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Use of kinematical determinant to locate the centre of cyclones and the ring of maximum wind enclosing the centre

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ABSTRACT. It is established that the kinematical determinant $[\mathbf{k}.\ \{(\nabla u)\times(\nabla v)\}]$ can be used to determine the centre of cyclones and the ring of maximum wind enclosing the centre.

1. Introduction

1·1. The streamlines spiral and terminate at the centre of a cyclone as illustrated in Fig. 1. The speed is zero at the centre. Proceeding from the centre outwards in any direction, it is noted that the speed increases, reaches a maximum whereafter it gradually decreases as shown in Fig. 2. The locus of such points of maximum speed is called the ring of maximum wind in this paper. A schematic illustration of the speed field (Fig. 3) where continuous lines are isotachs, numbers are speed in knots and ABCDA indicated by a broken line is the ring of maximum wind.

1·2. An excellent treatment of the properties of two dimensional linear vector field is given by Godske *et al.* (1957). Linear vector fields do not have maximum of speed/magnitude in the finite regions whereas the actual winds do have. After a study of the available literature, the author notes that there is a necessity for a critical examination of the kinematical properties of spiralling circulations which are essentially non-linear. Hence, the objectives of this paper are to study the kinematical parameters associated with spiralling circulations as in cyclones and establish that the kinematical determinant can be used to locate the centre of a spiral and the ring of maximum wind enclosing the centre.

2. Constraints

2·1. A two dimensional vector field $\mathbf{V}=u\mathbf{i}+v\mathbf{j}$ is studied. The magnitude V of the vector is given by the relation $V^2=u^2+v^2$ where u and v are called the components of the vector. If $\mathbf{V}$ is the wind, the magnitude is called the speed. $\mathbf{V}$, u, v and (V^2) are functions of x and y. An arbitrary scalar $\phi=\phi(x,y)$ also used in this paper.

2·2. The fields are constrained to be

(*a*) Single-valued

(*b*) Finite

(*c*) Continuous

(*d*) Differentiable

(*e*) Expandable as Taylor's series. The expansion from a point(x_0,y_0)taken as the origin is

$$\phi=\phi_{\text{at }(x_0,y_0)}+\left\{\left(\frac{\partial\phi}{\partial x}\right)_{(\text{at }x_0,\,y_0)}x+\left(\frac{\partial\phi}{\partial y}\right)_{\text{at }(x_0,\,y_0)}y\right\}+\left\{\left(\frac{\partial^2\phi}{\partial x^2}\right)_{(\text{at }x_0,\,y_0)}x^2+2\left(\frac{\partial^2\phi}{\partial x\,\partial y}\right)_{(\text{at }x_0,\,y_0)}x\,y+\left(\frac{\partial^2\phi}{\partial y^2}\right)_{\text{at }(x_0,\,y_0)}y^2\right\}+\ldots$$

$\mathbf{V}=$ [expansion of u] $\mathbf{i}$ + [expansion of v] $\mathbf{j}$

(*f*) At a point where the vector vanishes, the gradients of its components and its kinematical determinant do not vanish. If $\mathbf{V}$, for instance, vanishes at a point, then $\nabla u\neq0,\nabla v\neq0$ and $[(\nabla u)\times(\nabla v)]\neq0$ at that point.

