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Using dispersion modeling for ground level concentration

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सार – इस शोध पत्र में उन्नत स्त्रोतों से भू–स्तरीय सांद्रता का परिकलन करते हुए एक लघु अवधि निदर्श का अनुमान लगाया गया है जिससे एक फीकीयन–प्रकार का फार्मूला तैयार किया गया है। स्त्रोत और मिश्रित उँचाई को लें तो ये पवन वेग और भंवर–विस्तारित प्रोफाइलों की क्रियाएँ हैं। संवहन विसरण समीकरण के सही विलयन से आकलित किए गए निदर्श की तुलना मौसम विज्ञान आँकड़ों का उपयोग करते हुए धरातल के निकट एकत्रित किए गए प्रयोगात्मकर धरातलीय सांद्रताओं के साथ की गई है।

ABSTRACT. A short range model calculating ground-level concentration from elevated sources is estimated, which realized a Fickian-type formula. Taking the source and mixing height are functions of the wind velocity and eddy diffusivity profiles. The model estimated with an exact solution of the advection diffusion equation is compared with experimental ground level concentrations using meteorological data collected near the ground.

Key words – Ground-level concentration, Fickian-Type formula, Mixing height.

1. Introduction

Most of the estimates of dispersion from continuous point sources are pased on the Gaussian approach where the plume is dispersed by homogeneous turbulence. Turbulence is usually not homogeneous in the vertical direction. The dispersion parameters of the Gaussian plume model depend on downwind distance and stability classes. The effect of the three factors on the estimated ground-level concentration (glc) is investigated (Kretzchmar and Mertens 1984). The form of Gaussian plume solution and the mathematical problem associated with it have been widely discussed (Csanady, 1973; Seinfield, 1986).

The analytical solution of advection-diffusion equation by parameters of wind speed and eddy diffusivities as function of height above the ground is investigated (Lin and Hildmann 1997; Mangia *et al*. 2002 and Wortmann *et al.* 2005) in a finite or infinite vertical domain.

The planetary boundary layer (PBL) is often capped by an inversion, which tends to reflect back the air pollutions hitting the inversion base (Arya, 1999). The presence of inversion influences the ground level concentrations (Hanna *et al*. 1982) depending on the plume penetrating the elevated inversion or trapping blow it.

In this work, we introduce and validate a practical model for calculating the glc from elevated source that applies a new Gaussian formulation for transport and vertical diffusion. The model has previously been described in Lupini and Tirabassi (1981). In the present model the vertical source height and mixing height are expressed by simple functions of the vertical profiles of wind speed and turbulence diffusivity. The validation of the model with the data obtained from Copenhagen Experiment.

2. Mathematical Model

The steady state transport of a non-reactive contaminant released from a point source is described by the following partial differential equation.

$$u(z)\frac{\partial C}{\partial x} = \frac{\partial}{\partial z}\left[K(z)\frac{\partial C}{\partial z}\right] \quad (1)$$

$$C \rightarrow \delta(z-1) \quad as \;\; x \rightarrow 0$$

$$K\frac{\partial C}{\partial z} \rightarrow 0 \quad as \quad z \rightarrow 0,\ z \rightarrow h$$

where C is the concentration of the contaminant, u is the mean wind speed, $K(z)$ is the eddy diffusivity in z direction, x is the downwind distance and h is the mixing height.

We introduce non-dimensional variables u, C, x, z, K and h as follows:

$$z = z'/h_s\,,\ x = x'K_s/u_s h_s^2\,,\ C = C'u_s h_s/Q\,,$$

$$z = u'/u_s\,,\ K = K'/K_s\,,\ M = M'/h_s$$

where Q is the source emission rate, h_s is the emission height above the source height and M is the non-dimensional of mixing height.

One can estimate the concentration at the surface at any point using the standard Gaussian model for lateral concentrations as follows:

$$C(x, y, 0) = \frac{C_y}{\sqrt{2\pi\sigma_y}}\exp\left(-\frac{y^2}{2\sigma_y^2}\right) \tag{2}$$

where y is the crosswind distance and σ_y is the crosswind dispersion parameter.

To calculate C_y, one can suppose a Fickian type formula where the source and mixing heights are expressed by simple functions of the vertical profiles of wind and eddy diffusivity (Lupini and Tirabassi, 1981).

