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An aerodynamic method to compute evaporation

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सार—वाष्पन के आकलन के लिए अरेखीयसतहपरत मॉडल (शाहावी 1983) पर आधारित एक वायुगतिक विधि स्थापित की गई है। वाष्पन की दर को भूमि या जल सतह पर आकलित किया जा सकता है। निकाले गए संबंध से अनुमानित वाष्पन के मान, एक सहज प्राप्य गुणक एस से प्रभावित होते हैं जो कि पवनवेग, वायुमंडलीय स्थिरता, माप की ऊंचाई, सतह की रुक्षता, बेरोमीट्रिक दाब तथा वायु के घनत्व के संयोग को प्रदर्शित करता है।

व्युत्पन्न समीकरण के परिणामों तथा मापे गए मानों का तुलनात्मक अध्ययन करने पर पता चला है कि प्रस्तावित विधि काफी तर्कसंगत तथा सही है।

ABSTRACT. An aerodynamic method estimating evaporation on the basis of the non-linear surface layer model (Shahawi 1983), has been established. Evaporation rate, E, could be estimated over land or water surface. The values of evaporation estimated from the derived relation, are influenced by an easily obtainable coefficient, S, which represents a combination of wind speed, atmospheric stability, heights of measurements, roughness of the surface, barometric pressure, and density of the air.

Comparative studies between the results of the derived equation, and the measured values revealed considerable accuracy and validity of the proposed method.

1. Introduction

Turbulent water vapour flux can be evaluated directly by the eddy correlation method, using fast response sensors to record turbulent fluctuations and obtaining the mean product

$$E = \rho \, \overline{w' \quad q'} \tag{1}$$

where E is the water vapour flux. q' and w' are the fluctuating turbulent deviation from the mean value of specific humidity and vertical component of wind velocity. However, the equipment and techniques required are elaborate, and till now this method has been employed only by few specialized research groups.

Several methods have been established to evaluate evaporation in terms of easily measured quantities.

These methods may be classified to three main groups. The first is established on the basis of the conservation of heat or water within a system. This includes the heat budget and water budget methods. The second group is concerned wtih the diffusion of water vapour by turbulent processes. Methods of this group are known as bulk aerodynamic and profile methods. Combination of heat budget and bulk aerodynamic approaches yields the methods of the third group known as combination methods (Webb 196).

The object of this study is to establish a generalized formula to estimate evaporation rates with considerable precession from simple measurements of wind velocity, temperatures and humidities over land or water surface.

2. Theoretical approach

A mean property $X\,(Z)$ in the fully turbulent, constant flux layer is given by the following relation based on Monin-Obukhov similarity analysis (1954):

$$X = X_0 \; X_n\,(Z_n) \tag{2}$$

where X_0 is the scaling parameter for X, equal in individual cases to

$u_0 = -\dfrac{u_*}{k}$ for X = mean wind speed u, $X_n = u_n$

$\theta_0 = -\dfrac{H}{\rho\, c_p\, k\, u_*}$ for X = mean potential temperature θ, $X_n = \theta_n$

$q_0 = -\dfrac{E}{\rho\, k\, u_*}$ for X = mean specific humidity q, $X_n = q_n$, and $Z_n = Z/L$ the non-dimensional level of measurement

$L = -\dfrac{u^3_*}{kgH/\rho\, c_p\, \theta}$ the Monin-Obukhov length. Other notation is conventional.

The form of X_n (Z_n) has long been at issue. It is, however, well established that the eddy transfer coefficients for u, θ and q could be assumed equal. Therefore, similarity of the non-dimensional functions for wind (u_n), temperature (θ_n), and humidity (q_n) could be considered. Following the non-linear surface layer model (Shahawi 1983), one finds the expression for u_n and θ_n, and consequently q_n as follows :

$$u_n + C = \theta_n + C_1 = q_n + C_2 = Z_n + $$

$$+ 2\tan^{-1}\left(\frac{4}{4+Z_n}\right) + \ln\left(\frac{Z_n}{8+Z_n}\right) \quad (3)$$

