Mausam, (1979), 30, 2 & 3, 347-352

558.24 : 550.341.5

On stress accumulation near a finite rectangular fault

BARUN PROSAD PAL, SANJAY SEN*
and
ARABINDA MUKHOPADHYAY**

Geological Survey of India, Calcutta

ABSTRACT. The problem of stress accumulation near a locked vertical rectangular strike-slip fault is considered. The fault is taken to be situated in a visco-elastic half-space. It is assumed that tectonic forces maintain a steady shear stress far away from the fault. Exact solutions are obtained for the displacements and stresses in the system and it is shown that, in the absence of fault-slip, there would be a steady accumulation of shear stress near the fault., which would lead to a sudden slip on the fault, resulting in an earthquake, under suitable conditions, which are determined. The exact solutions also show that the accumulation of shear stress would again take place after the fault-slip.

It is shown that, under suitable conditions, this would lead to another slip on the fault after a sufficient time. The analytical solutions can be used to compute the influence of different factors, such as, the dimensions of the fault, the effective viscosity of the lithosphere and the stress system maintained by tectonic forces, on the time required for the shear stress accumulation to reach the level necessary to cause fault slip. It is also shown that, if adequate data are available on the ground deformation on the surface near the fault, it would be possible to obtain estimates of the probable times of fault slip. This may be of considerable use in earthquake prediction and in the estimation of changes in the seismic risk near the fault with time. Finally, it is shown that it would also be possible to obtain estimates of the effective viscosity of the lithosphere from a comparison of theoretically calculated results and suitable observational data, if available.

1. Introduction

In studying the problem of earthquake prediction it is important to note that, a fault-slip generating an earthquake would normally be preceded by the accumulation of stress near the fault over a considerable period of time. The fault slip would be expected to occur when this stress accumulation reaches a critical level. For strike-slip faults, such as the San Andreas fault, some theoretical models have been considered recently by Turcotte and Spence (1974), Savage (1975), Spence and Turcotte (1976) and Budiansky and Amazigo (1976). In the models proposed by Turcotte and Spence (1974), Spence and Turcotte (1976) and Savage (1975), the stress accumulation is taken to be the result of steady relative motion of the parts of the lithosphere on opposite sides of the locked fault. In these models the mechanism of the relative motion is not considered and the entire system is taken to be elastic. The models require the existence of large and steadily increasing lithospheric stresses far away from the fault. Budiansky and Amazigo (1976) considered a locked fault in a visco-elastic layer and assumed that tectonic forces maintain a constant shear stress in the layer far away from the fault. It is shown that steady accumulation of shear stress would occur in such a model. However, Budiansky and Amazigo considered a two dimensional problem in which the length of the fault is taken to be very large compared to its depth, moreover the effect of fault-slip on the stress system is not considered in detail. To obtain a more realistic representation of the process of stress accumulation and the interaction between the changes of stress due to fault slip and the stress accumulation under the action of tectonic forces, we consider a three dimensional problem in which a rectangular fault of finite length is situated in a visco-elastic half-space. We suppose that tectonic forces maintain a constant shear stress in the half-space far away from the fault. We then study the process of stress-accumulation in the system both before and after fault slip. We note in this connection that the problem of relaxation of stress in a visco-elastic half-space after fault slip has been considered earlier by Rosenman and Singh (1973 a, 1973 b) and Singh and Rosenman (1974). However the interaction between fault slip and creep

*River Research Institute, P. O. Mohanpur, West Bengal.

**Department of Applied Mathematics, University of Calcutta, University College of Science 92, Acharya Prafulla Chandra Road, Calcutta.

under the action of tectotic forces, leading to stress accumulation, does not seem to have been considered earlier.

2. Formulation

We consider a vertical strike-slip fault F, situated in a visco-elastic half-space which is taken to be elastic for purely dilatational changes and of the Maxwell type for deviatoric stresses and strains. We introduce rectangular Cartesian co-ordinates (x_1, x_2, x_3) with the free surface as the plane $x_1=0$, the plane of the fault as the plane $x_2=0$ and the x_3-axis along the trace of the fault on the free surface.

