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An analysis of infiltration in initial gradient soils

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ABSTRACT. Flow through loamy and clay soils starts only when the hydraulic gradients exceeds a certain value called initial gradient, thus giving rise to non-Darcian flow. An analytical solution of the problem of infiltration into homogeneous soil with initial gradient has been presented, which shall be quite useful in the area of irrigation and drainage. The effect of initial gradient on the advance of wetting front, the infiltration rate and accumulated depth of infiltration has been clearly brought out, and it is found that the non-recognition of initial gradient in soils leads to the overestimation of these values.

1. Introduction

In number of disciplines, and particularly in the field of irrigation, drainage and watershed studies, knowledge of how the infiltration and accumulated depth of infiltration varies with time is important. In case of watershed studies it is desirable to know the variation of infiltration with time in order to predict rainfall-runoff relations. The knowledge of relationship between the accumulated depth of infiltration and time is of great consequence in determining how much variation in inundation time can be allowed within a certain field so that irrigation efficiency can be maintained above a prescribed value.

In the practical situations arising in the above mentioned areas, one is often interested in the flow through loamy and clayey soils. It is now well recognized that for many clayey soils and to some extent sandy and silty soils, the flow starts only when the hydraulic gradient exceeds a certain value called the initial or threshold gradient; and the velocity gradient response for the such soils can be represented by the following equation :

$$v = K\,(i - i_0), \qquad i > i_0 \qquad (1)$$

where v is the macroscopic seepage velocity; K is the permeability coefficient; i is the hydraulic gradient and i_0 is the initial gradient. Kovacs (1967) reports the existence of initial gradient in case of flow of non-Newtonian fluids such as bentonite suspensions, oil water emulsions etc through porous media. In experiments conducted by Li (1963) with Houston clay (clay content of 72 per cent and void ratio of 0.85), he found the initial gradient around 50. In case of flow of water through clayey soils Pollubrinova-Kochina (1962) found that the magnitude of initial gradients may vary from 15 to 20. For dense clays Scott (1963) quotes the values of initial gradient to lie between 20 and 30. In experiments conducted by Roza (*vide* Kezdi 1974) measured values of initial gradient ranged from 0.2 to 0.5 for silts and 12 to 18 for clays. Karadi and Nagy (1961) and Kondon (1967) have reported that the initial gradient increases with the decrease in void ratio or water content for fully saturated soils. The experimental data showing the variation of initial gradient with water content is shown in Table 1.

Initial gradient also depends upon on such factors as the mineral composition and absorption complex of the soil, temperature etc.

Predominance of surface forces over the gravity forces in the case of clayey and other fine grained soils is said to be the cause of existence of initial gradient; these forces being strong enough to counter balance a certain portion of the applied hydraulic gradient called the initial gradient. It is known that higher the specific surface, the higher shall be the surface forces. As such, it is natural to expect increase in the magnitude of initial gradient with the decrease in the grain size and porosity.

The consequences of existence of initial gradient are of great interest in several areas such as suitable spacing of trenches or tile drains ; soil water movement to plant roots, and consolidation of clayey soils, infiltration of water into the soil etc etc. Recognising the existence of initial gradient many researchers in the last few years have investigated various physical problems of practical interest. Polubranova-Kochina (1969), Valsangkar and Subramanya (1972) and Arumagan (1975) studied the effect of initial gradient on various physical probems of interest in the area

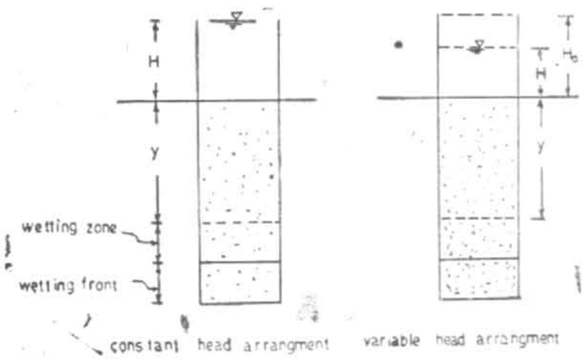

Fig. 1. Definition sketch for infiltration

TABLE 1

Variation of initial gradient of clayey soil with water content (*After* Karadj and Nagy 1961)