In order to eliminate the constants C, C_1 and C_2 of Eqn. (3), measurements of u, θ and q should be taken at two levels or more. The finite difference of the non-dimensional functions of Eqn. (3) corresponding to measurements of wind speed at two levels, a, and b, may be written as follows :

$$\triangle u_n = \left(\frac{b-a}{L}\right) + 2\left[\tan^{-1}\left(\frac{4L}{4L+b}\right)\right.$$

$$\left. - \tan^{-1}\left(\frac{4L}{4L+a}\right)\right] + \ln\left|\frac{b\,(a+8L)}{a\,(b+8L)}\right| \quad (4)$$

Eqn. (5) similar to Eqn. (4) is given below for $\triangle\theta_n$ and $\triangle q_n$ when temperatures and humidities are both measured at the levels l and h;

$$\triangle\theta_n = \triangle q_n = \left(\frac{h-l}{L}\right) + 2\left[\tan\left(\frac{4L}{4L+h}\right)\right.$$

$$\left. - \tan\left(\frac{4L}{4L+l}\right)\right] + \ln\left|\frac{h\,(l+8L)}{l\,(h+8L)}\right| \quad (5)$$

At sufficiently small heights or sufficiently large L, (neutral conditions), functions of Eqn. (4) and (5) approach the logarithmic form given below :

$$\triangle u_n = \ln\left(\frac{b}{a}\right) \quad (6)$$

$$\triangle\theta_n = \triangle q_n = \ln\left(\frac{h}{l}\right) \quad (7)$$

On the other side, the finite differences of the property X provided from application of Eqn. (2) to the levels of measurements (a) and (b) may be expressed as :

$$\triangle X = X_0 \; \triangle X_n \left(\frac{a}{L}\,,\; \frac{b}{L}\right) \quad (8)$$

Applying Eqn. (8) for the mentioned individual cases and substituting for the values of the corresponding scaling parameters we find :

$$u_* = k\,(u_b - u_a)/\triangle u_n \quad (9)$$

$$H = -\,\rho c_p k\, u_*\,(\theta_h - \theta_l)/\triangle\theta_n \quad (10)$$

$$E = -\,\rho k u_*\,(q_h - q_l)/\triangle q_n \quad (11)$$

Substitution among Eqns. (9) and (11) for u_* provides Eqn. (12) estimating evaporation by means of two level measurements ;

$$E = \frac{\rho k^2}{\triangle u_n \triangle q_n}\,(u_b - u_a)\,(q_l - q_h) \quad (12)$$

Eqn. (13) below is more convenient to use than Eqn. (12) because vapour pressures are more readily calculated than specific humidities

$$E = S\,(u_b - u_a)\,(e_l - e_h) \quad (13)$$

where,

$$S = 0.622\,\rho k^2/P\,\triangle u_n\,\triangle q_n \quad (14)$$

Considering the proper system of units; — gm/m³; P and ρ in mb; u — m/sec, taking Karman's constant, k, equals 0.4, and substituting for $\triangle q_n = \triangle\theta_n$ because humidities and temperatures are measured at the same levels $(l,\ h)$, we find :

$$S = 0.358\,\rho/P\,\triangle u_n\,\triangle\theta_n\ (\mathrm{mm/h})\,(\mathrm{m/sec})^{-1}\,\mathrm{mb}^{-1} \quad \mathbf{(15)}$$

When neutral conditions are considered, the coefficient S could be expressed as follows;

$$S = 0.358\,\rho/P \ln\left(\frac{b}{a}\right)\ \ln\left(\frac{h}{l}\right)(\mathrm{mm/h})\,(\mathrm{m/s})^{-1}\,\mathrm{mb}^{-1} \quad (16)$$

In order to evaluate, S, Eqns. (4) and (5) have been applied to estimate the values of $\triangle u_n$ and $\triangle\theta_n$ for different assumed values of L, at the given levels of measurement a, b, h and l. Therefore, the corresponding values of $\triangle u$ and $\triangle\theta$ could be estimated from Eqn. (17) which is derived after elementary substitutions for u_* and H in Eqns. (9) and (10), and the scale length expression;

$$L\,(\triangle u_n)^2/(\triangle\theta_n) = \frac{\theta}{g}\,(\triangle u)^2/\triangle\theta \quad (17)$$

The process is then reversed to determine the value of L, and consequently, $\triangle u_n$, and, $\triangle\theta_n$, corresponding to the measured, $\triangle u$, and $\triangle\theta$. Substitution in Eqn. (15) for, ,$\triangle u_n$, and $\triangle\theta_n$, yields the required value of the coefficient ,S. For example the values of ,S, are given in relation to differences of wind speed $\triangle u$ and, $\triangle\theta$, measured at the levels $a = l = 0.5$ m., and $b = h = 2$ m, while the values of ρ and P have been assumed 1200 g/m³ and 1000 mb respectively, Table .

