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Auto-regressive method of detection of weak signals in noise for monitoring precursors

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ABSTRACT. Among the several premonitory indications for an impending earthquake, the observation of precursory pulses, analogous to acoustic emission before a fracture, affords a direct method of studying the seismic stability of a given area. Since the area of observation cannot be expected to be a quiet one, the technique employed to detect precursory pulses embedded in a background of stationary noise must be capable of updating the information of "noise" and comparing it with that of incoming "signal". It is the aim of this paper to present a method of detection of weak signals embedded in stationary noisy background by means of array of sensors using Akaike criterion of final prediction error. Typical examples of real and artificial series are discussed to indicate the capability of the method.

1. Introduction

One of the several methods of prediction of an impending earthquakes is to monitor the forerunners (precursors as they are sometimes called). This method is analogous to the monitoring of acoustic emissions of stressed material before its failure (Sobolev 1975, Mjachkin *et al.* 1975, Pollock 1974). It has been found useful in the prediction of rockbursts (Brady 1976).

The crucial step in this method is to detect weak signals in a background of noise and locate their source. The usual procedure for enhancement of signal/noise ratio by array processing using the coherency properties of signal embedded in incoherent noise is not applicable in all practical situations (Birtill and Whiteway 1965). Therefore, one should search for a more sophisticated method to detect incoherent signals in a background of stationary noise. Unless this is done satisfactorily, the problem of detection of weak precursors remains unsolved and one is constrained to detect only big signals with the added risk of losing the chance of prediction of the main event sufficiently in advance to take protective measures.

This report gives some results of typical case studies of detecting weak signals in a background of noise. This has been done in two ways. First, by analysing noise data recorded at Gauribidanur array in which small transient signals were superimposed and secondly by analysing noise samples where weak signals are expected to exist but not detected by eyeball search or by the usual phase summation method.

2. Auto-regression method

The auto-regression method of detection of weak signals is based on an auto-regressive model of stationary noise which allows prediction of noise one "step" ahead into the future (Fryer *et al.* 1975). The "step" is the time step, equal to the sampling interval of the time series. If the noise is stationary, the predicted time series can be used to obtain the "error series" or "innovation" in the time series by subtracting the predicted series from the actual time series. The "error" is small if there is *no* signal and if the "error" is large, the noise is not stationary and hence, by definition, a signal is present. If the same operation is carried out on several sensors of an array, one can eliminate spurious signals and detect genuine ones by suitable methods as described below.

The auto-regressive model of noise sampled at discrete steps of time is developed in two stages. In the first stage, the given noise series is sampled at a suitable rate (usually 20 samples/second), depending on the expected periods in the noise. The auto-regression coefficients are determined by maximum entropy method (Burg 1972, Ulrych and Clayton 1976) for different orders. In the second stage, the final prediction error as defined by Akaike (1971) is computed for each order. The order which gives a minimum of the error is chosen as the correct order for auto-regression representation of the noise. We thus arrive at the optimum M-order auto-regression representation of the noise, x_i in the form :

$$x_i = \alpha_1 x_{i-1} + \alpha_2 x_{i-2} \, .. + \alpha_M x_{i-M} + \epsilon_i$$

where x_i is the value of x at time t_i and $\alpha_1, \alpha_2 .. \alpha_M$ are the auto-regression coefficients that will give minimum of final prediction error and $\triangle t$ is the sampling interval and ϵ_i is the error at time t_i, being the difference between actual and predicted values in time series. The coefficients $\alpha_1, \alpha_2, ..\, \alpha_M$ are determined for a suitably chosen length of noise series, usually of duration 10-15 seconds.