One supposes two virtual source (lower and upper) heights as follows:

$$\mu_s = \int_0^1 (u/k)^{0.5}\,\mathrm{d}z$$

$$\zeta_s = \int_0^1 u\,\mathrm{d}z$$

and two virtual mixing (lower and upper heights) as follows:

$$R = \int_0^M (u/k)^{0.5}\,\mathrm{d}z$$

$$N = \int_0^M u\mathrm{d}z$$

The crosswind- integrated concentration glc $C_y(x, 0)$ is introduced by means of Fickian - type formula with a source placed at the geometric average of the virtual source heights μ_s and ζ_s as follows:

$$C_y(x, 0) = \frac{1}{\sqrt{\pi x}}\exp(-\mu_s\zeta_s/4x) \tag{3}$$

For general profile of the wind and eddy diffusivity, one expect that $\zeta_s < 1 < \mu_s$ and $N<M<R$. This explains the physical meaning of the two levels and the importance of the u and K profiles in calculating the solution of the advection diffusion at the ground.

To get downwind distance at which maximum concentration occurs put $\partial C/\partial x = 0$. Differentiating equation (3) with respect to "x" and equating the result with zero, we get that:

$$x_{\max} = \frac{\zeta_s\eta_s}{2} \tag{4}$$

Pasquill (1974) gives a solution for the cases of u and K_z varying according to height raised to some power:

$$u = u_1(z/z_1)^\alpha \qquad K_z = K_{z_1}(z/z_1)^\beta$$

where u_1 is the wind speed at height z_1 and K_{z_1} is the eddy diffusivity at height z_1, then the solution has the form:

$$C(x, 0) = \frac{rz_1^\alpha}{2u_1\Gamma(S)}\left[\frac{u_1 z_1^{\beta-\alpha}}{r^2 K_{z_1} x}\right]^s \tag{5}$$

where the parameter $r = \alpha - \beta + 2$; the parameter $s = (\alpha + 1)/r$ and Γ is the gamma function. Then the values of the two virtual source heights are as follows:

$$\zeta_s = \frac{u_1}{z_1^\alpha(\alpha + 1)}$$

$$\mu_s = \sqrt{\frac{u_1 z_1^{\alpha-\beta}}{K_{z_1}}}\frac{2}{(\alpha - \beta + 2)}$$

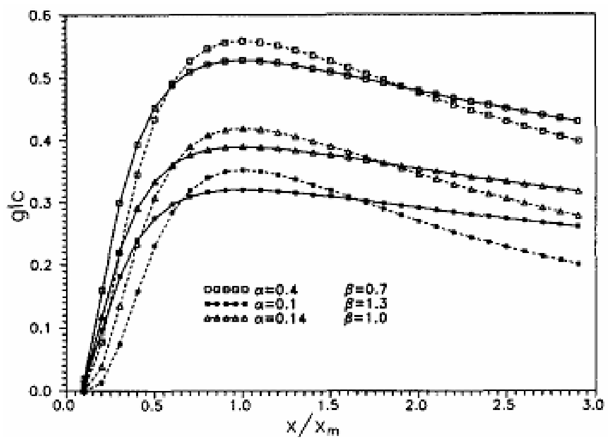

Fig. 1. Glc predicted by the proposed model (continuous line) Eqn. (6) and by analytical solution of the K- equation (dashed line) Eqn. (5) via downwind distance from the source (x) normalized by a maximum glc position (x_m)

The proposed approximated solution in the case of power law is given as follows:

$$C(x,0)=\frac{1}{\sqrt{\pi x}}\exp\left[-\frac{u_1^{3/2}z_1^{-\left(\frac{\alpha+\beta}{2}\right)}}{2\sqrt{K_{z_1}}(\alpha+1)rx}\right] \quad (6)$$

The maximum downwind distance takes the form:

$$x_m=-\frac{u_1^{3/2}z_1^{-\left(\frac{\alpha+\beta}{2}\right)}}{\sqrt{K_{z_1}}(\alpha+1)r}$$