Water content (Per cent)	Initial or threshold gradient I
29·1	22·8
32·2	12·6
34·5	7·3
39·3	3·1
42·1	0

of irrigation and drainage. Florin (1951), Roza and Kotov (1958), Girault (1960), Elnaggar *et al.* (1971) and Parlange (1973) investigated the role of initial gradient on the one-dimensional consolidation of clayey soils by vertical drainage.

The problem of infiltration of water into a homogeneous soil medium for a Darcian flow has been solved by many researchers (Green and Ampt 1911, Fok and Bishop 1965, Fok and Hansen 1966, Phillips 1969 etc). The same problem for the non-Darcy flow $[v=K(i-i_0)]$ has been analytically investigated in this paper.

2. Infiltration in soil with initial gradient

Consider a homogeneous and unsaturated soil medium (*see* Fig. 1). The medium is composed of a soil which exhibits the property of initial gradient and whose structure does not change after wetting. Further, it is assumed that hydraulic conductivity and moisture content remains constant in the wetted zone and constant (negative) pressure at the advancing front.

From the principle of continuity of flow one gets :

$$Q = n\ s\ A\ \frac{dy}{dt} \tag{2}$$

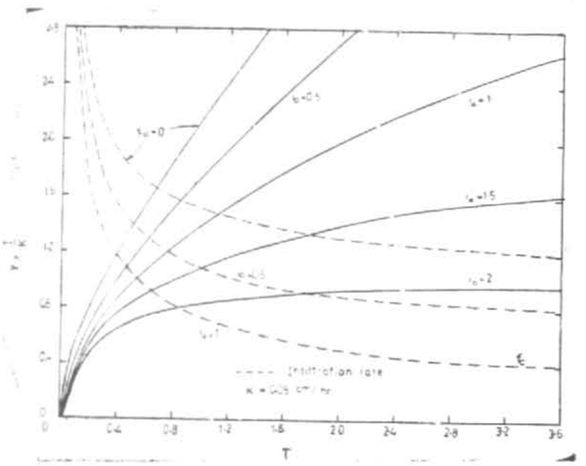

Fig. 2. Relation of movement of wetting front and infiltration rate with time

and the flow equation when the soil exhibits the property of initial gradient is given by the following equation :

$$v = K\,(i-i_0)\ ; \qquad i > i_0 \tag{3}$$

or

$$Q = A\,K\,(i-i_0)\ ; \qquad i > i_0 \tag{4}$$

Here Q = the rate of flow ; n = the porosity of the soil profile, s = net increment of degree of saturation, *i.e.*, $s = S_2 - S_1$, in which S_2 = the degree of saturation after wetting and S_1 = the degree of saturation before infiltration; A= the gross cross-sectional area through which flow occurs, y = the depth of wetting front; i = the gradient under which the flow occures ; *i.e.*,

$$i = \frac{H_0 + y + h_c}{y} \tag{5}$$

in which H_0 = the capillary potential head at the wetting front.

2.1. *Infiltration under constant head*

Assuming the infiltration head to be constant at $H=H_0$, and equating Eqn. 3 (after substituting the value of i from Eqn. 5) with Eqn. (2) one obtains :

$$\frac{dy}{dt} = \frac{K}{n\,s}\left[\frac{H_0 + h_c}{y} + (1 - i_0)\right] \tag{6}$$

Eqn. (6) in non-dimensional form can be written as :

$$\frac{dy}{d\tau} = \frac{1}{Y} - (i_0 - 1) \tag{7}$$

in which

$$Y = \frac{y}{H_0 + h_c} \tag{8a}$$

$$\tau = \frac{t\,K}{n\,s\,(H_0 + h_c)} \tag{8b}$$

Integration of Eqn. (7) with the boundary condition at $Y = 0$, $\tau = 0$ yields the following expression :

$$\tau = \frac{Y(1-i_0) + \ln\left(\frac{1}{1+(1-i_0)Y}\right)}{(1-i_0)^2} \quad (9)$$