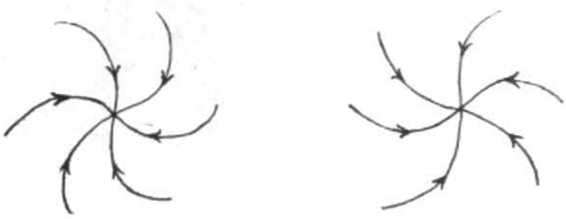


Fig. 1

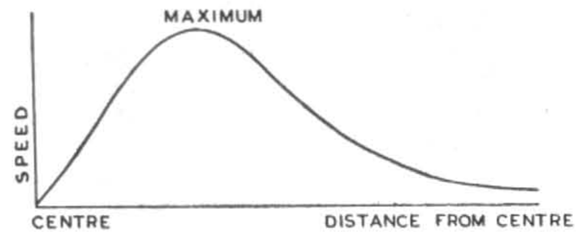


Fig. 2

3. Spiral centre and Col point

3.1. In a spiralling field, the vector lines/streamlines terminate at/originate from a point which in this paper is called the spiral centre. Two vector lines/streamlines intersect at a point with reversals in direction and all the other vector/streamlines in the immediate neighbourhood are hyperbolically shaped. This point is referred to as col point/neutral point.

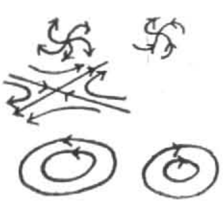
3.2. *Theorem I*

The magnitude of the vector must vanish

(*a*) at the spiral centre

(*b*) at a col point

(*c*) at a point where the area enclosed by vector lines vanishes in the limit.



(*d*) In short at a point where the direction of the vector is not single-valued, *i.e.*, the direction of the vector is multivalued.

Proof

$$\mathbf{V} = V(\cos\Psi\ \mathbf{i} + \sin\Psi\ \mathbf{j})$$

where

$$V^2 = u^2 + v^2 \quad \text{and}$$

$$\Psi = \tan^{-1}\left(\frac{v}{u}\right)$$

$$= \cos^{-1}\left(\frac{u}{V}\right)$$

$$= \sin^{-1}\left(\frac{v}{V}\right)$$

The only way by which Ψ can be ensured to have multi-values is by making $u = 0$ and $v = 0$, *i.e.*, the vector magnitude must vanish.

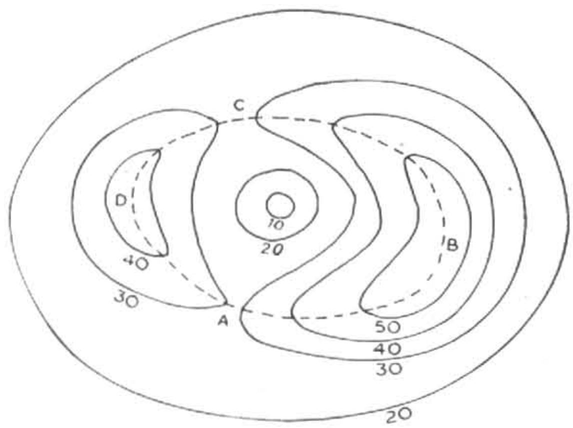


Fig. 3

3.3. The equiscalar line is usually called in meteorological literature as the isopleth. The isopleth of direction is called the isogon.

Theorem II

A point where the magnitude of the vector vanishes, *i.e.*, $u = v = V = V^2 = 0$ is a minimum enclosed by isopleths of magnitude.

Proof

At a point where $u = v = V = V^2 = 0$, the gradient of (V^2) vanishes and the Hessian of (V^2) is $4[(\nabla u)\times(\nabla v)]^2$. Further (V^2) is an essentially positive quantity. Under the constraint, 2·2 (f), $[(\nabla u) \times (\nabla v)] \neq 0$, *i.e.*, the Hessian is positive. Hence the point where the vector magnitude vanishes is a minimum enclosed by the isopleths of the magnitude.

3.4. In Theorem IV, a condition for spiral centre/col points is that the magnitude of the vector is zero. This is based on Theorem I. While defining the ring of maximum wind, the speed is said to increase in any direction starting from the centre initially. This is based on Theorem II.