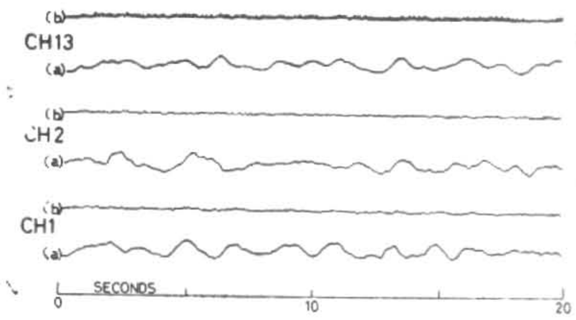

Fig. 1. Auto-regression of noise data corresponding to three channels 1, 2 and 13 respectively, (a) Actual noise data, (b) Residual error, on same scale. Auto-regression coefficients caculated from first 10 sec of data.

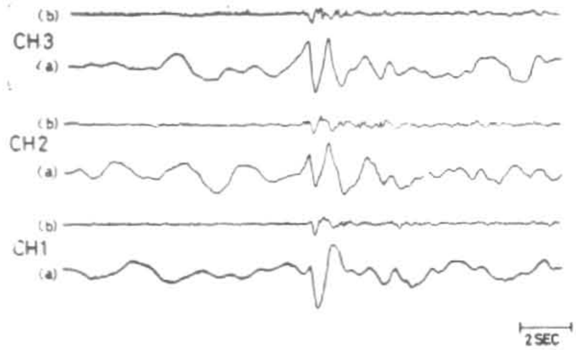

Fig. 2. Auto-regressive model for an event from Lake Baikal region (26 July 1977). (a) Actual data, (b) Residual error. A.R. coefficients calculated from first 10 seconds of data.

As stated before, if the noise is stationary, ϵ_i will be randomly distributed with a constant standard deviation. The prescription for detection of signal is as follows :

Compare the predicted error in the future time series with a fixed threshold which can be twice the standard deviation of the "error" series (calculated over a length equal to that used for finding the A.R. coefficients), and if the predicted error at any future time step exceeds the threshold, generate a spike at that time step. Thus, in all regions where the pre-set threshold is exceeded, spikes will be generated. Otherwise set it to zero. Thus one obtains a "BINARY" time series [(c)'s in Fig. 3, 4, 5 and 6]. The discrimination of genuine signal from spurious ones is made then by AND and ADD operation carried out on the binary series obtained from the time series corresponding to all the working sensors of the array after applying proper time delay. In the ADD series, the genuine signal will then appear as a spike or closely spaced spikes in a relatively noise free back ground [(e)'s in Figs. 3-6] while the AND series will be another binary series, in which non-zero values will confirm the existence of signals.

3. Case studies

We present some of the results obtained from the Gauribidanur array data, without going into the details of numerical computation. This array, established in 1965, is in continuous operation with the TDC-12 online system added to it in 1972. It has 20 short period seismometers with a gain of 180,000 at 1 sec. The outputs of all the 20 seismometers are recorded on a magnetic tape along with accurate time mark. The experimental details of this array are described by Singh *et al.* (1969).

The analogue record of noise on the magnetic tape is digitised at the rate of 20 samples/sec for