Eqn. (9) expresses the relation between non-dimensional depth, Y, and non-dimensional time, τ, when H_0+h_c ; K, n and s are assumed constant. When $i = 0$, *i.e.*, when the velocity gradient response is Darcian Eqn. (9) reduces to Eqn. (10)

$$\tau = Y - \ln(1+Y) \quad (10)$$

which is the same equation as obtained by Phillips (1969). When $i_0 = 1$, Eqn. (9) reduces to the indeterminate form of 0/0. Appliying La Hospital rule with the limit $i \to 1$, one gets :

$$\tau = \frac{Y^2}{2} \quad (11)$$

It can be seen from Eqn. (9) that when $Y(1-i_0) = -1$ or $Y(i_0-1)=1$, then τ becomes infinite, *i.e.*, time required to reach a finite depth of infiltration becomes infinite. In other words the infiltration would stop after a certain depth Y_{max} has reached and this Y_{max} is given by Eqn. (12) ;

$$Y_{max} = \frac{1}{(i_0-1)} \quad (12)$$

Looking at Eqn. (5) one observes that the value of gradient under which flow occurs shall always be greater than 1 and it shall have a value approaching one when Y approaches infinity. That means for values of $i_0<1$, there shall always be a resulting gradient (*i.e.*, $i-i_0$), or net gradient to cause flow and wetting front shall travel downwards. But for $i_0 > 1$, there can be a situation when applied gradient shall be equal to initial gradient and the flow would stop; the maximum depth to which the water shall infiltrate is given by Eqn. (12). Eqns. (9), (10) & (11) have been plotted and shown in Fig. 2 for various values of i_0.

The percentage error involved in the estimation of time in assuming the flow to be Darcian when it is non-Darcian, *i.e.*, $v=K(i-i_0)$, is shown in Fig. 3. This plot shows the variation of $(\tau_D-\tau)/\tau_D$ with i_0 for various values of Y; τ_D stands for non-dimensional time when flow is assumed Darcian, *i.e.*, when $i_0 = 0$.

2.2. *Infiltration under falling head*

Let Q be the volume of water poured instantaneously into the soil column (Fig. 1 b), then the discharge, Q_0 passing through the soil column is equal to Q/A, where A is the cross-sectional area of soil column. From the principle of continuity

$$Q_0 = H + nsy \quad (13)$$

Substituting the value of H from Eqn. (13) into Eqn. (5) and making use of Eqns. (2) and (4), the following expression is obtained :

$$\frac{dy}{dt} = \frac{K}{ns}\left(\frac{Q_0+h_c}{y} + (1-n) - i_0\right) \quad (14)$$

Non-dimensionalising Eqn. (14) one gets ;

$$\frac{dY}{d\tau} = \left(\frac{1}{Y} - (i_0 - 1)\right) \quad (15)$$

in which

$$Y = \frac{y}{Q_0+h_c} \quad (16a)$$

$$\tau = \frac{tK}{sn(Q_0+h_c)} \quad (16b)$$

and $\quad I_0 = (i_0+n) \quad (16c)$

Eqn. (15) is of the same form as Eqn. (7) whose solution is given by Eqn. (9), with the difference that Y, τ and i_0 are defined now by Eqn. (16). It is to be noted that falling head infiltration problem gets reduced to the problem of infiltration under constant head by suitable change in the composition of non-dimensional variables. As such the conclusions arrived at for the case of infiltration under constant head would hold good in this case also.