3.5. *Isogons*

In the immediate neighbourhood of a point (x_0, y_0) :

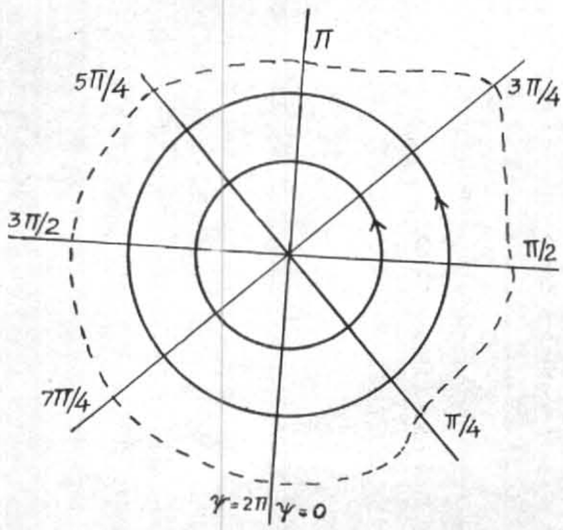


Fig. 4. Isogon field with a spiral centre

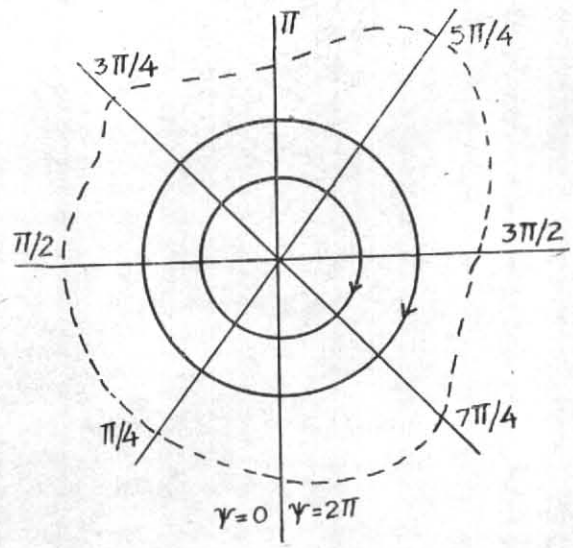


Fig. 5. Isogon field with a col point

—Isogons,— →Gradient, $\nabla\psi$: vector line, ...Contour on which integration $\oint \mathbf{n}\times(\nabla\psi)\ dl$ is performed, $\mathbf{n}$: Unit vector drawn outwards normal to contour, dl : Element of a line on the contour.

where $u = 0$ and $v = 0$,

$$\tan \Psi = \frac{\gamma x + \delta y}{\alpha x + \beta y}$$

where $\left(\frac{\partial u}{\partial x}\right)_{\text{at }(x_0,\, y_0)} = \alpha$, $\left(\frac{\partial u}{\partial y}\right)_{\text{at }(x_0,\, y_0)} = \beta$,

$$\left(\frac{\partial v}{\partial x}\right)_{\text{at }(x_0,\, y_0)} = \gamma \text{ and } \left(\frac{\partial v}{\partial y}\right)_{\text{at }(x_0,\, y_0)} = \delta$$

$$\nabla \Psi = \frac{(\alpha\,\delta - \beta\,\gamma)\,(\mathbf{k}\times\mathbf{r})}{(\alpha x + \beta\, y)^2 + (\gamma\, x + \delta\, y)^2}$$

Here $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j}$

From this we can easily enunciate.

Theorem III

In the immediate neighbourhood of a point (x_0, y_0) where the vector vanishes

(*a*) the gradient vector $\nabla\Psi$ lines are circles enclosing the point (x_0, y_0)

(*b*) the isogons are radial lines radiating out of (x_0, y_0)

(*c*) the gradient vector $\nabla\Psi$ line is anticlockwise provided $(\alpha\delta - \beta\gamma)$ is positive in which case :

$$\oint \mathbf{n} \times (\nabla\, \Psi)\, dl = +\, 2\,\pi\, \mathbf{k}$$

and the gradient vector line is clockwise provided $(\alpha\delta - \beta\gamma)$ is negative in which case :

$$\oint \mathbf{n} \times (\nabla\, \Psi)\, dl = -\, 2\,\pi\, \mathbf{k}$$

3.6. In Theorem IV, we will prove that at a spiral centre $(\alpha\delta - \beta\gamma)$ is positive and at a col point $(\alpha\delta - \beta\gamma)$ is negative. Fig. 4 gives a schematic representation of the isogon near a spiral point and Fig. 5 is for a col point.