The Fickian-type approximation presented is given by equation (3) in the case of $M\to\infty$ as follows:

$$\frac{1}{\sqrt{\pi x}}\exp\left(-h_s^2/4x\right)+\sum_{n=1}^{\infty}\left\{\exp\left[-(h_s-2nM)^2/4x\right]+\exp\left[-(h_s+2nM)^2/4x\right]\right\} \quad (7)$$

where

$$h_s=(\zeta_s\mu_s)^{0.5} \quad M=(RN)^{0.5}\prec\infty$$

3. Validation

Comparison was made in the case of an unstable atmosphere where an exponent for the wind profile α = 0.1 and eddy diffusivity β = 1.3; in the case of a neutral atmosphere α = 0.14 and β = 1; while in a stable atmosphere α = 0.4 and β = 0.7. Fig. 1 shows the glc calculated by the proposed model [Equation (6)] via the two-dimensional analytical solution of the K-equation (5) estimated by Pasquill (1974). From Fig. 1 we can see that the proposed model approximation represents a good agreement of the glc.

TABLE 1

Meteorological data used from Gryning *et al*. (1987)

Exp No.	u (m/s)	u_* (m/s)	L (m)	w_* (m/s)	H (m)	H/L
1.	3.4	0.37	-46	1.7	1980	-43
2.	10.6	0.74	-348		1920	-5
3.	5.0	0.39	-108		1120	-10
4	4.6	0.39	-173		390	-2.3
5.	6.7	0.46	-577		820	-1.4
6.	13.2	1.07	-569		1300	-2.3
7.	7.6	0.65	-136	2.1	1850	-1.4
8.	9.4	0.70	-72	2.1	810	-11
9.	10.5	0.77	-382		2090	-5.5

Table 1 shows the meteorological data, wind speed, friction velocity, Monin-Obukov length, vertical velocity scale, Mixing height, and stability parameter (Gryning *et al*. 1987). The meteorological data used were collected near the ground, so the comparison can be simulated the values given by a routine use the model.

The analytical approximation proposed in this paper is validated with the data sets obtained at Northern part of Copenhagen (Gryning and Lyck 1984). The tracer Sulfur hexafluoride (SF_6) was released with buoyancy from a tower at a height of 115 m and collected at the ground-level up to three crosswind arcs of tracer sampling units. The sampling units were positioned 2-6 km from the point of release. Tracer releases started 1h before the start of tracer sampling and stopped at the end of sampling period. The average sampling time was 1h and a roughness length was 0.6m.

Two different shapes of wind and eddy diffusivity have been used for the calculations from Table 1. We find that 2/3 of the mixing height is more than 1000 m, so we considered to be at approximately the top of the surface layer. For this reason we used two different wind and eddy diffusivity profiles calculated by using the similarity theory, one valid for the surface layer and the other for the top of the atmosphere.

For first one, we defined as follows:

$$u=\frac{u_*}{\mathrm{k}}\left[\ln(z/z_0)-\psi_m(z/L)+\psi_m(z_0/L)\right] \quad (8)$$

TABLE 2

Observed and calculated crosswind –integrated concentrations C_y/Q at different distances from the source. Proposed Model 1 uses equations (6), (7) and (8), while proposed Model 2 uses equations (6), (7), (9) and (10)

Exp.	Distance (m)	Observed data ($10^{-4}sm^{-2}$)	Predicted by proposed model 1 ($10^{-4}sm^{-2}$)	Predicted by proposed model 2 ($10^{-4}sm^{-2}$)
1	1900	6.84	5.85	6.25
	3700	2.31	4.60	5.52
2	2100	5.38	3.30	3.55
	4200	2.95	2.90	2.92
3	1900	8.20	5.85	6.45
	3700	6.22	4.90	5.30
	5400	4.30	4.10	4.56
4	4000	11.66	4.99	7.02
5	2100	6.72	5.15	5.93
	4200	5.84	4.50	5.25
	6100	4.97	4.10	4.67
6	2000	3.96	2.55	3.55
	4200	2.22	2.15	2.95
	5900	1.83	1.90	2.66
7	2000	6.70	3.70	4.35
	4100	3.25	2.90	3.22
	5300	2.23	2.65	2.85
8	1900	4.16	3.12	4.20
	3600	2.02	2.54	3.41
	5300	1.52	2.23	2.95
9	2100	4.58	3.25	3.40
	4200	3.11	2.70	2.90
	6000	2.59	2.40	2.49

$$K_z = \mathrm{k} u_* z / \phi_h \qquad (9)$$

where u_* is the friction velocity, k = 0.4 Von-Karman constant, z the height, z_0 the roughness height, L is the Monin-Obukhov length and Ψ_m, Φ_h are stability parameters defined as follows:

