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On stress accumulation and fault slip in the lithosphere

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ABSTRACT. A mechanism of accumulation of shear stress in the neighbourhood of a locked vertical strike-slip fault in the lithosphere is considered. The fault is taken to be situated in an elastic layer resting on and in welded contact with, a visco-elastic half space, in which shear stresses are maintained far away from the fault by tectonic forces. Exact solutions are obtained for the displacements and stresses in the system in the absence of fault slip, taking into account the displacements and stresses present initially. It is shown that stress accumulation would occur continuously in the upper layer near the fault till the fault slips suddenly, generating an earthquake. Exact solutions are next obtained for the displacements and stresses following the fault slip, taking into account the displacements and stresses present before fault slip, and it is shown that stress accumulation would again occur in the upper layer, till the fault slips again. The mathematical results are applied to some relevant observations on the accumulation of shear strain in the neighbourhood of the San Adreas fault. It is shown that a comparison of the mathematical results obtained and the observations on the ground deformations on the surface near the fault can be used to arrive at reasonable estimates for the times between consecutive slips on active strike slip faults. The results are also expected to lead to greater insight into the problem of earthquake prediction. It is shown that, if sufficient data on surface deformation are available, the results can be used to estimate the effective viscosity of the lower lithosphere.

1. Introduction

The problem of earthquake prediction has attracted the attention of many seismologists in recent years, due to the practical importance of earthquake prediction, and due to the fact that steady accumulation of relevant seismological data and the improvements in the techniques of analysis have made it possible to hope that effective programmes of earthquakes prediction might become possible in the near future. In this connection, a better understanding of the process of stress accumulation in the neighbourhood of faults, which eventually leads to a sudden fault slip, generating an earthquake, would be very useful. Effective quantitative analysis of the process of stress accumulation would be facilitated if it is possible to devise suitable theoretical models which incorporate the essential features of the mechanism of stress accumulation, and enable us to estimate the stress accumulation on the fault below the surface from the observed ground deformation on the surface. For strike-slip faults, some such theoretical models have been developed. In the theoretical models considered by Turcotte and Spence (1974), Savage (1975) and Spence and Turcotte (1976), the stress accumulation near locked faults is taken to be due to relative motion of the parts of the lithosphere on the two sides of the fault. The mechanism of this relative motion is not considered in such models, and the lithosphere is taken to be elastic. Budiansky and Amazigo (1976) considered a locked strike-slip fault situated in a visco-elastic layer, representing the lithosphere, which is free to slide on the asthenosphere below it. It is assumed that tectonic forces maintain a constant shear stress in the layer far away from the fault. It is shown that accumulation of shear stress would occur in the layer due to creep of the material. We note, however, that the effective viscosity of the lithosphere would be expected to depend on the depth, and the lower lithosphere, being at a much higher temperature and pressure, would be expected to undergo much greater creep than the upper lithosphere. We also note that the observed faulting on shallow strike-slip faults, such as the San Andreas fault, is often found to extend to depths of about 10 km or 15 km only. This appears to indicate that the accumulation of stress at greater depths does not reach sufficiently high values to cause further downward extension of the fracture. This phenomenon can be explained easily if we assume that the material of the lower lithosphere below the fault creeps under applied tectonic stresses without undergoing fracture. We again note that, according to the results of laboratory experiments on the deformation of rocks at high temperatures and pressures, as reported by Griggs and Handin (1960), Heard (1976) and others, at the pressures exceeding 3 kilobars and temperatures exceeding 300°C that may be expectd below such faults at depths exceeding 15 km (Heard 1976), and at the strain rates of the order of 0.1μ strain per year, observed in the neighbourhood of strike-slip faults (Savage and Burford 1973, Prescott and Savage 1976), the rocks would be sufficiently ductile to undergo large creeping deformations without fracture. Finally, we note that the tectonic forces causing stress accumulation are likely to be more prominent in the lower lithosphere. Keeping all these points in view, we consider a two-layer model. The upper layer is taken to be elastic, in welded contact with a viscoelastic half space below it.