4. Kinematical parameters

4.1. The partial derivatives of u and v with respect to x and y are set in the form of a matrix

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

Divergence, vorticity, stretching deformation and shearing deformation are obtained by adding or subtracting the diagonal elements of the matrix. Two more kinematical parameters L_1 and L_2 are proposed.

$$L_1 = 4 \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} = 4\mathbf{k}.\,[(\nabla u)\times(\nabla v)]$$

TABLE 1

Nomenclature	Abbreviation	Math. expression	Linear field
Divergence	DIV	$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$	$(\alpha + \delta)$
Vorticity	VOR	$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$	$(\gamma - \beta)$
Stretching deformation	D_{st}	$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$	$(\alpha - \delta)$
Shearing deformation	D_{sh}	$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$	$(\gamma + \beta)$
Kinematical determinant	L_1	$4\left[\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right]$	$4(\alpha\delta - \beta\gamma)$
L_2	L_2	$L_1 - (\text{DIV})^2$	$4(\alpha\delta - \beta\gamma) - (\alpha+\delta)^2$

$$= (\text{DIV})^2 + (\text{VOR})^2 - D_{st}^2 - D_{sh}^2$$

$$L_2 = L_1 - (\text{DIV})^2$$

4.2. A two dimensional vector field is defined as $\mathbf{V}=(\alpha x+\beta y)\mathbf{i}+(\gamma x+\delta y)\mathbf{j}$ where α, β, γ and δ are constants. Table 1 lists the parameters.

5. Patterns of vector/streamlines

5.1. In the case of linear two dimensional vector fields, the patterns of vector/streamlines are obtained by solving

$$\frac{dy}{dx} = \frac{\gamma x + \delta y}{\alpha x + \beta y}$$

5.2. The solution is $(y - \mu_1 x)^{\lambda} \; (y - \mu_2 x)^{\lambda'} = \text{constant}$

μ_1, μ_2, λ and λ' are related to α, β, γ and δ as follows :

$$\mu_1 = \frac{-(\alpha - \delta) + \sqrt{(\alpha+\delta)^2 - 4(\alpha\delta - \beta\gamma)}}{2\beta}$$

$$\mu_2 = \frac{-(\alpha - \delta) - \sqrt{(\alpha-\delta)^2 - 4(\alpha\delta - \beta\gamma)}}{2\beta}$$

$$\lambda = \left[\frac{(\alpha+\delta) + \sqrt{(\alpha+\delta)^2 - 4(\alpha\delta-\beta\gamma)}}{2\sqrt{(\alpha+\delta)^2 - 4(\alpha\delta - \beta\gamma)}}\right]$$

$$\lambda' = -\left[\frac{(\alpha+\delta) - \sqrt{(\alpha+\delta)^2 - 4(\alpha\delta-\beta\gamma)}}{2\sqrt{(\alpha+\delta)^2 - 4(\alpha\delta - \beta\gamma)}}\right]$$

$$\lambda + \lambda' = 1$$

5.3. *Case (i) λ and λ' are real and are of the same sign*

In such a case, two vector lines intersect at the origin with reversals in directions. The rest are hyperbolically shaped. The point of intersection is called a neutral point/a col point.

Case (ii) λ and λ' are real but their signs are opposite

All vector lines either terminate at/originate from the origin. The vector lines are parabolically shaped or resemble a point source/sink.

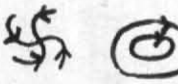
Case (iii) λ and λ' are complex conjugates

All the vector lines originate from/terminate at the origin. The vector lines are spiral shaped/ closed lines.

Details of derivations and discussions may be seen in the book *Dynamic Meteorology & Weather Forecasting* by Godske *et al.* (1957).