The stress-strain relations for deviatoric stresses τ_{ij} and deviatoric strains γ_{ij} for the viscoelastic half-space.

$$\left(\frac{1}{\eta}+\frac{1}{G}\frac{\partial}{\partial t}\right)\tau_{ij} = \frac{\partial}{\partial t}(\gamma_{ij}) \tag{1}$$

and for dilatations,

$$\sigma_{ii} = 3k.\triangle$$

where, $k=$ Bulk modulus, $G=$Rigidity, $\eta=$Newtonian viscocity and $\triangle=$Dilatation.

We note in this connection that in the process of long term deformation of the lithosphere under the action of tectonic forces we can expect the occurrence of secondary creep for which the Nabarro-Herring creep mechanism may be relevant (Heard 1976). The material would then be expected to have a Newtonian viscocity for which the simple constitutive Eqn. (1) would be reasonable.

We consider steady deformations of the system leaving out of consideration the comparatively short period immediately after a sudden fault slip when the seismic waves generated by fault slip exist in the neighbourhood of the fault. For such deformations, both before fault slip and after the seismic waves generated by faulting have propagated far away from the fault the inertial forces would be very small and can be neglected. This approach was adopted by Budiansky and Amazigo (1976) and also by Braslau and Lieber (1968), Rosenman and Singh (1973 a, 1973b) ; Singh and Rosenman (1974), Nur and Mavko (1974), Barker (1976) and Rundle and Jackson (1977) who considered stress relaxation in visco-elastic system following fault slip. On neglecting the inertial forces the stresses satisfy the relations;

$$\sigma_{ij,i} = 0 \tag{2}$$

where, σ_{ij} are the stresses, so that

$\tau_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}.\sigma_{kk}$

In Eqns. (1) and (2) the usual conventions are followed and i, j, k correspond to 1, 2, 3.

We assume that shear stress τ_{23} at a large distance from the fault has a constant value τ_∞ maintained by tectonic forces. Then the stresses satisfy the following boundary conditions ;

$$\left.\begin{array}{l}\sigma_{11},\ \sigma_{12},\ \sigma_{13} = 0 \text{ on } x_1 = 0\\ \tau_{23} \to \tau_\infty \text{ , as } x_2 \to \infty\\ \sigma_{11},\ \sigma_{12},\ \sigma_{13} \to 0 \text{ , as } x_1 \to \infty\end{array}\right\} \tag{3}$$

3. Deformation of the system in the absence of fault-slip

In the absence of any fault slip, the displacements and stresses would be continuous throughout. We assume that there is an initial displacement and stress field $(u_i)_0$ and $(\sigma_{ij})_0$ satisfying the relations (1) , (2), (3), where $(u_i)_0$, $(\sigma_{ij})_0$ are functions of (x_1, x_2, x_3).

To obtain the displacements and stresses, we take Laplace transforms of (1), (2) and (3) with respect to t. The resulting boundary value problem is solved easily as shown in the Appendix. On determining the inverse Laplace transforms of these solutions, we obtain exact solutions for displacements and stresses. It is found that the displacement u_3 parallel to the x_3 axis and the stress τ_{23} which controls the strike-slip faulting are given by :

$$u_3 = (u_3)_0 + \frac{\tau_\infty . x_2\ t}{\eta} \tag{A1}$$

and

$$\tau_{23} = (\tau_{23})_0 . e^{-Gt/\eta} + \tau_\infty . (1 - e^{-Gt/\eta}) \tag{A2}$$

The other deviatoric stresses τ_{ij} are of the form :

$$\tau_{ij} = (\tau_{ij})_0\, e^{-Gt/\eta}$$

Hence, all the shear stresses except τ_{23} relax completely as $t \to \infty$. If $(\tau_{23})_0 < \tau_\infty$ in the neighbourhood of the fault, gradual accumulation of the shear stress τ_{23} would occur near the fault and ultimately $\tau_{23} \to \tau_\infty$. If the critical value of the shear stress τ_{23} required for fault slip is less than τ_∞ a sudden fault slip would occur, generating an earthquake, when τ_{23} reaches the critical value.

The solution (A1) also shows that in addition to the initial displacement there would be a creeping displacement under the action of the stress τ_∞ maintained by tectonic forces.