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A locked vertical strike-slip fault is taken to be situated in the upper layer, reaching upto the surface. We assume that tectonic forces maintain a constant shear stress in the half space, at a great distance from the fault, and obtain exact solutions for the displacements and stresses in the system in the absence of fault slip, and also between two consecutive slips on the fault. The effect of taking into account any possible creep in the upper lithosphere, by taking the upper layer also to be visco-elastic, is considered briefly.

2. Formulation

We take H to be thickness of the upper elastic layer. We consider a plane vertical strike slip fault F of depth D, where $D \leqslant H$, so that the fault slip is in the upper layer to start with. We introduce rectangular cartesian coordinates (x, y, z) with the free surface of the upper layer as the plane $x=0$ and the plane of the fault as the plane $y=0$, as shown in Fig. 1, which represents the section of the model by the plane $z=0$. We assume that the length of the fault is large compared to the depth D. We therefore assume that the stresses and the displacements are independent of the coordinate z. We consider the displacement component $W(x, y, t)$ parallel to the z-axis, associated with strike slip faulting. Following the usual notation, we represent the relevant stress components by $\tau_{xz}(x, y, t)$ and $\tau_{yz}(x, y, t)$.

The corresponding displacements and stresses in the visco-elastic half space are represented by $w^1(x, y, t)$, $\tau^1_{xz}(x, y, t)$ and $\tau^1_{yz}(x, y, t)$. For the elastic layer the constitutive equations are taken to be :

$$\tau_{xz} = \mu_1 \frac{\partial \omega}{\partial x} \text{ and } \tau_{yz} = \mu_1 \frac{\partial \omega}{\partial y} \qquad (1)$$

The material of the half-space is taken to be linearly visco-elastic and of the Maxwell type. The constitutive equations are taken to be :

$$\frac{1}{\mu_2}\frac{\partial}{\partial t}\tau^1_{xz} + \tau^1_{xz}/\eta = \frac{\partial^2 \omega^1}{\partial t\, \partial x}$$

and

$$\frac{1}{\mu_2}\frac{\partial}{\partial t}\tau^1_{yz} + \tau^1_{yz}/\eta = \frac{\partial^2 \omega^1}{\partial t\, \partial y} \qquad (2)$$

as in Budiansky and Amazigo (1976).

We are interested in the slow accumulation of shear stress in the model over long periods of time. For such processes, the inertial forces are very small. In fact, the inertial forces would be significant only during the period just after fault slip, when the elastic waves generated by fault slip are still present near the fault. But this time would normally be small compared to the time of steady accumulation of stress between consecutive slips on the fault.

We, therefore, consider quasi-static deformations of the model, in which the internal forces can be

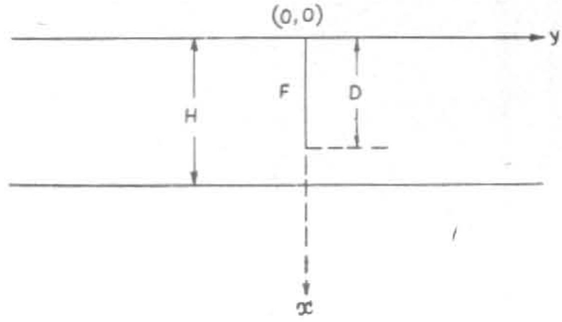

Fig. 1. Section of the system by the vertical plane $Z=0$

neglected. This approach was adopted by Budiansky and Amazigo (1976), and also be Braslau and Lieber (1968), Rosenman and Singh (1973a, 1973b), Singh and Rosenman (1974), Rundle and Jackson (1977), Nur and Mavko (1974) and Barker (1976), who considered stress relaxation in visco-elastic models following strike-slip faulting. For the quasi-static deformations we consider, the stresses satisfy the following relations (Fung 1964) :

$$\frac{\partial \tau_{xz}}{\partial x} \quad \frac{\partial \tau_{yz}}{\partial y} = 0, \quad 0 \leqslant x \leqslant H$$

and

$$\frac{\partial \tau^1_{xz}}{\partial x} + \frac{\partial \tau^1_{yz}}{\partial y} = 0, \quad x > H \qquad (3)$$

