any coastal engineering structure is constructed to tackle the problem of erosion. Hence, to record the tides and waves of the sea, a F. M. pressure recorder unit was installed in the sea at Digha in West Bengal. The transducer or sea unit was placed near the sea bed, over a tripod structure 3 ft high at a depth of 13.7 ft from the mean sea level at a distance of nearly 4000 ft from the shore into the sea at Lat. 21°36'N and Long. 87°37'E. This sea unit was connected by an armoured cable to the shore unit on the bank. With this F. M. recorder, waves and tides were successfully recorded for a period of over two years. The instrumental records were analysed to obtain in a unified form, some equations and diagrams, to enable an engineer to derive practically useful but theoretically sound parameters for designing a coastal engineering structure.

### 2. Notations

*Wave height and wave period parameters—Definitions*

$H$ —Denotes generally the wave height, i.e., vertical distance from crest to trough of the same wave.

$H_1 = A + C$

where,  $A$  = height of highest crest above mean water level (*vide* Fig. 8)

$C$  = depth of lowest trough below mean water level (*vide* Fig. 8)

$H_1'$ —True height (=  $H_1$  corrected for attenuation of waves with depth, and response of the instrument, as appropriate)

$H_s$ —Significant wave height. It is the mean of highest one-third of all the waves on the record according to Sverdrup and Munk (1947)

$\mu H_{max}$ —Probable maximum wave height for a specific period (2 hours)

$H_{rms}$ —The square root of the mean of the squares of the instantaneous distances of water level from the mean level

$N_z$ —The number of times the trace passes through the mean water level in an upward direction.

$T_z$ —The mean zero crossing period, obtained by dividing the duration of record (in sec) by the number of times ( $N_z$ ).

### 3. Choice of record

Instead of continuous recording of waves and tides, fortunately, systematic intermittent recording produces adequate informations for almost all purposes. It has been shown by Draper (1963) that a continuous recording for a whole day will only reduce the error in measurement by a factor of two when compared with only a ten minute recording, provided the wave conditions remain stationary. As this is not so, more frequent recording of ten to fifteen minutes duration at an interval of two hours is made. Wave records of ten minutes duration are taken with a chart speed of 1" per minute and tide records with a chart speed of 1" per hour. The time the recorder is 'on', is referred as recording period and the interval between the start of successive records as the recording interval.

### 4. Measurement of parameters from wave records

More than 2500 wave records for the period April 1968 to March 1970 have been analysed and the results in a unified form are given in this paper. A typical ten minutes length of a wave record of

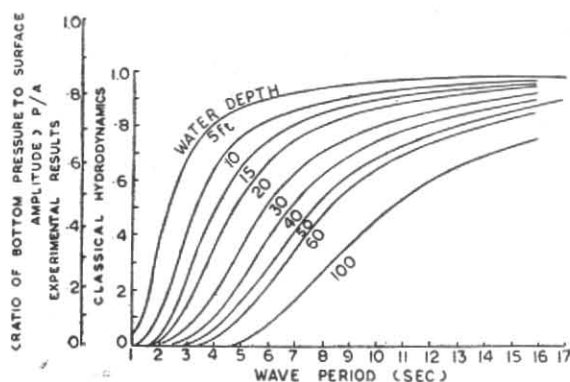


Fig. 1. Relations between pressure fluctuations on the sea bottom, surface wave ht., wave period and depth of water. The "Experimental results" scale is adopted from Draper L. attenuation of sea waves with depth "La Houille Blanc No. 6, Dec 1957 and refers to waves recorded on a gently sloping beach.

the sea waves at Digha is given in Fig. 8 from which the wave height parameters ( $H_1$ ) and the zero crossing period ( $T_z$ ) are determined.

(i) *The wave height parameter ( $H_1$ )* -- At first the mean water level is drawn by eye estimation. Each small vertical division of the recording paper corresponds to a water level variation of 0.2 ft over the transducer. From the mean water level the highest crest height ( $A$ ) and the lowest trough depth ( $C$ ) are measured. Then  $H_1$  is equal to  $A+C$ .