4. Deformation of the system after fault-slip

We suppose that, the critical value of τ_{23} for fault-slip is less than τ_∞, so that the fault slips after a sufficient time. We consider a dislocation model of the fault slip following Chinnery (1961, 1963, 1964), Maruyama (1966) and others, and assume that across the fault F, which is in the region $x_2=0$, $d \leqslant x_1 \leqslant D+d$; $-L \leqslant x_3 \leqslant L$, we have a discontinuity in the displacement u_3 given by

$$[u_3] = U.\ H(t) \tag{4}$$

where,

$$[u_3] = \underset{x_2 \to 0+}{Lt.}(u_3) - \underset{x_2 \to 0-}{Lt.}(u_3)$$

The other stresses and displacements are continuous throughout.

We assume that, just before fault slip, there exists a displacement field $(u_i)_p$ and a stress field $(\sigma_{ij})_p$ satisfying the relations (1), (2) and (3) where $(u_i)_p$, $(\sigma_{ij})_p$ are functions of (x_1, x_2, x_3).

To obtain the displacements and stresses, we take Laplace transforms of (1) to (4) with respect to t. The resulting boundary value problem can be solved as explained in the Appendix, on using the results given by Chinnery (1961).

The inverse Laplace transforms of these solutions can be determined, and finally we obtain solutions for the displacements and stresses valid for all x_1, x_2, x_3 and t.

The shear stress τ_{23} which controls the fault slip is found to be of the form :

$$\tau_{23}(x_1, x_2, x_3, t) = (\tau_{23})_p \exp\left(-\frac{Gt}{\eta}\right) + \tau_\infty (1 - e^{Gt/\eta})$$

$$+ A(x_1, x_2, x_3)\, U \exp(-Gt/\eta)$$

$$+ B(x_1, x_2, x_3)\, U \exp\left[-\frac{G(3\lambda + 2G)}{3\eta(\lambda + 2G)} \cdot t\right]$$

$$+ C(x_1, x_2, x_3)\, U \exp\left[-\frac{G(3\lambda + 2G)}{3\eta(\lambda - G)} \cdot t\right]$$

$$+ \{ D(x_1, x_2, x_3) + E(x_1, x_2, x_3)\, t \} \times$$

$$U \exp\left[-\frac{G(3\lambda + 2G)}{3\eta(\lambda + G)}\, t\right] \quad (5)$$

where $\lambda = K - \dfrac{2G}{3}$ and A, B, C, D, E are functions of x_1, x_2, x_3 and λ, G, η.

The expressions for A, B, C, D, E are very complicated and are not given here.

From Eqn. (5) we find that, the initial shear stress $(\tau_{23})_p$ relaxes completely. Just after fault-slip there is a drop in the stress τ_{23} given by $\tau_D = -(A+B+C+D) \times U$ which is found to be positive near the fault. However this coseismic stress-drop relaxes completely as $t \to \infty$. The term $\tau_\infty (1 - e^{-Gt/\eta})$ represents the accumulation of shear stress under the action of the stress τ_∞ maintained by tectonic forces. For large values of t, this term becomes predominant, so that the accumulation of stress under the action of tectonic forces determines that ultimate level of spehear stress near the fault. Ultimately the stress τ_{23} in the neighbourhood of the fault again approaches τ_∞. Since the critical value of τ_{23} for fault slip is taken to be less than τ_∞, the fault would slip again after a sufficient time. Apart from τ_{23} the other shear stresses are found to relax completely as $t \to \infty$. If T be the time from the first fault slip to the next, T can be calculated if we compute the change in $(\tau_{23})_F$ on the fault with time. To do this it is necessary to have estimates of τ_∞ and η. To obtain some idea about the magnitude of these quantities, we consider the stresses and displacements near the middle of a fault whose length is fairly large compared to its depth and which extends upto the surface so that $d=0$ and $L >> D$. In this case we find that close to the middle of the fault, and close to the surface $x_1=0$

$$u_3 \approx (u_3)_p + \frac{\tau_\infty \cdot x_2\, t.}{\eta} + \frac{U.\, H(t)}{2\pi}.$$

$$\left[\tan^{-1}\left(\frac{D+x_1}{x_2}\right) + \tan^{-1}\left(\frac{D-x_1}{x_2}\right)\right]$$

and

$$\tau_{23} \approx (\tau_{23})_p\, e^{-Gt/\eta} + \tau_\infty (1 - e^{-Gt/\eta}) -$$

$$-\frac{GU}{2\pi}\left[\frac{D+x_1}{x_2^2 + (D+x_1)^2} + \frac{D-x_1}{x_2^2 + (D-x_1)^2}\right] e^{-Gt/\eta}$$