5.4. We can state the conditions in terms of L_1, and L_2. Case (*i*) L_1 is negative and L_2 is negative. This ensures λ and λ' to be real and of the

TABLE 2

λ and λ'	L_1	L_2	Patterns	Schematic diagram
Real and of same sign	Negative	Negative	Hyperbolically shaped	
Real but of opposite signs	Positive	Negative	Parabolic shaped/ point source or sink	
Complex conjugates	Positive	Positive	Spiralling/ closed lines	

same signs. Case (*ii*) L_1 is positive and L_2 is negative. This ensures λ and λ' to be real but of opposite signs. Case (*iii*) L_1 is positive and L_2 is positive. This ensures λ and λ' to be complex conjugates. It may be noted that the Case (*iv*) L_2 positive and L_1 negative is impossible since $L_2 = L_1 - (\text{DIV})^2$

5.5. Table 2 lists the patterns in terms of L_1 and L_2.

The graphical representations in column 5 of Table 2 are merely illustrative. For a near exhaustive patterns, please refer Godske's book (pp. 200-203).

5.6. Taking a non-linear vector field expandable as a Taylor's series we get

$$\mathbf{V} = \left[u + \left\{ \left(\frac{\partial u}{\partial x} \right)_{\text{at } (x_0,\, y_0)} x_{\text{at } (x_0,\, y_0)} + \left(\frac{\partial u}{\partial y} \right)_{\text{at } (x_0,\, y_0)} y \right\} + \ldots \right] \mathbf{i}$$

$$+ \left[v + \left\{ \left(\frac{\partial v}{\partial x} \right)_{\text{at } (x_0,\, y_0)} x_{\text{at } (x_0,\, y_0)} + \left(\frac{\partial v}{\partial y} \right)_{\text{at } (x_0,\, y_0)} y \right\} + \ldots \right] \mathbf{j}$$

Here (x_0, y_0) is taken as the origin of the coordinate. Under the constraint (f) of 2.2, *viz.*, at a point where $u=v=0$, $(\nabla u) \neq 0$, $(\nabla v) \neq 0$ and $(\nabla u) \times (\nabla v) \neq 0$, we note that the vector field has the characteristics of a linear vector field in the immediate neighbourhood of a point where $u=0$ and $v=0$. Therefore we can easily enunciate the following.

Theorem IV

(*a*) The vector/streamlines in the immediate neighbourhood of a point where $u=v=0$, L_1 is negative and L_2 is negative, will be hyperbolically shaped as at a col point with two vector/streamlines intersecting at that point and having reversals in direction.

(*b*) The vector/streamlines in the immediate neighbourhood of a point where $u=v=0$, L_1 is positive and L_2 is negative will be parabolically shaped/resemble a point source or sink. All the vector/streamlines will either terminate at/originate from the point.

(*c*) The vector/streamlines in the immediate neighbourhood of a point where $u=v=0$, L_1 is positive and L_2 is positive will be spiral shaped/closed lines. All vector lines which are spiral shaped either terminate at or originate from the point and closed lines enclose the point where $u=0$ and $v=0$.

5.7. The case where $u=v=0$, $L_1=0$ and $L_2=0$ is complex. We can have in this case all patterns enumerated in Theorem IV and many more patterns not discussed. For instance, at $x=0$, $y=0$ $u=v=L_1=L_2=0$ in the specific case of $\mathbf{V}=(x^2+y^2)[(x+y)\,\mathbf{i}+(-x+y)\mathbf{j}]$ and the vector/streamlines are spiral shaped.

Similarly, at $x=0$, $y=0$, $u=v=L_1=L_2=0$ in the specific case of $\mathbf{V}=(x^2+y^2)\ (x\,\mathbf{i}+y\,\mathbf{j})$ and the vector/streamlines are hyperbolically shaped.

6. Ring of maximum wind

6.1. We define the ring of maximum wind enclosing the spiral centre as follows. Proceeding from the spiral centre in any direction, the speed initially increases, reaches a maximum whereafter it decreases. The locus of such points is a continuous curve enclosing the spiral centre. This locus is defined as the ring of maximum wind.