3. Estimation of evaporation rates from water surfaces

Deration of a working equation estimating evaporationrates from lakes, reservoirs and other water surfacs could be of large practical importance especial when the different parameters are represented. It is cnvenient to consider the roughness of the water surfac,Z_0, very close to the water surface itself because the vae of ,Z_0, ranges between 0.003 cm (Mills 1974) and 0 cm (Webb 1960). Therefore, the temperature measued at the water surface, Ts, could be accepted as rohness level temperature.

TABLE 1

Estimated values of the coefficient $S=0.358\ \rho/P\triangle U_n\ \triangle\theta_n$(mm/h) (m/s)$^{-1}$ mb^{-1}, in relation to differences of wind velocities and temperatures measured at the levels $b=h=2$m and $a=l=0.5$ m for $\rho=1200$ g/m^3, $P=1000$ mb (Tabulated values are $S\times10^3$)

$\triangle\theta$°C	Unstable					Neutral	Stable				
Δu m/s	-2.00	-1.50	-1.00	-0.50	-0.25	0.00	$+0.25$	$+0.50$	$+1.00$	$+1.50$	$+2.00$
0.1	1038	934	741	537	606	360	173	58	46	33	20
0.2	864	751	678	512	443	360	279	247	204	190	63
0.3	624	557	512	437	406	360	317	288	253	230	215
0.4	532	480	446	407	384	360	336	314	279	256	243
0.5	467	437	416	393	371	360	346	329	303	281	266
0.6	432	411	393	378	364	360	347	332	317	301	286
0.7	416	397	384	368	362	360	349	339	329	314	300
0.8	407	384	380	364	360	360	351	342	337	323	311
0.9	395	380	376	362	360	360	353	346	339	332	319
1.0	384	378	368	360	360	360	355	347	342	338	326
1.1	380	376	364	360	360	360	356	350	343	340	331
1.2	376	373	360	360	360	360	358	351	345	341	332
1.3	372	370	360	360	360	360	360	353	347	342	336
1.4	368	369	360	360	360	360	360	356	349	343	337
1.5	366	368	360	360	360	360	360	358	352	345	340
1.6	365	367	360	360	360	360	360	360	353	348	344
1.7	364	365	360	360	360	360	360	360	356	350	346
1.8	363	361	360	360	360	360	360	360	358	352	347
1.9	362	360	360	360	360	360	360	360	360	354	349
2.0	361	360	360	360	360	360	360	360	360	358	352
2.0	360	360	360	360	360	360	360	360	360	360	360

In order to reduce the number of apparatus used to measured wind velocity, air temperature and humidity we consider that the roughness level is the lower level of measurements.

The following equalities could be given :

$a = l = Z_o$

$u_a = u_{(Z_0)} = 0$

$T_e = T_s$

$e_l = e_s$ — the saturation water vapour pressure at T_s.

Considering the above stated, the required equation could be derived by substitution in Eqn. (15), we find;

$$E = S_w\, u_b\,(e_s - e_h) \quad \text{mm/h} \tag{18}$$

where,

$$S_w = 0.358\ \rho/P\triangle u_n\ \triangle\theta_n\ \text{(mm/h) (m/s)}^{-1}\ \text{mb}^{-1},$$

$$\triangle u_n = u_n(b/L) - u_n(Z_0/L) \qquad \text{and}$$

$$\triangle\theta_n = \theta_n(h/L) - \theta_n(Z_0/L)$$

For neutral stratification, Eqn. (18) could be written as follows;

$$E = S_n\, u_b(e_s - e_h) \qquad \text{mm/h} \tag{19}$$

where,

$$S_n = 0.358\ \rho/P\ \ln\left(\frac{b}{Z_0}\right)\ln\left(\frac{h}{Z_0}\right)\ \text{mm/h}\ \text{(m/s)}^{-1}\text{mb}^{-1}$$

