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On stress accumulation in the lithosphere and interaction between two strike-slip faults

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ABSTRACT. The problem of stress accumulation near locked vertical strike-slip faults in the lithosphere, and the interaction between two parallel strike slip faults are considered. The two parallel faults are taken to be situated in a visco-elastic half space. It is assumed that tectonic forces maintain a steady shear stress far away from the fault. Exact solutions are obtained for the displacements and the stresses in the system. It is shown that in the absence of fault slip, there would be a steady accumulation of the shear stress near the faults, which would lead to a sudden slip on one of the faults, resulting in an earthquake. The subsequent changes in shear stress in the neighbourhood of the fault system are investigated and exact solutions are obtained for the displacements and stresses. It is seen that the fault slip on one fault affects the accumulation of shear stress on the other and the effect leads to significant changes in the rate of stress accumulation in the neighbourhood of faults if the faults are close to each other. This inturn, affects significantly the return time between consecutive slips on the fault. It is shown that the theoretical results obtained may be useful in the estimation of seismic risk and also in earthquake prediction.

1. Introduction

Strike-slip faulting is considered to be a major cause of tectonic earthquakes. It would be expected that fault slip on strike-slip faults would be preceded by accumulation of shear stress over a considerable period of time, and fault slip would occur when the stress accumulation reaches a critical level. Turcotte and Spence (1974), Spence and Turcotte (1976), Savage (1975) and Budiansky and Amazigo (1976) have recently considered some theoretical models to explain the accumulation of stress. In the models proposed by Turcotte and Spence (1974), Savage (1975) and Spence and Turcotte (1976) the stress accumulation is supposed to be due to steady relative motion of the adjacent parts of the lithosphere, leading to accumulation of shear stress on the locked fault.

The mechanism of the relative motion is not considered in these models, and the lithosphere is taken to be elastic. These models require the existence of very large and steadily increasing lithospheric stresses far away from the fault. In the model considered by Budiansky and Amazigo (1976), a locked fault in a visco-eleastic layer is considered, and it is supposed that tectonic forces maintain a constant shear stress in the layer far away from the fault. It is shown that steady accumulation of shear stress would occur in such a model. In all these models, the presence of a single fault is considered. However, in a fault system, there is often more than one fault along which fault slip can occur. One may mention in this connection the Hayward and Calvaeras faults which are approximately parallel to the San Andreas fault. In such cases, fault slip on one fault may have considerable influence on the stress accumulation on a neighbouring fault, and hence on the time of slip on the neighbouring fault. We try to study this effect by using a simple theoretical model. We consider two vertical parallel faults, situated in a half space. We assume that the half space is visco-elastic, and that tectonic forces maintain a constant shear stress far away from the faults. We study the accumulation of shear stress in this model, and the effect of slip on one fault on stress accumulation on the other.

2. Formulation

We consider two vertical parallel faults F and F^1, reaching upto the surface we assume that the lengths of the faults are large compared to their depths D and D^1. We introduce rectangular cartesian co-ordinates (x, y, z), with the z-axis, parallel to the length of the fault F, the free surface as the plane $x=0$, the x-axis vertically downwards, and the origin on the line of intersection of the fault F and the free surface. The faults F and F^1 are shown in Fig. 1, which represents a section of the system by the plane

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$z=0$. Since the lengths of the faults are large compared to their depths, we consider a two dimensional problem, in which the displacements and stresses are functions of (x, y, t) and are independent of Z. Let $w(x, y, t)$ be the displacement component parallel to the Z axis, associated with strike-slip faulting and let the relevant stress components in the usual notation be τ_{xz}, and τ_{yz}. We take the half space to behave as a linearly visco-elastic material of the Maxwell type for the process of stress accumulation we are considering. The constitutive equations for the half space are taken to be ;