Hence, on the surface $x_1=0$ and close to the middle of the fault, the rate of accumulation of shear strain would be τ_∞/η. In this connection, Savage and Burford (1973) have reported that, in the neighbourhood of a locked section of the San Andreas fault close to Ross mountain, the rate of accumulation of shear strain has been estimated to be $(0.55 \pm 0.05)\mu$ strain per year, from the results of geodetic surveys. We therefore take $\tau_\infty/\eta = 0.55\mu$ strain per year for this part of the fault. This is a rough estimate, and a more reliable estimate may be obtained on comparing the theoretically calculated ground displacement with data on the observed ground displacement at different distances from the fault over a sufficiently long time, provided such data are available. To obtain an estimate for η, we note that Cathles, III (1975) has estimated the effective viscosity of the lower lithosphere for slow deformations to be of the order of 10^{21} poise, from a comparison of theoretical models and observational data on the post-glacial uplife in Fennoscandia and Canada. Budiansky and Amazigo (1976) also obtain an estimate for η of the same order from a comparison of theoretical and observational data. We use the estimate $\eta = 10^{21}$ poise. This gives us an estimate for τ_∞. With these estimates of τ_∞ and η we compute the variations of τ_{23} with time at points on the fault. This enables us to determine the time taken by τ_{23} to reach the critical level for fault slip. To illustrate this process, we compute the changes in τ_{23}/τ_∞ with time, near the centre of

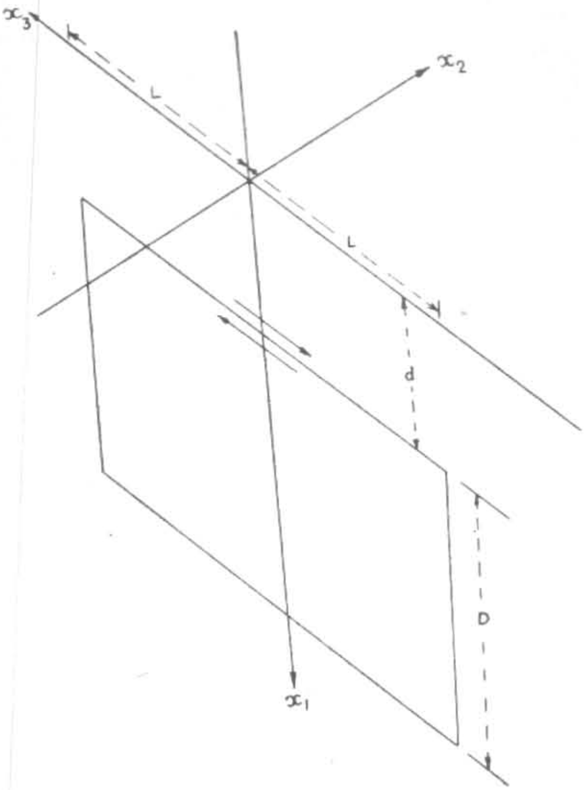

Fig. 1. Finite rectangular fault and the co-ordinate axes.