(ii) *The zero crossing period ( $T_z$ )* -- The number of times, the record crosses the zero line moving in an upward direction are counted. This gives the value of  $N_z$ . Then  $T_z = 600/N_z$  sec.

#### 5. Processing and presentation of the wave parameters

5.1(a). *Corrected wave height ( $H_1'$ )* -- As the wave height sensor was an under water pressure measuring unit, the recorded values of wave heights are to be corrected for (i) attenuation of waves with depth and for (ii) the frequency response of the instrument. The sensor is placed at 13.7 ft below the mean sea level. The depth of water is determined by adding to or subtracting from 13.7 ft, the water level as observed from tide records (Fig. 9).

Fig. 1 gives the relations between pressure fluctuation on the sea bottom, surface wave height, wave period and depth of water. The experimental results scale is adopted from Draper (1957). This is applicable to waves recorded on a gently sloping beach like Digha beach. The correction

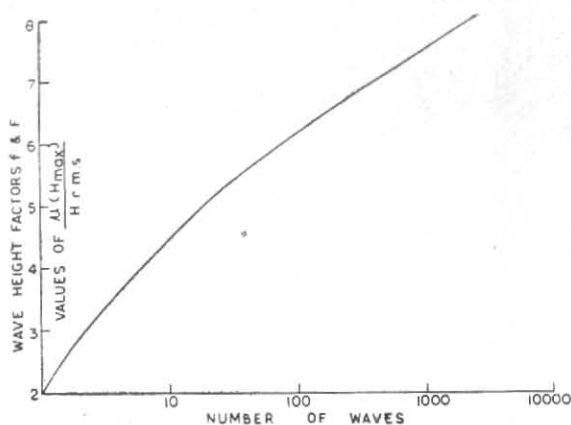


Fig. 2. Relationship between the No. of waves on a record and the normalized maximum wave height  
For  $N=1, 2, 5, 10, 20, 50, 100, 200, 500, 1000, 2000, 5000, 10000, 20000, 50000$  and  $100000$  the  $\mu(H_{\max}/D_{rms})$  are 2.00, 2.91, 3.86, 4.47, 5.02, 5.68, 6.14, 6.57, 7.09, 7.47, 7.82, 8.28, 8.61, 8.92, 9.32 and 9.61 respectively.

factor for a particular wave height is obtained from the curve given in Fig. 1, when the corresponding wave period  $T_z$  and the depth of water above the sensor are known.  $H_1$  is to be divided by the corresponding factor obtained from Fig. 1 to get the corrected wave height  $H_1'$ .

5.1(b). *Highest waves in a specific period* -- The recording unit is programmed to take a ten minutes wave record for every two hours. So the largest wave height will probably occur during the inoperative period. To find out this parameter it is assumed that there will never be a storm worse than those which were present during the recording period and there is no relation between successive waves in a record of narrow frequency band. Longuet-Higgins (1952) developed a relationship between the number of waves in a record and normalized maximum wave height in a specific period. The normalized maximum wave height is expressed as  $\mu H_{\max}/H_{rms}$  where,  $H_{rms}$  is the square root of the mean of the squares of the instantaneous distances of water level from the mean level and is computed from surface elevation and depression at all points. The  $H_{rms}$  which is also expressed as  $D_{rms}$  is a measure of the mean energy per unit area of the sea surface. For a given wave system with specific  $H_{rms}$  the relation between this normalized maximum wave height and the wave number is expressed graphically in a Fig. 2 [adopted from Longuet-Higgins (1952) -- Table 2].

Fig. 2 may be explained with an example. Say, in a ten minute wave record at an interval of two

hours we have the zero crossing period of this record to be ten seconds, *i.e.*, there will be 60 of these waves during this recording period. And in the two hours interval there will be 720 of ten second waves.

$$\begin{aligned} &\text{Therefore, } \mu H_{\max} \text{ (2 hours)} \\ &= \frac{\mu H_{\max} / H_{\text{rms}}}{\mu H_{\max} / H_{\text{rms}}} \quad \text{for 720 waves} \\ &\quad \times H_{\max} \text{ (observed)} \\ &= \text{say, } \frac{F}{f} \times H_{\max} \text{ (observed in the record)} \end{aligned}$$

But we take here  $H_1'$  for  $H_{\max}$  for the record. Thus the value of  $F$  and  $f$  are determined with the help of Fig. 2.