For $L \leq 0$

$$\phi_h = (1 - 16z/L)^{-1/2}$$

$$\psi_m = 2\ln\left[\left(\frac{1+A}{2}\right)\right] + 1n\left[\left(\frac{1+A^2}{2}\right)\right] - 2\tan^{-1} A + \frac{\pi}{2}$$

where

$$A = (1 - 16z/L)^{1/4}$$

For $L > 0$

$$\phi_h = 1 + 5z/L \qquad \psi_m = -5z/L$$

In the second case from (Sharan and Yadav 1998) during stable and near neutral case $M/L \geq -10$, we defined as follows:

$$K_z = k u_* z (1 - z/M)^2 / \phi_h \qquad (10)$$

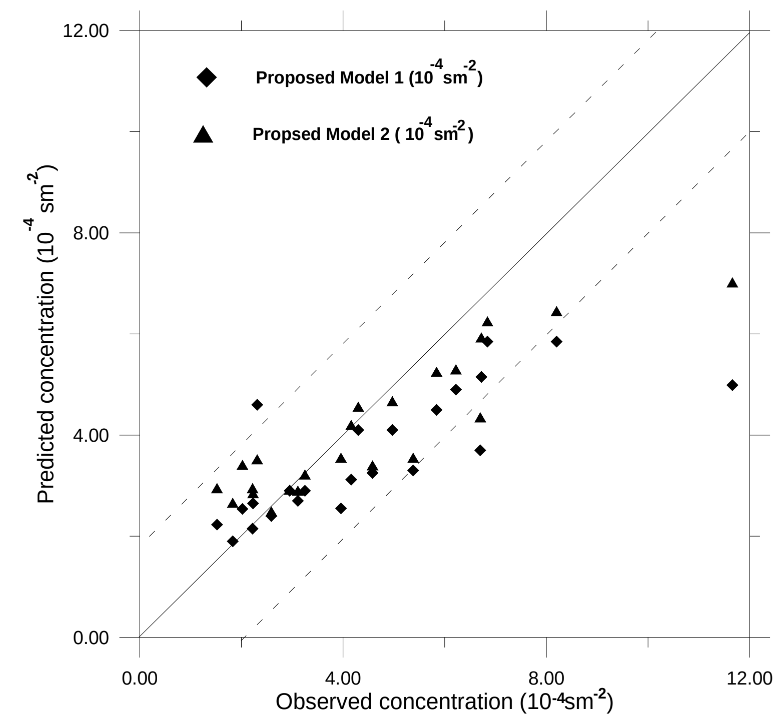

Fig. 2. Observed *versus* predicted concentrations (10^{-4} sm^{-2})

For convective condition ($M/L \leq -10$) we used convective velocity (w_*) instead of friction velocity (u_*) (Arya, 1999) to give

$$K_z = kw_* z(1 - z/M) \quad (11)$$

Table 2 shows that the measured ground level concentration and predicted concentration by proposed model 1 used ones of the Fickian-type model (6) and two equations (7) and (8) and predicted proposed model 2 used ones of the Fickian-type model (6) and three equations (7), (9) and (10). We find that the predicted proposed model 2 model is nearer to one to one observed concentrations than predicted proposed 1.

Fig. 2. Show that the relations between observed and predicted concentrations by the two proposed models. We find that the predicted glc by proposed model 2 is agreement with the observed concentration than predicted by proposed model 1, however predicted concentrations by proposed model 1 are within factor of two with observed concentrations.

4. Conclusions

One can show that the glc predicted by an exact analytical solution of the advection-diffusion equation for elevated source can be approximated by a Fickian type formula where the source and the mixing heights are expressed by simple functions of the vertical profiles of wind and eddy diffusivity profiles. We can see that the proposed model approximation represented a good agreement of the glc.

Predicted of the model performance based on Fickian type formula, using SF_6 tracer data using meteorological data collected near the ground alongside wind and eddy diffusivity profiles calculated by using the similarity theory produced good result.

The predicted proposed model 2 is agreement with one to one observed concentration than predicted proposed model 1, but predicted concentrations by proposed model 1 are within factor of two with observed concentrations.

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