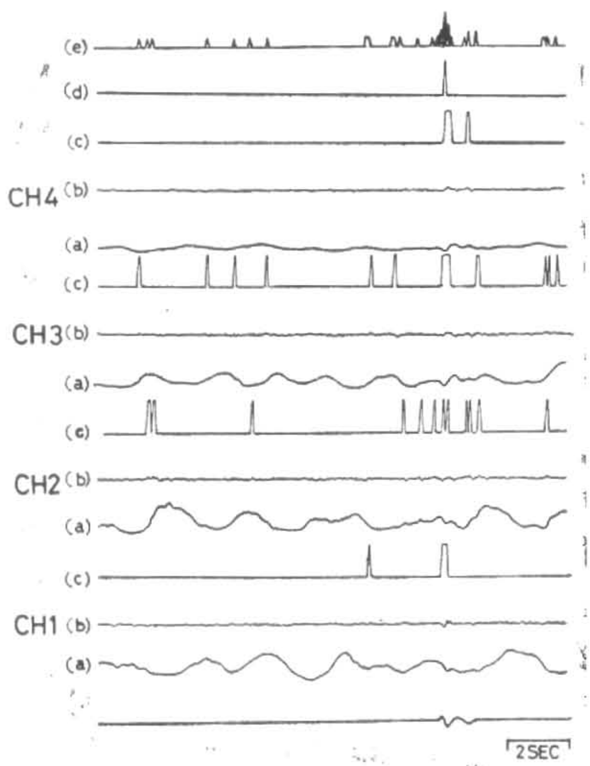

Fig. 3. Multichannel detection of weak synthesized signal superimposed on actual noise with zero lag. Case when signal period is less than that of the predominant noise. The bottom curve represents synthetic signal.

(a) Actual noise data plus signal, (b) Residual error,

(c) The trace is zero except where error values are more than 2 s.d. (s.d. calculated from first 10 seconds of residual error), (d) Logical ANDing of all (c)'s

(e) Normalized sum of all (c) 's, A.R. coefficients are calculated from first 10 seconds of data.

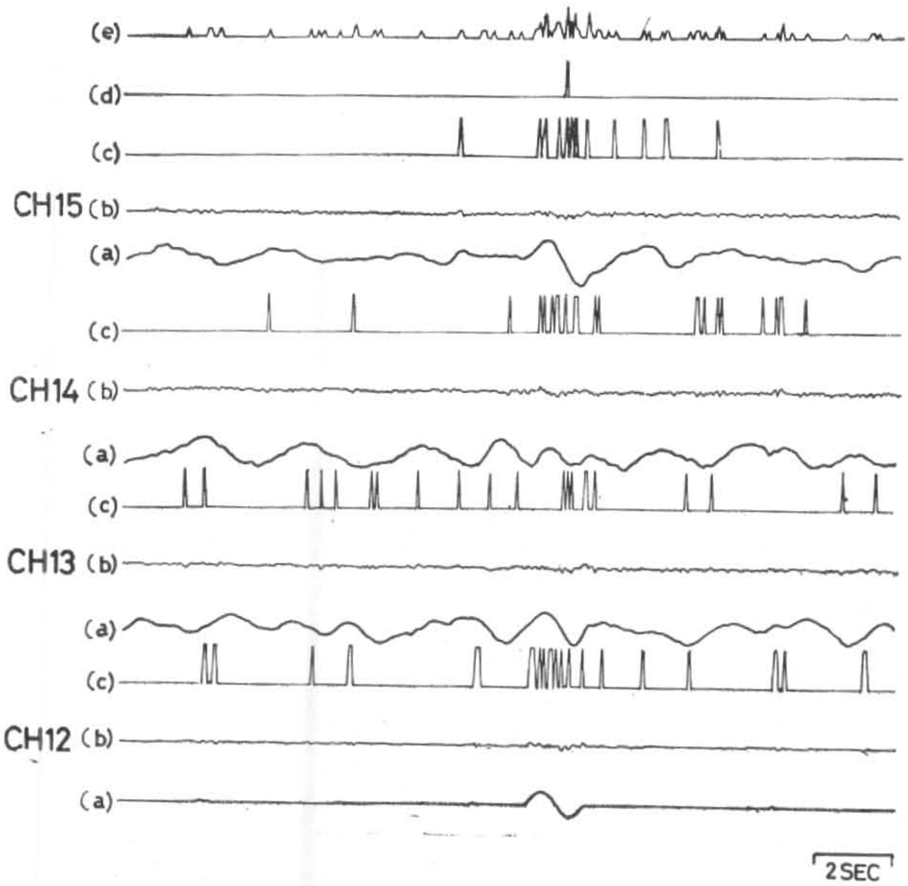

Fig. 4. Same as Fig. 3, only the synthetic signal period (lowest trace) is comparable to the predominant noise period.