The absolute values of  $F$  and  $f$  will not be necessary in such applications.

5.1(c). *Significant wave height* ( $H_s$ ) — This is defined (according to Sverdrup-Munk 1947) as average height of 1/3 of the highest waves. It is calculated from the following relationship :

$$H_s = H_1' \cdot 2 \cdot (2\theta)^{-1/2} (1 + 0.289\theta - 1 - 0.247\theta^2)^{-1}$$

where,  $\theta = \log_e N_z$

The relationship is expressed graphically in Fig. 3. This figure is a modification of Fig. 2 of Tucker (1963).

5.2. *Presentation of the data*

For constructional and operational purposes it is often important to know how long a given condition will last. The cumulative percentage diagram is found to be most useful. From this diagram it is possible to decide at a glance how often and for how long wave conditions of certain specific heights and above are liable to persist in a certain period (in a season or a year). In other words, the exceedance graphs give the percentage of time for which wave conditions exceeded any particular height. To draw the percentage of exceedance graph for significant wave height the range of significant wave height is split up into a number of small ranges (*e.g.*, 0-0.49, 0.5-0.99, 1-1.49, 1.5-1.99 etc). The wave height data is then inspected and for each value, one count is recorded in each of the lower ranges. This is done for every record. The number of counts in each range is then totalled and divided by the total number of wave height figures in the whole data, giving the proportion of time for which waves exceeded the given heights. The cumulative distribution of significant wave height ( $H_s$ ) is presented in Fig. 4. From this figure the proportion of time for which  $H_s$  exceeded any given height may be determined.

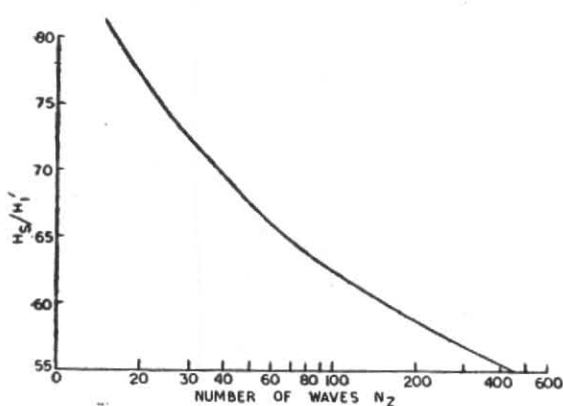


Fig. 3. Determination of  $H_s$

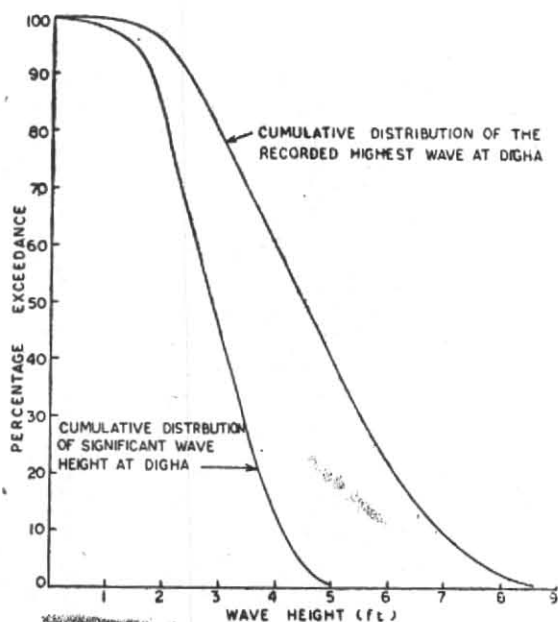


Fig. 4. Cumulative distribution of the recorded highest wave and significant wave height at Digha

In a similar manner the range of probable maximum wave heights in the recording interval (*i.e.*, 2 hours) has also been split up and following the above process cumulative distribution curve for probable maximum wave height has been drawn and shown in Fig. 4.