3. Accumulated infiltration related to flow time

Accumulated amount of infiltered water, d_a shall be equal to the fillable pore space as volume fraction of soil multiplied by the length of the wetted zone, y, *i.e.*,

$$d_a = nsy \quad (17)$$

Non-dimensional depth Y, therefore, in case of infiltration under constant and variable head would respectively be as :

$$Y = \frac{d_a}{ns(H_a+h_c)} \quad (18)$$

and

$$Y = \frac{d_a}{ns(Q_0+h_c)} \quad (19)$$

Substitution of these values of Y, in Eqn. (9) would lead to the relationship between accumulated depth of infiltration and time for infiltration under constant and falling head; the non-dimensional time factor, τ, being defined by Eqns. (8 b) and (16 b) respectively.

4. Infiltration rate related with flow time

The flow rate, Q through the soil medium shall be equal to the infiltration rate, I, multiplied by the cross-sectional area of flow, *i.e.*,

$$Q = I.A \quad (20)$$

Equating Eqn. (20) with Eqn. (4) and using appropriate expressions for hydraulic gradient defined in Eqn. (5), Eqns. (21) and (22) are obtained for infiltration under constant and variable head respectively as :

$$Y = \frac{K}{I + K(i_0-1)} \quad (21)$$

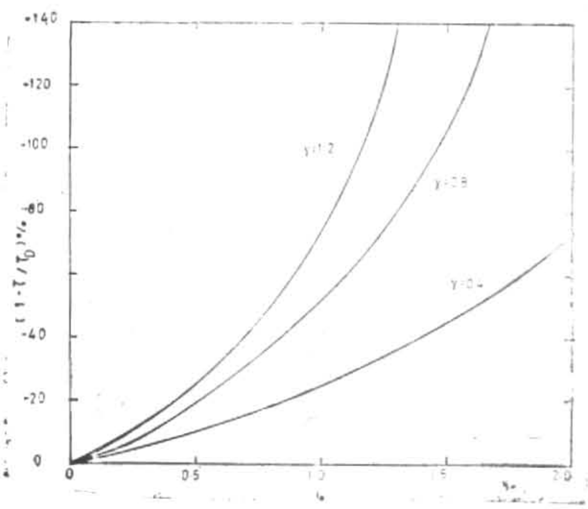

Fig. 3. Percentage error *versus* initial gradient for various values of 'Y'

$$\text{and} \qquad Y = \frac{K}{I + K\,(I_0 - 1)} \qquad (22)$$

Substitution of these values of Y in Eqn (9) would give a relation between infiltration rate and time for infiltration taking place under constant and variable head; τ being defined respectively by Eqns. (8 b) and (16b). The variation of infiltration rate with time is also shown in Fig. 2 for different values of i_0.

The variation of accumulated infiltration, d_a, with time can be obtained from Fig. 2, d_a being equal to ordinate of curve for any time τ multiplied by $n\ s\ (H_0+h_c)$ and $n\ s\ (Q_0+h_c)$ for infiltration taking place under constant and variable head respectively.

5. Conclusions

From the present investigation, the following conclusions be drawn :

(1) The presence of initial gradient causes the movement of wetting front at a slower rate, and this is true for both constant and variable head (*see* Fig. 2). In other words, the time for the infiltration to reach a certain level is always under-estimated for a Darcian flow within initial gradient, and the error increases with increasing depth of infiltration and magnitude of initial gradient (Fig. 3). Typically for $i_0 = 0.5$, the error involved in the estimation of time is 26 per cent, 53 per cent and 75 per cent at $Y = 0.4$, 0.8 and 1.2 respectively.

For values of i_0 greater than one, the infiltration would stop at a fixed depth depending upon the exact value of initial gradient (Eqn. 9).

(2) The infiltration rate would be slower with the presence of initial gradient (*see* Fig. 2). Typically, for $\tau = 0.4$ and $K = 0.05$ cm/hr the infiltration rate (cm/hr) for $i_0 = 1$, 0.5 and 0; would be 0.058, 0.078 and 0.095 respectively. With the increase in initial gradient, the magnitude of constant rate of infiltration, f_c, reduces.

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