$$\left.\begin{aligned} &\frac{1}{\mu}\frac{\partial}{\partial t}\tau_{xz}+\frac{\tau_{xz}}{\eta}=\frac{\partial^2 w}{\partial t\,\partial x} \\ \text{and}\quad &\frac{1}{\mu}\frac{\partial}{\partial t}\tau_{yz}+\frac{\tau_{yz}}{\eta}=\frac{\partial^2 w}{\partial t\,\partial y}\end{aligned}\right\} \tag{1}$$

as in Budiansky and Amazigo (1976). We note in this connection that the process of long-term deformation in the lithosphere is likely to be characterised by secondary creep for which the Nabarro Herring creep mechanism may be relevant (Heard 1976) and the material would have Newtonian viscosity, so that the constitutive equations (1) would be reasonable.

We consider the steady accumulation of shear stress in the model, leaving out the period just following fault slip when the elastic waves generated by faulting exists in the neighbourhood of the fault. Both before fault slip, and after the waves generated by faulting have propagated far away from the fault, the inertial forces would be very small and can be neglected. This was done earlier by Budiansky and Amazigo (1976) and also by Braslau and Lieber (1968). Rosenman and Singh (1973a , 1976), Singh and Rosenman (1974), Nur and Mavko (1974), Barker (1976). Rundle and Jackson (1977) who studied post-seismic stress relaxation in visco-elastic systems following fault slip. For such quasi-static deformations, the stresses satisfy the relation :

$$\frac{\partial}{\partial x}\tau_{xz}+\frac{\partial}{\partial y}\tau_{yz}=0 \tag{2}$$

We assume that the shear stress τ_{yz} at a large distance from the fault has a constant value τ_∞ and the upper surface of the half space is free. Then the stresses would satisfy the following boundary conditions :

$$\left.\begin{aligned} &\tau_{xz}=0 \quad \text{at } x=0 \\ &\tau_{xz}\to 0 \quad \text{as } x\to\infty \\ \text{and}\quad &\tau_{yz}\to\tau_\infty \quad \text{as } y\to\infty\end{aligned}\right\} \tag{3}$$

3. Displacements and stresses in the absence of fault slip

We first consider the deformation of the medium under the action of the shear-stress, τ_∞, and in the absence of fault slip when the stresses and displacements would be continuous throughout. We assume that there is an initial displacement and stress field w_0, $(\tau_{xz})_0$, $(\tau_{yz})_0$, satisfying the conditions (1)—(3). We take Laplace transforms of (1)—(3) with respect to t, and then solve the resulting boundary value problem, as explained in the Appendix; on inverting the Laplace transforms we find the exact solutions:

$$\left.\begin{aligned} &w(x, y, t)=w_0+\frac{\tau_\infty\, y\, t_1}{\eta} \\ &\tau_{xz}(x, y, t)=(\tau_{xz})_0\, e^{-ut_1/\eta} \\ \text{and}\quad &\tau_{yz}(x, y, t)=(\tau_{yz})_0\, e^{-ut_1/\eta} \\ &\qquad +\tau_\infty\,(1-e^{-ut_1/\eta})\end{aligned}\right\}\text{(AI)}$$

where t_1 is the time, measured from some suitable instant when the relations (1)-(3) become valid.

The solutions (A1) show that in addition to the initial displacement there would be a creeping displacement of the medium under the applied stress τ_∞. The initial stresses relax completely and tend to zero as $t_1\to\infty$. Hence $\tau_{xz}\to 0$ as $t_1\to\infty$ and τ_{yz} increases steadily from $(\tau_{yz})_0$ to τ_∞, assuming that $(\tau_{yz})_0<\tau_\infty$. Let us assume that the critical stress τ_c for fault slip on F and F^1 is less than τ_∞ and greater than the value of $(\tau_{yz})_0$ on the fault. Then after a sufficient time, fault slip would occur on the fault. Let us suppose that F slips first. We consider a dislocation model of the fault slip, following Chinnery (1961, 1963, 1964), Maruyama (1966), Rybicki (1971) and others, and asume that across the fault F, which is in the region $y=0$, $0\leqslant x\leqslant H$, We have,

$$[w]=u\,H(t) \tag{4}$$

$$\text{and}\qquad [\tau_{xz}]=0=[\tau_{yz}] \tag{5}$$

where for any function $f(x, y, t)$,

$$[f]=\underset{y\to 0+0}{\mathrm{Lt.}}\, f(x, y, t)-\underset{y\to 0-0}{\mathrm{Lt.}}\, f(x, y, t) \tag{6}$$