TABLE 2

Estimated* values of coefficient, S_w, (mm/h) $(m/s)^{-1}$ mb^{-1} for roughness parameter, $Z_0 = 0.05$ cm, $\rho = 1200$ g/m^3, $P = 1000$ mb, in relation to wind velocities measused at $b = 10$ m and temperature differences $\triangle T = T_2 - Ts$(°C)

T°C / u_{10} m/s	Unstable							Neutral	Stable					
	−3	−2.5	−2	−1.5	−1.0	−0.5	−0.25	0	+0.25	+0.5	+1.0	+1.5	+2	+2.5
0.50	1250	1200	1150	1100	1050	750	650	523	280	140	80	60	40	20
1.00	720	705	690	670	650	590	560	523	480	420	270	180	150	120
2.00	630	610	590	575	555	540	530	523	510	500	480	455	420	380
3.00	565	560	553	542	535	530	525	523	520	518	500	485	480	470
4.00	545	540	533	530	529	527	524	523	521	520	515	510	500	480
5.00	535	532	530	528	526	524	523	523	522	520	518	516	513	510
6.00	532	530	528	527	525	523	523	523	523	521	519	517	515	511
7.00	530	528	526	525	523	523	523	523	523	523	520	518	516	513
8.00	528	526	524	523	523	523	523	523	523	523	523	521	518	517
9.00	526	525	523	523	523	523	523	523	523	523	523	523	520	518
9.00	524	523	523	523	523	523	523	523	523	523	523	523	523	520
10.00	523	523	523	523	523	523	523	523	523	523	523	523	523	523

*Tabulated values are $S \times 10^5$

For example, the values of S_w have been estimated for $Z_0 = 0.05$ cm, wind velocity measured at $b = 10$ m, and air temperature measured at $h = 2$ m. Results are shown in Table 2.

4. Estimation of evaporation rates

An attempt to test the accuracy and validity of the present method for practical applications has been performed. Lake Albert values of wind velocity measured at 10 m above water surface, air temperature and humidity measured at 2 m, and water temperature, have been employed to estimate the rate of evaporation adopting Eqn. (18). The daily mean values of the above stated elements have been found from the hourly observations during the four periods mentioned in the published Ph. D. thesis by Cheng Wan—li (1976). The results are illustrated in Table 3 together with the corresponding results provided from different methods.

It is shown that the results provided from the bulk aerodynamic method are very close to the results achieved from the present method. This is convenient since the bulk aerodynamic method could be special case of the method described in this paper. Compare, for instance Eqn. (19) to the form of Dalton relationship used for investigations at Lake Albert;

$$E = C\,u_{10}\,(e_s - e_2) \ \ \text{mm/3h} \qquad (20)$$

The tabulated values of the coefficient ,S, for neutral stratification gives $S_n = 1.57 \times 10^{-3}$, (cm/3h), while lake Albert value of $C = 1.56 \times 10^{-3}$ (cm/3h) defined by independent reliable measurements of evaporation applying energy budget approach.

TABLE 3

Evaporation results (cm/day) from variable coefficient method, E, in relation to results found by Cheng Wan—Li (1976)

Date	$S \times 10^3$	E	E_{ba}	E_{eb}	$\bar{E}$	$E/\bar{E}\%$	$E_{ba}/\bar{E}\%$	$E_{eb}/\bar{E}\%$
1 Jul	1.59	0.10	0.10	—	0.13	77	77	—
2	1.64	0.17	0.16	0.06	0.15	94	107	40
3	1.57	0.29	0.29	0.21	0.10	290	290	210
4	1.55	0.05	0.05	0.23	0.06	83	83	380
5	1.55	0.11	0.11	0.06	0.50	22	22	12
6	1.50	0.17	0.18	0.01	0.12	142	150	83
4 Oct	1.53	0.28	0.29	0.13	0.31	90	94	42
5	1.52	0.36	0.37	0.27	0.11	327	336	245
6	1.41	0.15	0.17	0.38	0.21	71	81	181
7	1.43	0.11	0.12	0.17	0.21	52	57	81
8	1.50	0.36	0.37	0.37	0.42	86	88	81
21 Jan	1.58	0.42	0.41	0.57	0.75	56	55	76
22	1.59	0.66	0.64	0.39	0.73	90	88	53
23	1.61	0.67	0.65	0.50	0.71	94	92	70
24	1.60	0.50	0.49	0.47	0.56	89	88	84
25	1.57	0.77	0.77	0.51	0.67	115	115	76
1 Apr	1.58	0.42	0.41	0.42	0.31	135	132	135
2	1.58	0.24	0.24	0.19	0.33	73	73	58
3	1.50	0.38	0.39	0.86	0.37	103	105	232
4	1.41	0.09	0.10	—	0.35	26	29	–
5	1.55	0.30	0.30	0.30	0.37	81	81	81
6	1.57	0.55	0.55	0.42	0.42	131	131	100