We assume that the upper surface of the upper layer is free, and the shear stress τ^1_{yz} in the half space at a large distance from the plane $y=0$ is τ_∞, which is a constant. We assume that the share stress τ^1_{xz} in the half space vanishes at a great depth (*i.e.*, as $x \to \infty$). Then the shear stresses would satisfy the following boundary conditions :

$$\tau_{xz} = 0, \text{ at } x=0, \quad \tau_{xz} = \tau^1_{xz} \text{ at } x = H,$$

$$\tau^1_{xz} \to 0 \text{ as } x \to \infty \text{ and } \tau^1_{yz} \to \tau_\infty \text{ as } y \to \infty \text{ in } x \geqslant H$$

Since the upper layer and the half space are assumed to be in welded contact, we have,

$$w = w^1 \text{ at } x=H \qquad (5)$$

3. Displacements and stresses in the absence of fault slip

We first study the deformations of the system in the absence of fault slip. We assume that the displacements and stresses are continuous throughout the system. We also assume that three is a displacement field w_0, w_0^1 and a stress field $(\tau_{xz})_0$ $(\tau_{yz})_0$, $(\tau^1_{xz})_0$, $(\tau^1_{yz})_0$ in the system at time $t = 0$, satisfying the relations (1)-(5).

Since the system has been undergoing purely shearing deformations in the absence of fault slip, we assume that w_0, w_0^1 are independent of x, so that $(\tau_{xz})_0 = 0 = (\tau^1_{xz})_0$. To obtain the solution, we take Laplace transforms of both sides of the equations (1)-(5) with respect to t. This gives a boundry value problem whose solution is obtained without difficulty. The inverse Laplace trasform of that solution is also obtained without difficulty, and we obtain exact solutions for the displacements and the stresses. We finally have, for the half space,

$$\left.\begin{aligned} &w^1(x, y, t_1) = w^1_0 + \frac{\tau_\infty y t_1}{\eta},\ \tau^1_{xz}(x, y, t_1) = 0 \\ &\text{and} \\ &\tau^1_{yz}(x, y, t_1) = (\tau^1_{yz})_0 \times \exp(-\mu_2 t_1/\eta) \\ &\qquad + \tau_\infty[1 - \exp(-\mu_2 t_1/\eta)] \end{aligned}\right\} \text{(A1)}$$

where t_1 is the time measured from any suitable instant after the relations (1)—(5) become valid for the system. For the layer :

$$\left.\begin{aligned} &w(x, y, t_1) = w_0 + \frac{\tau_\infty y t_1}{\eta},\ \tau_{xz}(x, y, t_1) = 0 \\ &\text{and} \\ &\tau_{yz}(x, y, t_1) = (\tau_{yz})_o + \frac{\mu_1 \tau_\infty t_1}{\eta} \end{aligned}\right\} \text{(A2)}$$

This solution shows that, apart from the initial displacement, there would be a steady creeping displacement in the visco-elastic medium under applied stress τ_∞. The elastic layer, in welded contact with the visco-elastic half-space, would be deformed continuously as a result, giving rise to a steady increase in the shear stress τ_{yz} corresponding to the term $\mu_1 \tau_\infty t_1/\eta$. After a sufficient time, the increase in shear stress τ_{yz} in the upper layer would be sufficient to cause a sudden fault slip, generating an earthquake. However, in the half space, the initial shear stress decays with time and the additional shear stress due to the applied stress τ_∞ never exceeds τ_∞. Hence, it may be expected that no significant accumulation of shear stress would occur in the half space, and the fracture would not extend into the half space, if τ_∞ is not large.

4. Changes in displacements and stresses after fault slip

We consider the quasi-static deformations in the system after the elastic waves generated by fault slip have propagated far away from the fault. For such quasi-static deformations, equation. (1)-(5) are valid. We consider a dislocation model of the fault, following Chinnery (1961, 1963) Fung (1964), Maruyama (1966), Rybicki (1971) and others, and assume that, across the fault F in the region $y=0$, $0 \leqslant x \leqslant D \leqslant H$, we have

$$[w] = uH(t) \tag{6}$$

where u is a constant, and

$$[\tau_{xz}] = 0 = [\tau_{yz}] \tag{7}$$

where, for any function $f(x, y, t)$,

$$[f] = \underset{y\to 0+0}{\text{Lt.}} f(x, y, t) - \underset{y\to 0-0}{\text{Lt}} f(x, y, t) \tag{7a}$$

for any particular pair of values of x and t.