this analysis. Fig. 1 shows a typical record of short period noise. The first half of the noise record is used to obtain optimum auto-regression coefficients. The whole series is then 'predicted' in steps of $\triangle t$, using these coefficients. The "error" defined as the difference between the predicted and the observed value is shown along with the observed time series, and is plotted in the same scale. We see that the "error" amplitude is small compared to the noise amplitude in the whole time window. Fig. 2 deals with the analysis of the signal recorded at Gauribidanur (GBA) from an event in the Lake Baikal region. Though the signal is clearly seen in the actual record, there is an improvement in the S/N ratio of about 1.5 in the "error" series.

It is not at all surprising that this method detects a signal amplitude which is big compared to that of noise. The interesting case is the one where signal amplitude is small compared to noise. In the following two examples the noise recorded by a seismometer is modified by adding a transient signal for the purpose of illustrating the effectiveness of auto-regression method. In Fig. 3, the combined noise *plus* signal series does not show any discernible indication of the signal in most of the time series corresponding to different channels of the array. However, by applying the auto-regression method, followed by the AND and ADD operation on different channels, single spike in the AND series is obtained where the signal is present, while in ADD series, the signal is prominently displayed. Evidently signal "period" is less than that of the predominant noise period in this example, thereby one can argue that the signal detection could be achieved by filtering the noise suitably. However, the efficiency of he auto-regression method lies in picking up signals whose periods and amplitudes are comparable to those of noise as illustrated in the Fig. 4. Here one cannot see any indication of the signal, but by proceeding on similar lines, the existence of a weak signal is revealed in the 'ADD' series, and is doubly confirmed by the presence of a single spike in the 'AND' series.

Fig. 5 illustrates the results of the analysis of a weak *PcP* signal of a shallow focus earthquake from Honshu, Japan. Some of the channels do suggest a signal while others do not. The conventional phase-summation gives no better indication of the signal than the individual channels. The auto-regression method shows up signal at the