S– cm/3h (m/sec)$^{-1}$ mb^{-1} , E_{ba}– bulk aerodynamic, E_{eb}–energy budget, $\bar{E}$– mean value of evaporation measurements by class A pans at Dumandang and Pelican Point.

The form of Dalton relationship determined by comparison with water budget measurements at lake. Hafner (MarCiano and Harbeck 1954), and supported by investigations at lake Mead (Harbeck *et al.* 1958) and at lake Eucumbene (Webb 1960) is another special case of Eqn. (19);

$$E = C\, u_a\, (e_s - e_a) \quad \text{cm/3h} \tag{21}$$

The numerical coefficient C has the values 1.39×10^{-3} and 1.62×10^{-3} (cm/3h) (m/s)$^{-1}$mb^{-1}, corresponding to $a = 4$ m and $a = 2$ m respectively, while substitution for $b = h = 4$ m and $b = h = 2$ m, and the chosen value of roughness parameter $Z_o = 0.05$ cm, in Eqn. (19) gives the values $S = 1.60$ and 1.89. The overestimation could be due to the utilization of certain prefixed value of roughness parameter.

5. Conclusion

A method for estimation of evaporation rates over land or water surface in different atmospheric stratifications has been proposed. The cencerned flux is expressed in terms of more easily measured quantities.

The coefficient ,S, of the present method includes a combination of wind speed; atmospheric stability; roughness length; barometric pressure; air density; and the heights of measurements. It could be claimed that the aerodynamic methods of prefixed coefficients are special cases of the present method. Moreover, estimations by the present method seems realistic since it accounts for most of the parameters affecting evaporation processes.

References

Cheng Wan- Li, 1976, "Evaporation from lake Albert during 1974-1975", Research Report No. 20, Flinders Institute for Atmospheric and Marine Sciences, Flinders Univ. of South Australia.

Harbeck, G.E. Jr., Kohler, M.A., Koberg and others, 1958, "Water-Loss Investigations-lake Mead studies", U.S. Geol. Survey Prof. Paper 298, 100 p.

Marciano, J.J. and Harbeck, G.E. Jr., 1954, "Mass-transfer studies, in Water-loss Investigations-lake Hafner studies," U.S. Geol. Survey prof. Paper 269, 46-70.

Mills, G.A., 1974, "Atmospheric Momentum and Energy Transfer over lake Albert", Research Report No. 15, Flinders Institute for Atmospheric and Marine sciences, Flinders University of South Australia.

Monin, A. S. and Obukhov, A. M., 1954, "Basic Lows of Turbulent Mixing in the surface layer of the Atmosphere, *Tr. Geofiz. Inst. Akad. Nauk USSR*, 24 (151).

Shahawi, M.A., 1983, "A non-Linear surface-layer Model", *Mausam*, **34**, 4.

Webb, E.K. 1960a, "An Investigation of Evaporation from Lake Eucumbene", C.S.I.R.O., Division of Met. Physics, Technical Paper No. 10, 75 pp.

Webb, E.K., 1960 b, "On Estimating Evaporation with Fluctuaing Bown Ratio", *J. Geophys. Res.*, **65**, 3415-3417.

Webb, E.K., 1965, "Aerial Microclimate", Agri. Met., Met. Mono; American Met. Soc., 6, 28, pp. 27-58, July 1965.