We assume that, after the coseismic slip, the fault remains locked till the shear stress τ_{yz} in the neighbourhood of the fault becomes sufficiently large, after which the fault slips again. We assume that the displacements and stresses remain continuous everywhere in the model, except on F, implying that fault slip does not occur elsewhere in the system. We also assume that, just prior to fault slip, there is a displacement field $(w)_p$, $(w^1)_p$ and a stress field $(\tau_{xz})_p$, $(\tau_{yz})_p$, $((\tau^1_x)_p$, $(\tau^1_{yz})_p$, satisfying the relations (1)-(5). Noting that the displacements w, w_1 given by (A1) and (A2) are independent of x, we assume that w_p, w^1_p are independent of x, so that $(\tau_{xz})_p = 0 = (\tau^1_{xz})_p$.

To obtain solutions for the displacements and the stresses, we take Laplace transforms of both sides of the relations (1)—(7) with respect to t. The solution of the resulting boundary value problem can be obtained on using results given by Maruyama (1966) and Rybicki (1971). Finally, on inverting the Laplace transforms, we obtain exact solutions for the displacements and the stresses, valid for all times and distances. For the elastic layer $0 \leqslant x \leqslant H$, we obtain,

$$\left.\begin{aligned} &w(x, y, t) = (w)_p + \tau_\infty yt/\eta + w_{01}(x, y) \\ &\qquad + w_{02}(x, y, t), \\ &\tau_{xz}(x, y, t) = -\tau_{xz1}(x, y) + \tau_{xz2}(x, y, t) \\ &\text{and } \tau_{yz}(x,y,t) = (\tau_{yz})_p + \mu_1\tau_\infty t/\eta - \tau_{yz_1}(x,y) \\ &\qquad + \tau_{yz_2}(x, y, t) \end{aligned}\right\} \text{(8)}$$

For the half space we obtain

$$\left.\begin{aligned} &w^1(x,y,t) = (w^1)_p + \tau_\infty yt/\eta + w^1_1(x,y) \\ &\qquad + w^1_2(x,y,t) \\ &\tau^1_{xz}(x,y,t) = -\tau^1_{xz1}(x,y) + \tau^1_{xz2}(x,y,t) \\ &\text{and } \tau^1_{yz}(x,y,t) = (\tau^1_{yz})_p \exp(-\mu_2 t/\eta) \\ &\quad + \tau_\infty[1 - \exp(-\mu_2 t/\eta)] - \tau^1_{yz1}(x, y) + \\ &\quad + \tau^1_{yz2}(x, y, t) \end{aligned}\right\} \text{(9)}$$

In (8) and (9), the first two terms in the expressions for w, w^1, τ_{yz} and τ^1_{yz} represent the displacements and stresses that would have been present in the absence of fault slip, as in (A1) and (A2). The last two terms in each of the expressions in (8) and (9) depend on the dislocation u, and represent the effect of fault slip. In (8), the complete expressions for the terms w_{01} and w_{02} are given by :

$$w_{01}(x, y) = \frac{u}{2\pi}[\tan^{-1}(D_5/y) + \tan^{-1}(D_6/y)] + \frac{u}{2\pi}\sum_{m=1}^{\infty} R^m f_m(x, y) \tag{10}$$

$$\text{and } w_{02}(x, y, t) = \frac{u}{2\pi}\sum_{m=1}^{\infty} R^m D_m^1(t) f_m(x, y)$$

where $s = \mu_2/\mu_1$, $R = (1-s)/(1+s)$, $D_5 = D - x$, $D_6 = D + x$, $f_m(x, y) = \tan^{-1}(D_{1m}/y) - \tan^{-1}(D_{2m}/y) + \tan^{-1}(D_{3m}/y) - \tan^{-1}(D_{4m}/y)$ (11)

$$\text{and } D_m^1(t) = \sum_{r=1}^{m} A_{rm}\left[\, 1 - e^{-a_1 t} e_{r-1}\,(a_1 t)\,\right] \quad (12)$$