the fault $[x_1 = (d+D)/2;\ x_2 = x_3 = 0]$. Keeping the San Andreas fault in view, we take $d=0$; $D=10$ km, $L=200$ km, $U=4$ metres; $\lambda=G=3.7\times10''$ dynes/cm^2 (using estimates given by Knopoff 1958 and Aki 1967). For the points under consideration, we take $(\tau_{23})_p$ to be equal to the co-seismic stress drop τ_D on the fault. We study the variations of the ratio τ_{23}/τ_∞ which starts from zero at $t=0$ and tends to 1 as $t\to\infty$ under the assumptions we have made. The variations of this ratio with the time t, which is in years, are shown in Fig. 2. If the value of τ_{23} at the point in the critical configuration is known, we can use this curve to estimate the time of the next slip on the fault. We find, from Fig. 2, that τ_{23} reaches a value very close to τ_∞ (about $0{\cdot}96\ \tau_\infty$) after 300 years. The time T from one fault slip to the next, is found to depend significantly on the critical value τ_c which τ_{23} reaches at the time of fault slip near the centre of the fault. If $\tau_c=0{\cdot}5\tau_\infty$, it is found that the return time T is about 72 years. But if $\tau_c=0{\cdot}8\tau_\infty$, the return time is about 160 years, and if $\tau_c=0{\cdot}9\ \tau_\infty$, the return time is about 225 years. The difficulty regarding the unknown value of τ_c can be removed to some extent, if we assume that the fault slips when the post-seismic stress accumulation $\tau_a=\tau_{23}-(\tau_{23})_p+\tau_D$ becomes equal to the coseismic stress drop τ_D near the centre of the fault. With the values of the parameters, which we have taken for the model, we find, on using Fig. 2, that the return time would be about 205 years. Such estimates of the return time are expected to be useful in long-term earthquake prediction, and also in estimating the time-dependence of seismic risk, which increases with increase in time, measured from a fault slip. Moreover, the estimate which we can obtain for the magnitude of the post-seismic stress accumulation may give us some idea about the magnitude of the stress release in an earthquake. This may again be helpful in estimating the magnitude of seismic risk.

The estimates we have obtained are subject to considerable uncertainty, since the value of η has been chosen here on the basis of results obtained from other theoretical models and from significantly different considerations. The estimates would be more reliable if the value of η can be obtained by comparing theoretically calculated results for the model we are considering with relevant observational data. In this connection, we note that, we have obtained exact solutions for the ground displacement on the free surface, as a function of x_2, x_3 and t and depending on η, τ_∞ and the fault parameters. Hence, if detailed observational data are available for the ground deformations at different distance

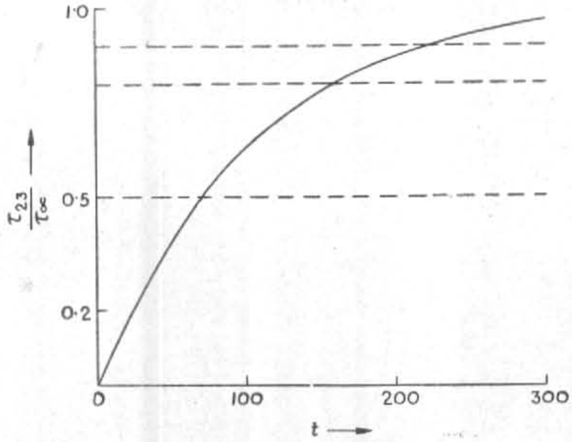

Fig. 2. Variation of shear-stress τ_{23} with time; the time t is in years.

from the fault over a sufficient length of time, it would be possible to obtain more reliable estimates of the parameters η and τ_∞, so that we can have an estimate of the effective viscosity of the lower lithosphere. This would lead to more reliable estimates of the return time T and the time-dependence of seismic risk. Reliability of the estimates may also be improved if we consider changes in the average stress accumulation over the fault as a whole instead of the stress accumulation at a particular point. Such an exercise would involve very large computations and would be worthwhile if more reliable estimates of the model parameters can be calculated from more detailed observational data.

Acknowledgements

A part of this work has done by one of the authors (Dr. Arabinda Mukhopadhyay) in the Department of Applied Mathematics and Theoretical Physics, University of Cambridge, and he thanks the Association of Commonwealth Universities, U. K. and British Council for financial assistance which enabled him to work in Cambridge as senior visitor.

One of the authors (Barun Prosad Pal) is grateful to the Director General, Geological Survey of India (G.S.I.) and the Chief Geophysicist, G.S. I. for giving him an opportunity to present the paper.

One of the authors (Sanjay Sen) thanks the Director, River Research Institute, W. B. for his kind interest and encouragement.