6.2. *Theorem V*

Given a spiralling field $\mathbf{V}$ in which :

(*a*) $\nabla\Psi \neq 0$ and the isogons radiate outwards from the spiral centre.

(*b*) the spiral centre is in a positive region of L_1 and this positive region of L_2 is enclosed by a zero isopleth of L_1 outside which L_1 is negative.

(*c*) the isogons radiating outwards cut the zero isopleth of L_1, only once, then the zero isopleth of L_1 is the ring of maximum wind.

Proof

Step I

We note that the vector lines of $(\nabla\Psi)\times\mathbf{k}$ are isogons. If n is the distance measured along the isogons from the spiral centre

$$\left\{\left(\nabla\,\Psi\right)\times\mathbf{k}\right\}\cdot\nabla=|\nabla\,\Psi|\,\frac{\partial}{\partial\,n}$$

where $|\nabla\,\Psi|=\left\{\left(\frac{\partial\Psi}{\partial x}\right)^2+\left(\frac{\partial\Psi}{\partial y}\right)^2\right\}^{1/2}$

and $\frac{\partial}{\partial\,n}$ is differentiation along the isogon.

Step II

We note that $\nabla\Psi=\dfrac{u\,\nabla\,v-v\,\nabla\,u}{(V)^2}$

and $\nabla\,(V^2)=2\,(u\,\nabla\,u+v\,\nabla\,v)$

Hence $(\nabla\,u)\times(\nabla\,v)=\nabla\left(\dfrac{V^2}{2}\right)\times\nabla\,\Psi$

$$L_1=4\,\mathbf{k}\,.\left[\ (\nabla\,u)\times(\nabla\,v)\ \right]$$

$$L_1=4\,\mathbf{k}\,.\left[\ \nabla\,\left(\frac{V^2}{2}\right)\times\nabla\,\Psi\ \right]$$

Step III

We note that $\mathbf{A}\,.\,(\mathbf{B}\times\mathbf{C})=(\mathbf{C}\times\mathbf{A})\,.\,\mathbf{B}$

Replacing $\mathbf{A}$ by $\mathbf{k}$, $\mathbf{B}$ by $\nabla\left(\dfrac{V^2}{2}\right)$ and $\mathbf{C}$ by $\nabla\,\Psi$, we get

$$\mathbf{k}.\left[\nabla\left(\frac{V^2}{2}\right)\times\nabla\,\Psi\ \right]$$

$$=\left[\left(\nabla\Psi\right)\times\mathbf{k}\ \right]\,.\,\nabla\left(\frac{V^2}{2}\right)$$

$$=\frac{1}{2}\left|\nabla\,\Psi\right|\frac{\partial}{\partial n}\left(\ V^2\ \right)$$

$$i.\,e.,\quad L_1=2\left|\nabla\,\Psi\right|\frac{\partial}{\partial n}\left(V^2\right)$$

Differentiating L_1 with respect to n, we get

$$\frac{\partial L_1}{\partial n}=2\left|\ \nabla\Psi\ \right|\frac{\partial^2}{\partial n^2}\left(V^2\right)+$$
$$+\,2\,\frac{\partial}{\partial n}\left(V^2\right)\frac{\partial}{\partial n}\left|\nabla\Psi\right|$$

Step IV

At a point where $L_1=0$ and $\dfrac{\partial L_1}{\partial n}$ is negative $\dfrac{\partial(V^2)}{\partial n}=0$ and $\dfrac{\partial^2\,(V^2)}{\partial n^2}$ is negative.