In (11) and (12),

$$\left.\begin{array}{l} A_{rm} = \left(\frac{m}{r}\right)\left(\frac{2s}{1-s}\right)^r, \; a_1 = \frac{\mu_2}{\eta\,(1+s)}, \\ e_n\,(a_1 t) = 1 + \sum_{p=1}^{n} \frac{(a_1 t)^p}{p!}, \quad (n \geqslant 1) \\ e_0\,(a_1 t) = 1, \\ D_{1m} = 2mH + D + x,\; D_{2m} = 2mH - D + x, \\ D_{3m} = 2mH + D - x \text{ and } D_{4m} = 2mH - D - x. \end{array}\right\} \quad (13)$$

We also have, in (8),

$$\tau_{xz1}\,(x, y) = \frac{\mu_1 u y}{2\pi}\left[\, f_1\,(y, D_5) - f_1\,(y, D_6)\,\right] - \frac{\mu_1 u y}{2\pi}\sum_{m=1}^{\infty} R^m\, \Psi_m\,(x, y) \quad (14)$$

$$\tau_{xz2}\,(x, y, t) = \frac{\mu_1 u y}{2\pi}\sum_{m=1}^{\infty} R^m\, D^1{}_m\,(t)\, \Psi_m\,(x,y) \quad (15)$$

$$\tau_{yz1}\,(x, y) = \frac{\mu_1 u}{2\pi}\left[\, f_2\,(y, D_5) - f_2\,(y, D_6)\,\right] - \frac{\mu_1 u}{2\pi}\sum_{m=1}^{\infty} R^m\, \phi_m\,(x, y) \quad (16)$$

$$\text{and } \tau_{yz2}\,(x, y, t) = \frac{\mu_1 u}{2\pi}\sum_{m=1}^{\infty} R^m\, D^1{}_m\,(t)\, \phi_m\,(x, y) \quad (17)$$

$$\text{where, } \Psi_m\,(x, y) = f_1\,(y, D_{4m}) - f_1\,(y, D_{2m}) + f_1\,(y, D_{1m}) - f_1\,(y, D_{3m}) \quad (18)$$

$$\text{and } \phi_m\,(x, y) = f_2\,(y, D_{2m}) + f_2\,(y, D_{4m}) - f_2\,(y, D_{1m}) - f_2\,(y, D_{3m}) \quad (19)$$

$$\text{In (14) - (19), } f_1\,(y, D_{im}) = \frac{1}{y^2 + D^2{}_{im}} \quad (19a)$$

$$\text{and} \quad f_2\,(y, D_{im}) = \frac{D_{im}}{y^2 + D^2{}_{im}} \quad (19b)$$

where $D_{im}(i=1, 2, 3, 4, 5, 6)$ have been defined earlier in (10)—(13). The expressions for $w_1{}^1$, $w^1{}_2$, $\tau^1{}_{xz1}$, $\tau^1{}_{xz2}$, $\tau^1{}_{yz1}$, and $\tau^1{}_{yz2}$ in (9) are similar in form to those of w_{01}, w_{02}, τ_{xz1}, τ_{xz2}, τ_{yz1}, and τ_{yz2} respectively, as given in the equations (10)-(19). It is found that the terms $-\tau_{xz1}$, $-\tau_{yz1}$, $-\tau^1{}_{xz1}$ and $-\tau^1{}_{yz1}$, representing the co-seismic stress drops, are all negative throughout the model. The last term in each expression in (8) and (9) is zero at $t=0$, and starts increasing as t increases, finally approaching a finite limit as $t \to \infty$. In each of the expressions for the stresses, the sum of the last two terms $\to 0$ as $t \to \infty$, indicating that the post-seismic stress drops relax completely as $t \to \infty$.