The percentage of occurrence of a specific wave period may be determined in the same way as above. The range of measured zero crossing periods is split up into a number of small ranges (5.5-5.49, 5.5-5.99, 6.0-6.49 etc). The time period data is then inspected and for each one count is recorded in the corresponding range. This being done for every record, the number of counts in each range is then totalled and divided by the total number of time period figures in the whole data; this gives the percentage of occurrence of some specific period of waves. The distribution of zero crossing period is shown in Fig. 5.

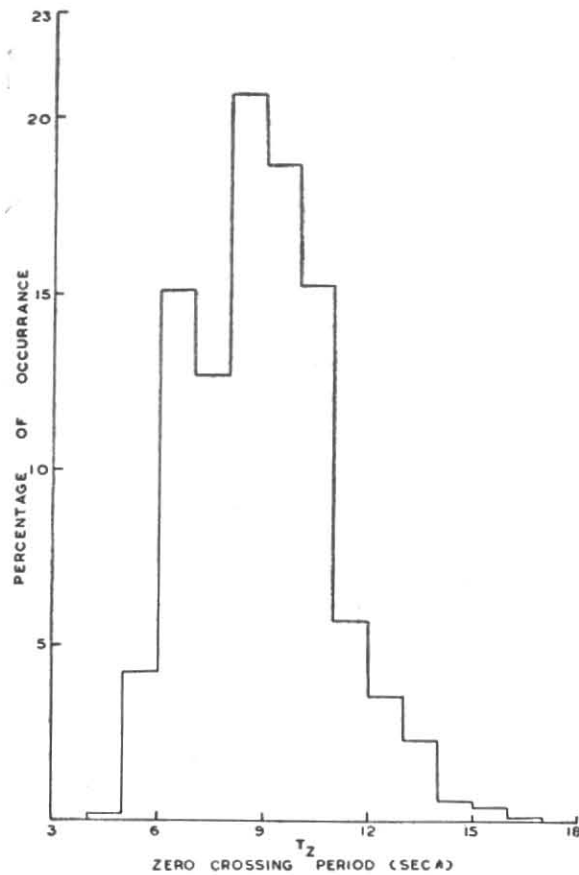


Fig. 5

#### 6. Design wave height and wave period

It is often necessary to determine the highest wave which is likely to occur in the life time of a structure. Estimating the wave conditions for hundred years from one year's data is obviously hazardous. But as an estimate has to be made, the statistics of extreme values provides a theoretical basis on which this may be done. This extreme value statistics is concerned directly with the occurrence of rare events.

Wave height values are plotted on logarithmic normal probability graph paper. The proportion of time for which a wave height is exceeded is plotted against that wave height. Darbyshire (1956) has shown that this representation makes a linear relationship which can be extrapolated to estimate the probability of occurrence of any greater wave height.

The range of measured wave height is split up into a number of small ranges (*e.g.*, 0.0-0.49; 0.5-0.99; 1.0-1.49 etc). The wave height data are then inspected and for each value one count is recorded in its proportional ranges and also one count is recorded in each of the lower ranges. This is done for every record. The number

of units in each range is then totalled and divided by the total number of wave height figures in the whole data; this gives the proportion of time for which the wave exceeded the given height. This data is then plotted on probability graph paper (Fig. 6). The best fitting straight line is then drawn through points. Points representing the higher waves have been deduced from very few observations and are, therefore, less reliable, and should be accorded less weight than the other points further down the line.

Each ten minutes record is assumed to be representative of the wave during its recording interval of two hours; one unit of time is therefore equal to one recording interval, *i.e.*, two hours. If a wave of certain height has a probability density  $P$ , this means that its probability of occurrence is once in  $(1/P)$  times the recording interval.