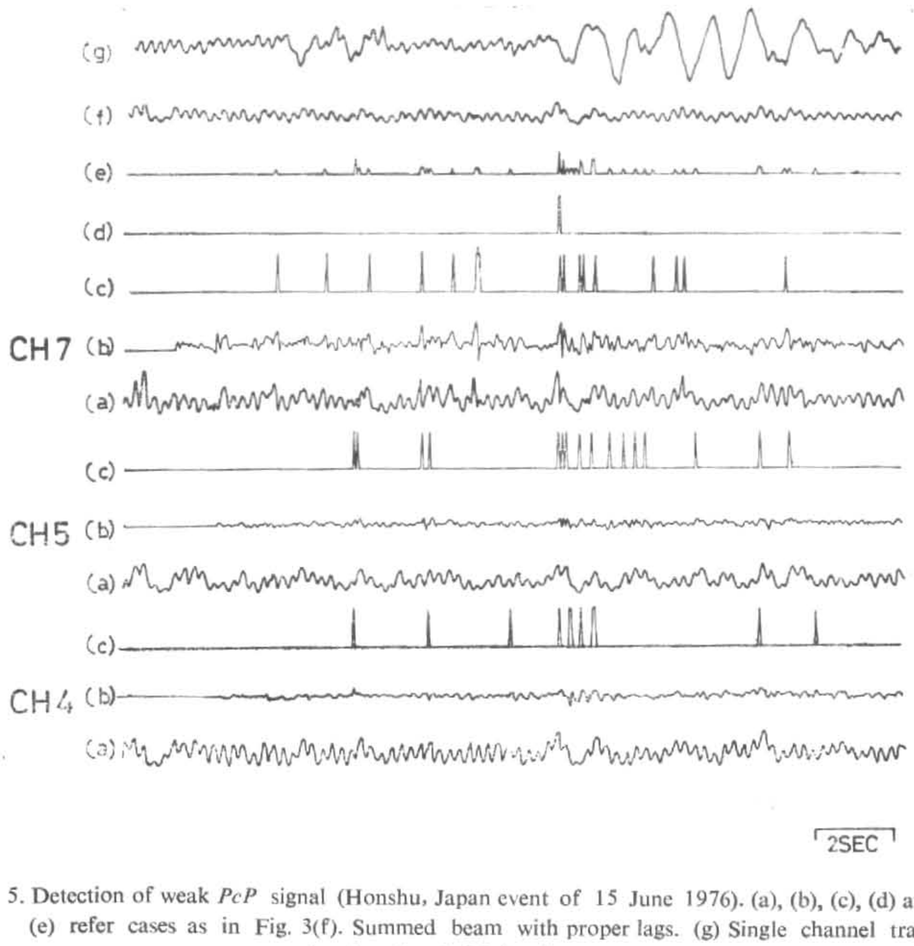

Fig. 5. Detection of weak *PcP* signal (Honshu, Japan event of 15 June 1976). (a), (b), (c), (d) and (e) refer cases as in Fig. 3(f). Summed beam with proper lags. (g) Single channel trace showing *P* and *PP* signals.

expected time as can be seen in this figure (traces d and e). It must be emphasised that this prescription does not need *a priori* knowledge of the shape of the signal, in contrast to the usual Wiener filter where the shape of the expected signal is needed to construct a suitable filter to remove the noise.

Fig. 6 shows another case of an actual time series containing a signal from Nevada test site. The seismogram does not show clearly the presence of a signal, but by standard beam steering procedure and digital filtering, it was possible to detect the signal (Varghese and Roy 1976). The present method also succeeds in picking up the signal as can be seen in traces (d) and (e).

4. Discussion

We see from these case studies that the auto-regression method has the ability to detect weak signals in a background of noise. The output of an array of sensors monitoring an area can be connected to a computer which processes the data for locating the source of signal. Since many prediction problems are faced mainly with the task of detecting weak incoherent signals recorded by different sensors, this method is suitable, since it does not demand signal coherency from sensor to sensor. This procedure may be employed to detect premonitory signals from an impending earthquake, or acoustic emission of any structure before failure.

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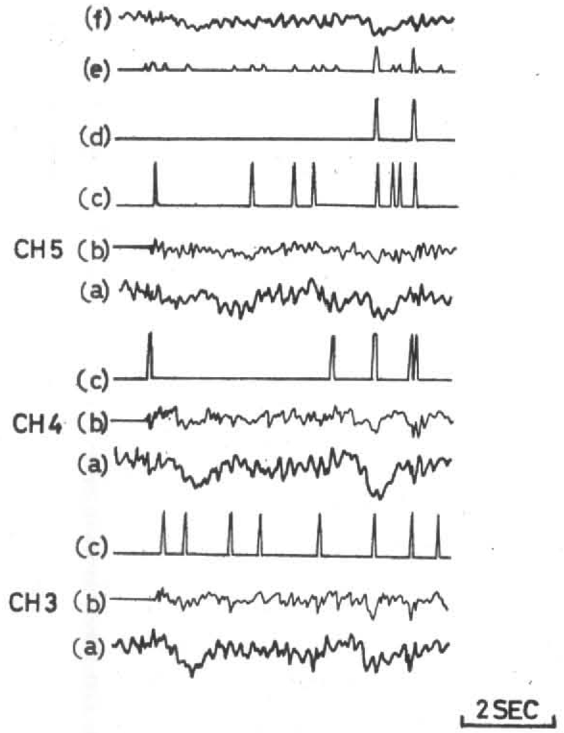

Fig. 6. Detection of weak NTS signal of 26 August 1976 by A.R. method. (a), (b), (c), (d), (e) and (f) have the same meaning as in Fig. 5. It is seen that (d) and (e) shows the presence of the signal.

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