5. Discussion of the results and applications

We consider the complete expression for the stress τ_{yz}, which controls the fault slip in the upper layer. We find that, just after fault slip, the value of τ_{yz} falls below the critical value for fault slip due to the coseismic stress drop—τ_{yz1}. However, for $t>0$, τ_{yz} increases continuousy due to two reasons. The first reason is the increase due to creep associated with relaxation of the coseismic stress drop, represented by the term τ_{yz2}. To explain this physically, we note that, following fault slip, there would be coseismic stress drops both in the layer and in the half space. In the visco-elastic half space, the coseismic stress drop would relax, and the material would creep, as indicated by the increasing displacement $w^1{}_2(x, y, t)$. The layer, being in welded contact with the half space, would be deformed continuously as a result, as indicated by the increasing displacement $w_{02}\,(x, y, t)$. This increasing displacement leads to gradual accumulation of stress in the layer. The second reason is the increase in τ_{yz} due to creep associated with the effect of the stress τ_∞ maintained by tectonic forces. This is represented by the term $\mu_1 \tau_\infty\, t/\eta$ in (9). If τ_∞ is positive, the stress τ_{yz} would increase continuously with t in the neighbourhood of the fault, till it reaches the value required for fault slip. Since τ_{yz} has no upper bound if $\tau_\infty > 0$, the fault slip would definitely occur after a finite time T. However, if $\tau_\infty = 0$, τ_{yz} has the upper bound $(\tau_{yz})_p$, which it approaches as $t \to \infty$. Hence if $\tau_\infty = 0$, the fault slips again after a finite time T, if and only if the stress accumulation required for fault slip is less than the coseismic stress drop τ_{yz1}.

We next consider the stress $\tau^1{}_{yz}$ in the lower half space given by (9). The initial stress $(\tau^1{}_{yz})_p$ relaxes completely as $t \to \infty$, and the coseismic stress drop also relaxes completely as $t \to \infty$ and $\tau^1{}_{yz}$ finally $\to \tau_\infty$ as $t \to \infty$ at all points in the layer, including points below the fault. Hence, if the shear stress τ_∞ is not sufficiently large to cause downward extension of the fault into the half space, the fault remains confined to the upper layer only. The fault slip in the upper layer and the fact that the fault slip does not extend downwards into the lower medium can thus be explained in this model. We also note that τ_{xz} and $\tau^1{}_{xz} \to 0$ as $t \to \infty$.

After the second fault slip at $t=T$, the system would again reach the quasi-static state of stress accumulation after the elastic waves generated by faulting have propagated far away from the fault. The time required for this would normally be small compared to T. Stress accumulation would occur again, as before, and the fault would slip again when the stress τ_{yz} becomes sufficiently large in the neighbourhood of the fault. The results we have obtained would again be

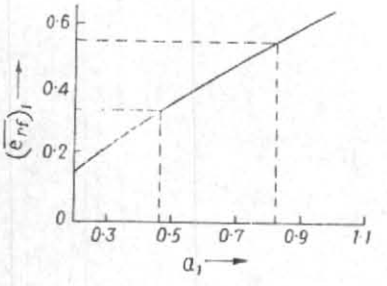

Fig. 2. Variation of $(\bar{e}_{rf})_1$ with a_1: The unit for a_1 is 10^{-2}/year and the unit for $(\bar{e}_{rf})_1$ for is μ strain year

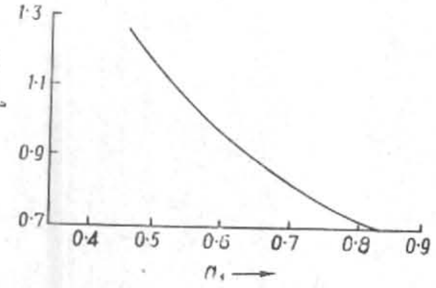

Fig. 3. Variation of η with a_1: The unit for η is 10^{21} Poise and the unit for a_1 is 10^{-2}/year

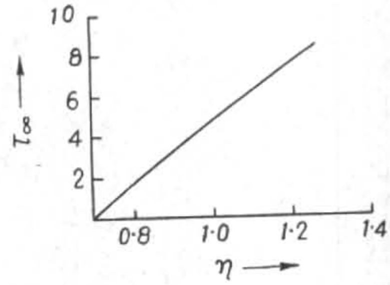

Fig. 4. Variation τ_∞ with η: The unit for η is 10^{21} Poise and τ_∞ is in bars.

expected to represent approximately the process of stress accumulation between any two such consecutive slips on the fault. We note, however, that since τ_{yz} becomes large in the upper layer at points below the fault as t increases, the fault slip would be expected to extend downwards, after a few slips, to the boundary $x = H$. However, in the half space, the shear stress τ^1_{yz} would $\to \tau_\infty$ as $t \to \infty$ even at points below the fault. Hence, no significant stress accumulation occurs below the faults if τ_∞ is not very large, and the fault does not extend into the half space.