For simplicity we assume that $u=$ constant, *i.e.*, the dislocation is a constant; we assume that, after the coseismic slip, the fault F becomes locked again. When the elastic waves generated by faulting have propagated far away from the fault, the system would again reach the quasi-static state, when the relations (1)-(5) would be satisfied. We also suppose that, just before fault slip, there are displacements and stresses in the system, given by w_p, $(\tau_{xz})_p$, $(\tau_{yz})_p$ satisfying (1), (2) and (3).

4. Displacements and stresses after fault slip

To determine the displacements and stresses after fault slip, we take Laplace transforms of both sides of (1)-(5), with respect to t. The resulting boundary value problem can be solved, as explained in the Appendix, on using results given by Chinnery (1963). Inversion of the Laplace transforms finally gives us exact solutions for the displacements and

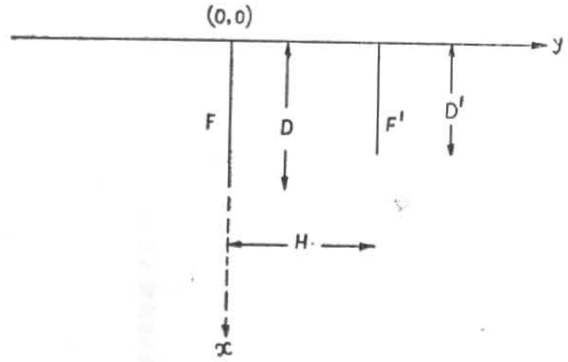

Fig. 1. Section of the fault system by the plane $Z=o$

the stresses at time t after fault slip, in the form :

$$\left.\begin{aligned} w(x, y, t) &= w_p + \frac{\tau_\infty\, y\, t}{\eta} + \frac{u\, H(t)}{2\pi} \times \\ &\left[\tan^{-1}\left(\frac{D+x}{y}\right) + \tan^{-1}\left(\frac{D-x}{y}\right)\right] \\ \tau_{xz}(x, y, t) &= (\tau_{xz})_p\, e^{-\mu t/\eta} - \frac{\mu\, u\, y}{2\pi} \times \\ &\left[\frac{1}{y^2+(D-x)^2} - \frac{1}{y^2+(D+x)^2}\right] e^{-\frac{\mu t}{\eta}} \\ \tau_{yz}(x, y, t) &= (\tau_{yz})_p\, e^{-\mu t/\eta} + \tau_\infty (1 - e^{-\mu t/\eta}) \\ &- \frac{\mu u}{2\pi} e^{-\mu t/\eta}\left[\frac{D+x}{y^2+(D+x)^2} + \frac{D-x}{y^2+(D-x)^2}\right] \end{aligned}\right\} \quad (7)$$

In (7), the terms containing τ_∞ as a factor represent the displacements and stresses due to creep of the half space, under the stress τ_∞ maintained far away from F and F^1. The term containing u as a factor represents the effect of the fault slip. We find that the initial stresses $(\tau_{xz})_p$, $(\tau_{yz})_p$, relax with time, and finally $\rightarrow 0$ as $t \rightarrow \infty$. The stress drops due to fault slip, represented by the terms containing u in the expressions for the stresses also relax completely and $\rightarrow 0$ as $t \rightarrow \infty$, and $\tau_{yz} \rightarrow \tau_\infty$ as $t \rightarrow \infty$ at points in the neighbourhood of F and F^1. The initial displacements w_p and the elastostatic displacement due to the fault slip on F persists, and the creeping displacement $\tau_\infty yt/\eta$ due to the stress τ_∞ maintained far away from the fault increases steadily, and the rate of increase of the shear strain e_{yz} near the fault systems is τ_∞/η. Such a uniform rate of increase of shear strain in the neighbourhood of locked sections of the San Andreas fault has been reported by Savage and Burford (1973) and Prescott and Savage (1976).