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Appendix

Taking Laplace transforms of (1), (2), (3) we get

$$(\bar{\tau}_{ij}) = \left(\frac{S}{\frac{S}{G}+\frac{1}{\eta}}\right)(\bar{\gamma}_j) + \frac{1}{\frac{S}{G}+\frac{1}{\eta}} \times \left\{\frac{1}{G}(\tau_{ij})_0 - (\gamma_{ij})_0\right\} \quad (1a)$$

$$\bar{\sigma}_{ii} = 3k.\,\bar{\triangle} \quad (2a)$$

$$\bar{\sigma}_{ij,\,i} = 0 \quad (3a)$$

$$\bar{\tau}_{ij} = \bar{\sigma}_{ij} - \tfrac{1}{3}\,\delta_{ij}\,\bar{\sigma}_{kk} \quad (4a)$$

$$\left.\begin{array}{l}\bar{\sigma}_{11},\ \bar{\sigma}_{12},\ \bar{\sigma}_{13},\ = 0,\ \text{on } x_1 = 0\\ \bar{\sigma}_{11},\ \bar{\sigma}_{12},\ \bar{\sigma}_{13} \to 0,\ \text{as } x_1 \to \infty\end{array}\right\} \quad (5a)$$

and

$$\bar{\tau}_{23} \to \tau_\infty/S, \text{ as } x_2 \to \infty \quad (6a)$$

It is easily seen that (la)—(6a) are satisfied by the solutions :

$$(\bar{u}_3) = \frac{(u_3)_0}{S} + \frac{\tau_\infty \cdot x_2}{\eta.\,S^2}$$

$$(\bar{u}_1) = \frac{(u_1)}{S}$$

$$(\bar{u}_2) = \frac{(u_2)_0}{S}$$

$$(\bar{\tau}_{23}) = \frac{(\tau_{23})_0}{S+G/\eta} + \tau_\infty\left(\frac{1}{S} - \frac{1}{S+G/\eta}\right)$$

and, $\bar{\tau}_{ij} = (\tau_{ij})_0 \,/\, (S + G/\eta)$ for all deviatoric stresses τ_{ij} except τ_{23}

$$(\bar{\sigma}_{ii}) = \frac{3k}{S}\cdot(\triangle)_0$$

The inverse Laplace transforms are obtained easily giving the displacements and stresses.

After fault slip we have to satisfy the additional condition $[\bar{u}_3] = U/S$ on F (7a)
In (la), $(\tau_{ij})_0$ and $(\gamma_{ij})_0$ would be replaced by $(\tau_{ij})_p$ and $(\gamma_{ij})_p$, the values just before fault slip.

We find that, the conditions (1a)—(7a) would be satisfied if,

$$(\bar{u}_i) = (\bar{u}_i)_1 + (\bar{u}_i)_2$$

$$(\bar{\tau}_{ij}) = (\bar{\tau}_{ij})_1 + (\bar{\tau}_{ji})_2$$

$$(\bar{\sigma}_{ij}) = (\bar{\sigma}_{ij})_1 + (\bar{\sigma}_{ij})_2$$

where, $(\bar{u}_i)_1$, $(\bar{\tau}_{ij})_1$ and $(\bar{\sigma}_{ii})_1$ satisfy equations (1a) to (6a) and $(\bar{u}_i)_2$, $(\bar{\tau}_{ij})_2$ and $(\bar{\sigma}_{ii})_2$ satisfy equations (2a), (3a), (4a), (5a), (7a) and the following equations which replace (1a) and (6a) :

$$(\bar{\tau}_{ij}) = \frac{S}{\frac{S}{G}+\frac{1}{\eta}}(\gamma_{ij}) \quad (1b)$$

and

$$(\bar{\tau}_{23}) \to 0, \text{ as } x_2 \to \infty \quad (6b)$$

The boundary value problem for $(\bar{u}_i)_1$, $(\bar{\tau}_{ij})_1$ and $(\bar{\sigma}_{ii})_1$ has been solved already. The boundary value problem for $(\bar{u}_i)_2$ $(\bar{\tau}_{ij})_2$, $(\bar{\sigma}_{ii})_2$ reduces to the problem of determining the elasto-static displacements and stresses due to a dislocation U on F where F is situated in an elastic half-space. This elasto-static problem has been solved by Chinnery (1961). Hence the solutions for $(\bar{u}_i)_2$, $(\bar{\tau}_{ij})_2$, $(\bar{\sigma}_{ii})_2$ are obtained. Finally on inverting he Laplace transforms we obtain the displacements and stresses.