$$\text{Let } e_{rf} = \frac{1}{\mu_1}\frac{\partial}{\partial t}\left(\tau_{yz2} \right), \text{ and } e_{rc} = \frac{\tau_\infty}{\eta} \quad (22)$$

where τ_{yz2} is given by (17). Then e_{rf}, and e_{rc} represent the rates of accumulation of shear strain due to creep associated with relaxation of coseismic stress drop and the creep due to the shear stress τ_∞.

We shall try to compare e_{rf} and e_{rc} with the observed rates of strain accumulation on the surface near strike slip faults. For the San Andreas fault, Savage and Burford (1973) have reported that, in the neighbourhood of a section of the San Andreas fault near Ross mountain, which has remained locked since the San Francisco earthquake of 1906, the averge rate of strain accumulation near the fault from 1906 to 1969 has been $(0.55 \pm 0.05)\mu$ strain per year. It has also been reported by Prescott and Savage (1976) that, during the period 1971-1975, the average rate of strain accumulation near another locked section of the southern part of the San Andreas fault near Palmdale, along which fault slip occurred in 1857, was about $(0.21 \pm 0.003)\mu$ strain per year. These two observational results give :

$$(\overline{e_{rf}})_1 + e_{rc} = 0.55\mu \text{ strain/year}$$
$$\text{and } (\overline{e_{rf}})_2 + e_{rc} = 0.21\mu \text{ strain/year} \quad (23)$$

Here $(\bar{e}_{rf})_1$ is the average value of e_{rf} near the fault on the surface ($x \approx 0$, $y \approx 0$) for the locked part of the fault near the rupture of 1906, from 1906 to 1969, and $(\overline{e_{rf}})_2$ is the average value, from 1971 to 1975, of the rate of accumulation of strain near the rupture of 1857 on the surface ($x \approx 0$, $y \approx 0$). In writing (23), we assume that τ_∞/η has the same value for the two parts of San Andreas fault near the ruptures of 1906 and 1857. From our analysis, it is clear that $(\overline{e_{rf}})_1$, e_{rc} and $(\overline{e_{rf}})_2$ are non-negative in our model, assuming that $\tau_\infty \geqslant 0$. Hence $e_{rc} \leqslant 0.21\ \mu$ strain/year, and $(\overline{e_{rf}})_1$ lies between 0.34μ strain/year and $0{\cdot}55\mu$ strain/year. We compute the values of $(\overline{e_{rf}})_1$ for different values of η, taking $D = H = 10$ km and $u = 4$ metres, following Knopoff (1958), $\mu_2 = 3.7 \times 10^{11}$ dynes/cm², following Aki (1967) and $s = 1$, $x = 0$, $y = 0$.

It is found that $(\overline{e_{rf}})_1$ is a monotonic function of a_1, as shown in Fig. 2. Since (22) is satisfied by $(\overline{e_{rf}})_1$, we find, from Fig. 2, that $0.0045/\text{year} \leqslant a_1 \leqslant 0.0081/\text{year}$. Taking $\mu = 3.7 \times 10^{11}$ dynes/cm², following Aki (1967), we find that

$$0.71 \times 10^{21} \text{ poise} \leqslant \eta \leqslant 1.26 \times 10^{21} \text{ poise} \quad (24)$$

as shown in Fig. 3. This gives an estimate of the effective viscosity of the lower lithosphere below the fault. We note in this connection, that Cathles III (1975) has also obtained estimates of the order of 10^{21} poise for the effective viscosity of the uppermost part of the mantle from the analysis of data on post-glacial uplift in Fenno-Scandia and Canada.

For any value of η in the range (24), we find the corresponding values of a_1 and $(\overline{e_{rf}})_1$ and hence by (23), the corresponding value of e_{rt}. This gives the corresponding values of $\tau_\infty = \eta\, e_{rt}$. These estimates of τ_∞ are given in Fig. 4. It is found that τ_∞ does not exceed 8.4 bars in this model. Hence, in the lower medium, the maximum shearing stress would not become greater than 8.4 bars as $t \to \infty$. This may explain the fact that the fault slip does not extend into the lower medium.