6.3. In Fig. 6, ABCDA represents schematically the zero isopleth of L_1. Inside ABCDA, L_1 is positive and outside L_1 is negative. The spiral is centred at O. The isogons radiate outwards from O. They cut ABCDA only once. Starting along any isogon we note that the magnitude of the vector increases as we proceed outwards since L_1 is positive and hence $\partial(V^2)/\partial n$ is also positive. At the point where the isogon intersects ABCDA, $\partial V^2/\partial n=0$ since L_1 is zero and $|\nabla\Psi|\neq 0$. Further $\partial^2(V^2)/\partial n^2$ is negative since $\partial L_1/\partial n$ is negative because L_1 changes from positive to negative. We note that the points of intersections of the isogons and the zero isopleth of L_1 constitute a locus of maxima of (V^2). Hence the zero isopleth of L_1 is the same as the ring of maximum wind.

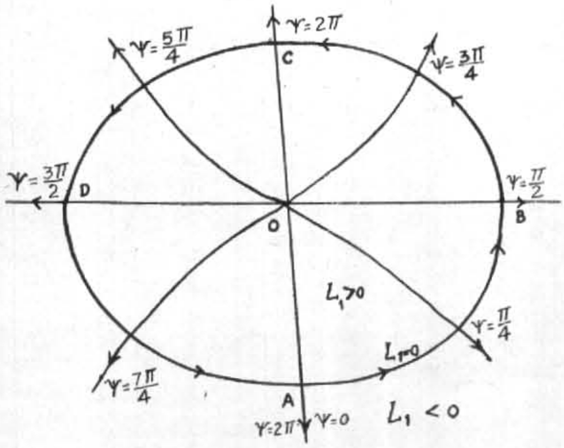


Fig. 6. ABCDA is the zero isopleth of L_1 inside which $L_1>0$ and outside which $L_1<0$

6.4. A relationship between the mean of the square of maximum wind and the totality of the kinematical determinant can easily be established.

Theorem VI

$$\iint L_1 \, ds = 4\pi \, \overline{(V_m^2)}$$

The area of integration is enclosed by the ring of maximum wind and $\overline{V_m^2}$ is the mean of the square of the maximum wind.

Proof

$$L_1 \mathbf{k} = 4\left[\nabla\left(\frac{V^2}{2}\right) \times \nabla \Psi\right]$$

$$\iint \nabla\left(\frac{V^2}{2}\right) \times \nabla \Psi \, ds = \oint\left[\mathbf{n} \times \left(\nabla \Psi\right)\right]$$

$$\times \frac{V^2}{2} dl = \left(\frac{\overline{V_m^2}}{2}\right) \oint \mathbf{n} \times \nabla \Psi \, dl$$

Here we make use of Theorem III (c) and get $\iint L_1 \, ds = 4\pi \, \overline{(V_m^2)}$. This relation can be of use to get an estimate of the mean of the maximum wind in the context of cyclones with sparse data distribution near the centre.

7. Models

7.1. A linear two dimensional vector field can be obtained from two scaler fields $\phi_1 = \frac{1}{2}(\alpha x^2 + \delta y^2)$ and $\phi_2 = \frac{1}{2}(\beta y^2 - \gamma x^2)$ as $\mathbf{V} = \nabla \phi_1 + (\nabla \phi_2) \times \mathbf{k}$. We will proceed in an analogous way for non-linear vector fields.

7.2. $\nabla \phi_1$ is an irrotational field and $(\nabla \phi_2) \times \mathbf{k}$ is a non-divergent field. ϕ_1 and ϕ_2 are chosen such that non-linear vector fields are obtainable, *i.e.*, ϕ_1 and ϕ_2 are not linear or homogenous quadratic function of x and y. We note that

Case (i)

$$L_1 = (\text{DIV})^2 - D_{st}^2 - D_{sh}^2$$
$$= 4 \text{ Hessian of } \phi_1$$
$$L_2 = - D_{st}^2 - D_{sh}^2$$

in the case of irrotational fields. Hence, spiralling patterns are not possible. Further at a point where $\nabla \phi_1 = 0$ and the Hessian of ϕ_1 is positive, all $\nabla \phi_1$ vector lines terminate at or originate from that point.