5. Interaction between strike-slip faults

We now consider the effect on F^1 of the slip on the fault F. We shall study the shear stress τ_{yz} in the neighbourhood of F^1 which would control the possibility of a slip on F^1. We note that in the absence of F, the stress τ_{yz} on F^1, assuming the same driving stress τ_∞ and the same initial stress $(\tau_{yz})_p$ would have been given by (AI) with $y=H$ and $(\tau_{yz})_0=(\tau_{yz})_p$. Let us represent this stress by $(\tau_{yz})^1{}_{F^1}$. The actual shear stress $(\tau_{yz})_{F1}$ on F^1 following the fault slip on F would be given by (7) with $y=H$. We therefore have from (A1) and (7) :

$$(\tau_{yz})^1{}_{F1} - (\tau_{yz})_{F1} = \frac{\mu u e^{-\mu t/\eta}}{2\pi} \times \left[\frac{D+x}{H^2+(D+x)^2} + \frac{D-x}{H^2+(D-x)^2}\right] \quad (8)$$

The expression in (8) is always positive on F^1 if $D^1 \leqslant D$, so that $x \leqslant D^1 \leqslant D$ on F^1. Hence $(\tau_{yz})_{FI} < (\tau_{yz})^1{}_{F1}$, and the effect of the fault slip on F is to reduce the stress accumulation on F^1. This would reduce the possibility of slip on F^1. However, since the stress on F^1 also $\rightarrow \tau_\infty$ as $t \rightarrow \infty$ as is evident from (7) the fault F^1 would slip eventually if the critical stress for slip on $F^1 < \tau_\infty$, unless F slips again before the slip on F^1 occurs. We also note that $(\tau_{yz})^1{}_{F1} - (\tau_{yz})_{FI} \rightarrow 0$ as $t \rightarrow \infty$, so that the stress drop on F^1 due to the slip on F also relaxes and tends to disappear with time. Thus the main effect of the fault slip on F is to delay fault slip on F^1. The stress drop in (8) becomes small if H is large, so that the stress drop on F^1 due to slip on F is small if the faults are far away from each other.

Let $(\tau_{yz})_p = K\tau_\infty$ and let the initial stress drop at $t=0$, at the mid point of F^1 due to slip on F be $K^1\tau_\infty$. Let us take $H=D=D^1$ for simplicity. Then from (8) we find that,

$$\frac{\mu u}{2\pi} = \frac{3K^1 D\tau_\infty}{8} \quad (9)$$

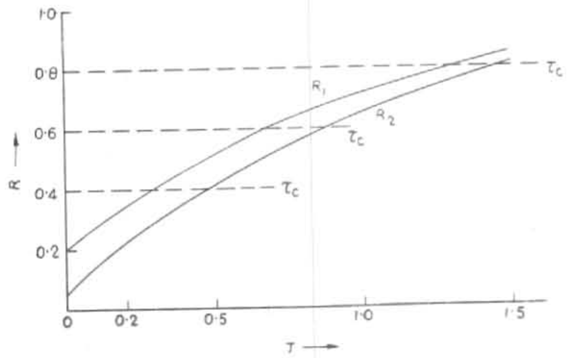

Fig. 2. Variation of R with increase in T

We now study the changes in the ratios

$$R_1 = \frac{\overline{(\tau_{yz})}^1{}_{F^1}}{\tau_\infty} \text{ and } R_2 = \frac{\overline{(\tau_{yz})}_{F^1}}{\tau_\infty} \text{ with time,}$$