We next use the estimates for τ_∞ and η we have obtained to compute the variations of the ratio τ_v/τ_D, where $\tau_v = \tau_{yz2} + \mu_1 \tau_\infty\ t/\eta$ represents the post-seismic stress accumulation at time t, for points on the fault near the surface ($x \approx 0$, $y \approx 0$). In Fig. 5, the curves I, II, III and IV correspond to $\tau_\infty = 8.4$ bars, 6.0 bars, 3.0 bars and zero, with the corresponding values of η, which are $\eta = 1.26 \times 10^{21}$ poise,

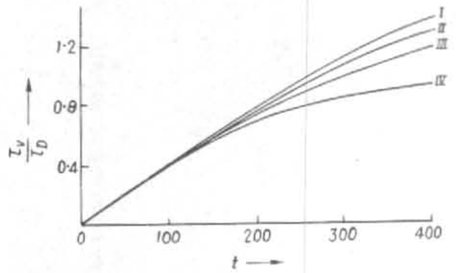

Fig. 5. Variation of the ratio of the post-seismic stress accumulation τ_v and the coseismic stress drop τ_D. The time t in years and the curves I, II, III and IV correspond to $\tau_\infty = 8\cdot4$ bars, $6\cdot0$, $3\cdot0$ bars and zero.

$1\cdot1\times10^{21}$ poise, 0.92×10^{21} poise and 0.71×10^{21} poise respectively, from Fig. 4. It is seen that the rate of accumulation of stress depends significantly on the values of τ_∞ and η. For $\tau_\infty = 0$, the stress accumulation τ_v never reaches the seismic stress drop τ_D, and hence fault slip does not occur again, unless the stress accumulation required for fault slip is less than τ_D. However, if $\tau_\infty > 0$, τ_v becomes greater than τ_D after a sufficiently long time, and then there is a possibility of a fault slip similar to the one which occurred at $t=0$. For the locked part of the San Andreas fault near the rupture of 1906, Fig. 5 shows that this would occur at a time exceeding 260 years after the fault slip. The exact time depends on the values of τ_∞ and η. When τ_v/τ_D exceeds 1, there would be the possibility of a major fault movement generating an earthquake with a magnitude of the same order as that of the San Francisco earthquake of 1906. If the fault slips at a smaller value of τ_v/τ_D, the magnitude of the fault slip would be expected to be smaller, since the stress accumulation is smaller. If more reliable data become available on the ground deformation on the surface over a long period of time at different distances from the fault, it would be possible to obtain more definite and reliable estimates of τ_∞ and η, and this would enable us to determine more definitely the variation of τ_v/τ_D with time. This, in turn, would enable us to obtain more reliable estimates of the return times of major earthquakes on the fault, and of the time-dependence of seismic risk near the fault.

If, instead of taking the upper layer to be perfectly elastic, we assume that it is linearly viscoelastic and of the Maxwell type, with coefficient of viscosity $\eta_1 >> \eta$, we find that the method used here would again give exact solutions for the displacements and the stresses. However, τ_{yz} would be bounded, and would $\rightarrow (\eta_1/\eta)\ \tau_\infty$ as, $t \rightarrow \infty$, while $\tau^1_{yz} \rightarrow \tau_\infty$, as $t \rightarrow \infty$, as in the model considered here. Since $\eta_1 >> \eta$, τ_{yz} attains values $>> \tau_\infty$. Hence, the stress accumulation is much greater in the upper layer. This would again explain the fact that the fault slip is confined to the upper layer. Estimates for the return time of fault slip can be obtained, as in the case of the model considered here, and the estimates are found to be slightly greater than those we have obtained here.

In conclusion, we note that the simple model we have considered cannot be expected to represent all the features of the complex process of stress accumulation and fault slip. However, this model appears to explain some relevant observational results on shallow strike-slip faults, and also gives estimates of the effective viscosity of the lower lithosphere near the fault for the process of stress accumulation. This model would also enable us to obtain estimates of the return times of major earthquakes due to strike-slip faulting, provided sufficient data on the ground deformation near the faults are available, and to estimate the changes in seismic risk with time.

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