Case (ii)

$$L_1 = (\text{VOR})^2 - D_{st}^2 - D_{sh}^2 = L_2$$
$$= 4 \text{ Hessian of } \phi_2$$

in the case of non divergent fields. Hence parabolic shaped streamlines are not possible. Further at a point where $\nabla \phi_1 = 0$ and the Hessian of ϕ_2 is positive, the vector lines enclose the point.

7.3. A spiralling field appears to be a combination of both non-divergent and irrotational fields. Under certain boundary conditions, a vector field can be expressed as the sum of an irrotational and a non-divergent field by using Helmholtz theorem.

$$\mathbf{V} = A \nabla \phi_1 + B (\nabla \phi_2) \times \mathbf{k}$$

We note that such a field is preferable for real fluids since the velocity fields are both divergent and rotational. A specific case is where $\phi_1 = \phi_2 = \phi$.

$\mathbf{V} = (A \nabla \phi) + B (\nabla \phi) \times \mathbf{k}$. Taking ϕ as $\exp[-\{(x^2/2\sigma_1^2)+(y^2/2\sigma_2^2)\}]$ the author (Lakshminaraynan 1975) has modelled cyclones and has shown a good correspondence between the observation and the model. The ring of maximum wind can easily be shown to be the zero isopleth of L_1 in this model.

8. Discussion

8.1. Six constraints have been used. The constraints of single-valuedness and finiteness are experimentally established to be tenable in a number of physical phenomena like cyclones etc. Continuity, differentiability and expandability as Taylor's series are basis to mathematical analysis and these constraints have been commonly used in the context of cyclones as well. However it would be preferable to have a preliminary experimental verification to determine the extent upto which these constraints are tenable in specific cases. The last constraint 2·2 (f), *viz.*, at a point where $u = v = 0$, $\nabla u \neq 0$, $\nabla v \neq 0$ and $(\nabla u) \times (\nabla v) \neq 0$ is imposed mainly with a view to exclude a number of theoretically complex vector line pattern which are theoretically feasible but are not practically relevant for atmospheric motion.

8.2. Godske *et al.* (1957) have noted that vector/streamline patterns which are hyperbolically shaped with a col point change into spiralling elliptical patterns when the rotation exceeds a critical value. They have not specified the critical value. It is established that if L_1 and L_- are positive, the spiralling/elliptical patterns occur. We can interpret that positive L_1 means that the rotational *plus* divergent characteristics as given by $(\mathrm{DIV})^2 + (\mathrm{VOR})^2$ must be greater than the deformation characteristics given by $(D_{st}^2 + D_{sh}^2)$. Similarly positive L_1 means that the rotational characteristics given by $(\mathrm{VOR})^2$ must be greater than deformation characteristics given by $D_{st}^2 + D_{sh}^2$. In short, a field with predominant rotational and divergent characteristics has spiralling patterns and that with predominant deformation characteristics has a col point with hyperbolically shaped lines.

8.3. An appropriate orthogonal coordinate system for the evaluation of the kinematical determinant is the isogons and normals to the isogons. Construct the isogon chart and draw lines normal to it. Evaluate $|\nabla \Psi|$ from this chart. Noting that the positive direction of the isogon is from the spiral centre/towards a col point, $\partial (v^2)/\partial n$ is evaluated. L_1 is equal to $2[\partial (V^2)/\partial n]\,|\nabla \Psi|$ as shown in step III of Theorem V. L_1, is positive if the speed increases along the isogon. and negative if the speed decreases along the isogon. L_1 is zero at points of maxima/minima/inflexion. We easily note that the ring of maximum wind is the zero isopleth of L_1. The converse, *viz.*, the zero isopleth of L_1 is the ring of maximum wind, is established in Theorem V. Complex cases of a spiral centre enclosed by more than one ring of maximum wind can be dealt with by the methods outlined in this paper.

9. Conclusion

With the help of a few theorems, it is established that the kinematical determinant can be used to locate the centre of cyclones and the ring of maximum wind enclosing the centre.

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