The probable wave height which will occur once in any particular length of time (say 100 years) can be obtained from this curve. The recording interval is divided by the time for which the forecast is required using the same unit of time. This gives the probability  $P$  of one occurrence. Corresponding to this probability value  $P$  the

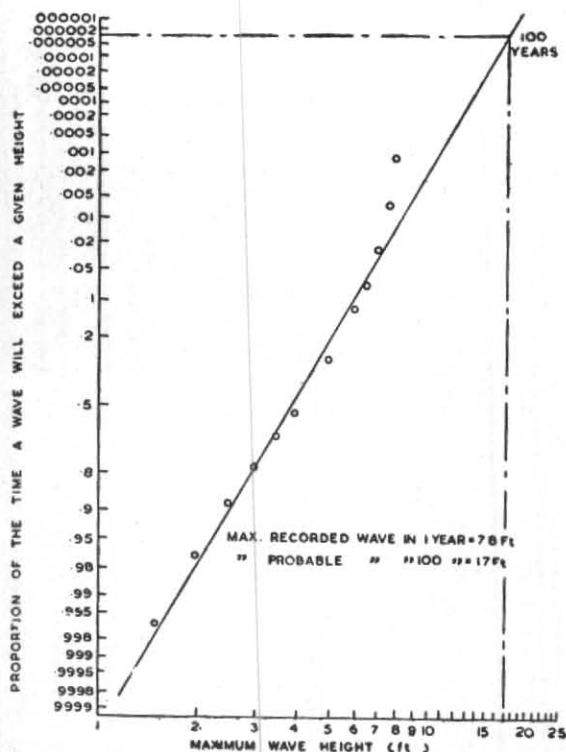


Fig. 6. Maximum wave height plotted against the proportion of the time a wave will exceed a given height

most probable wave height which will occur once during the period under consideration, is found out from this graph. The period of waves in a storm is a function of the severity of the storm.

Jasper (1956) has shown that the predominant period of waves also appears to follow the same type of distribution as wave height, so by the same method one can predict the most probable maximum period. It is reasonable to assume that the largest wave height may be associated with the largest wave period. Analysis with zero crossing period is done in the same way as above, and the resulting curve as drawn is shown in Fig. 7. The probable wave period corresponding to design wave height (*i.e.*, for 100 years) may be found out from this curve.

#### 7. Discussion of results

From Fig. 4 the proportion of time for which  $H_s$  and  $H_{max}$  (2 hours) exceeded any given height may be determined. For example, the significant wave height exceeded 2 feet for 80 per cent of the time. The height of the highest wave exceeded 4 feet for 62 per cent of the time.

In Fig. 5 the absence of waves with a zero crossing period less than 5 sec suggests that the shorter locally generated waves were almost absent. It also shows that most common zero crossing period

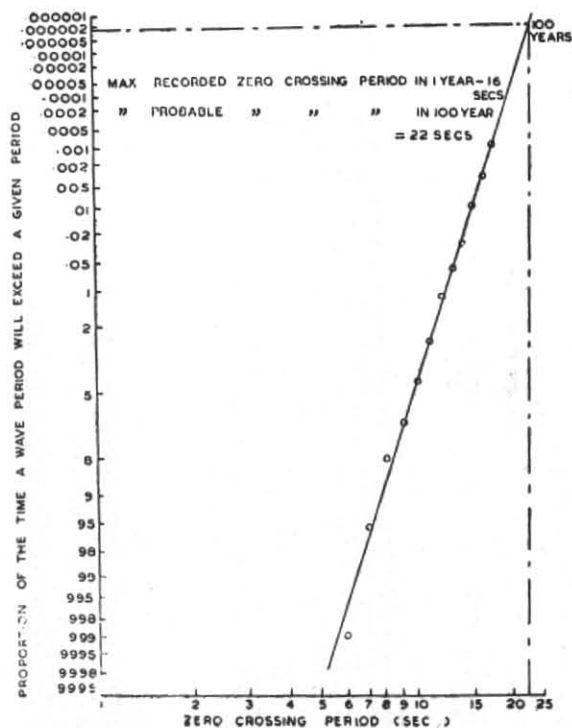


Fig. 7. Zero crossing period plotted against the proportion of the time a wave record will exceed a given period

ranges from 6 sec to 11 sec approximately.

In Fig. 6, the data used are the calculated values of wave height ( $H_1'$ ) from the records. This presentation suggests that the highest wave of all which will occur at Digha once in 100 years will be about 17 feet high.

For engineering purposes,  $T_s$  is probably the most significant wave period, since this tends to be the period of the largest waves in the record. It is also one which enters more often into the statistical formulae. Fig. 7 shows that the maximum zero crossing wave period which will occur at Digha once in 100 years will be about 22 seconds.

#### 8. Conclusion

An estimate of the design wave height and wave period obtained by this method should not be over emphasized as there is no guarantee that a wave of certain height and period will actually occur in a given length of time. On the other hand the reliability of the derived values of the design wave height and period depends on the quality of the recording on which it is based. Thus, where the data for longer period are not available, the wave height and period calculated from the above method may be used as a general guide in erecting any coastal structure,

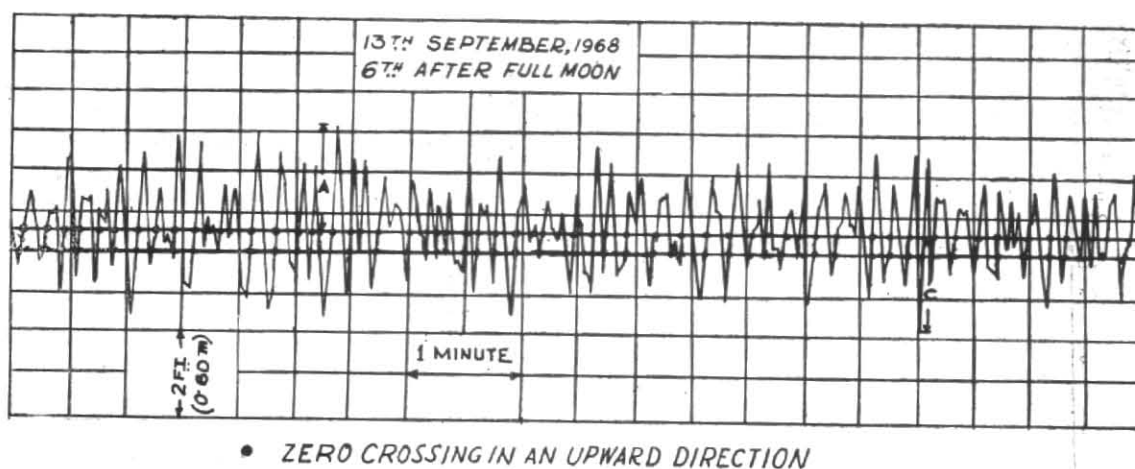


Fig. 8. A typical wave record of sea waves at Digha

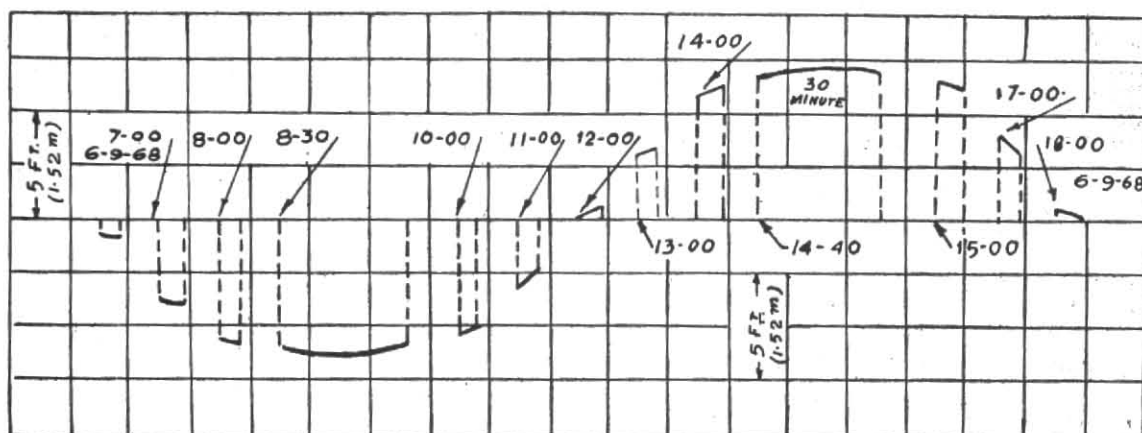


Fig. 9. A typical tide record of sea at Digha

*Acknowledgement*

The authors would like to express their gratitude

to Shri P.B. Ray, Director, River Research Institute, Govt. of West Bengal for his keen interest and encouragement in this work.

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