using the results (A_1), (7) and (9) and taking $H=D=D^1$, $K=0.2$ and $K^1=0.5$, where $(\bar{\tau}_{yz})^1{}_{F^1}$ and $(\bar{\tau}_{yz})_{F^1}$ represent the average values of $(\tau_{yz})^1{}_{F^1}$ and $(\tau_{yz})_{F^1}$ over the fault F^1. Fig. 2 shows the variations of R_1 and R_2 with the dimensionless quantity $T=\mu t/\eta$. It is seen that the difference between the ratios decreases with time. If the critical stress τ_c which required for fault slip on $F^1<\tau_\infty$, then Fig. 2 shows that the fault F^1 would slip eventually, but the time of slip on F^1 is delayed due to the fault slip on F. This delay depends on the critical stress τ_c and is found decrease slowly with increase in τ_c. To obtain an idea of the magnitude of the delay we note that for the region San Andreas fault, the effective viscosity of the lithosphere has been estimated to be about 10^{21} poise (Budiansky and Amazigo 1976). Taking $\mu = 3.7\times10^{11}$ dynes per square cm following Aki (1967) we find from Fig. 2 that the slip on the fault F^1 would be delayed as a fault of the slip on F by 16 years, 14.5 years or 12.5 years, according as the critical stress τ_c is given by $0.4\,\tau_\infty$, $0.6\,\tau_\infty$ or $0.8\,\tau_\infty$. Thus the time between the slip on F and the slip on F^1 would be increased significantly due to the effect of F on F^1. This increase in the time between the slips on F and F^1 is about 30 per cent if $\tau_c = 0.4\,\tau_\infty$, 14 per cent if $\tau_c = 0.6\,\tau_\infty$ and 8 per cent if $\tau_c = 0.8\tau_\infty$. Thus the time of slip on a strike-slip fault is influenced significantly by the existance of another strike-slip fault in its neighbourhood. Hence, in making any prediction regarding the time of slip or in estimating the changes in seismic risk with time, the possible influence of other faults in the neighbourhood have to be considered to improve the estimate.

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References

Aki, K., 1967, *J. geophys. Res.*, **72,** 1217-1231.

Barker, T.G., 1976, *Geophys. J.R. Astr. Soc.*, **45,** 689-705.

Braslau, D. and Lieber, P., 1968, *Bull. seis. Soc. Amer.*, **58,** 613-628.

Budiansky, B. and Amazigo, J.C., 1976, *J. geophys. Res.*, **81,** 4897-4900.

Chinnery, M.A., 1961, *Bull. seism. Soc. Amer.*, **51,** 355-372.

Chinnery, M.A., 1963, *Bull. seism. Soc. Amer.*, **53,** 921-932.

Chinnery, M. A., 1964, *Bull. seis. Soc., Amer.*, **54,** 2085-2089.

Fung, Y.C., 1964, *Foundations of solid Mechanics*, Prentice Hall.

Heard, H. C, 1976, *Phil. Trans. Roy. Soc. Lond;* **A, 283,** 173-186.

Maruyama, T., 1966, *Bull. Earthquake Res. Inst.*, Tokyo Univ., **44,** 811-871.

Nur, A. and Mavko, G., 1976, *Science*, **183,** 204-206.

Prescott, W. H. and Savage, J. C., 1976, *J. geophys Res.*, **81,** 4901-4908.

Rosenman, M. and Singh, S. J., 1973(a), *Bull. seis. Soc. Amer.*, **63,** 2145-2154.

Rosenman, M. and Singh, S. J., 1973(b), *Bull. seis. Soc, Amer.*, **63,** 1737-1752.

Rundle, J. B. and Jackson, D.D., 1977, *Geophys. J. R. Astr. Soc.*, **49,** 575-592.

Rybicki, K., 1971, *Bull. seis. Soc. Amer.*, **61,** 79-92.

Savage, J. C., 1975, *J. geophys. Res.*, **80,** 4111-4114.

Savage, J. C. and Burford, R. O., 1973, *J. geophys. Res.*, **78,** 832-845.

Singh, S.J. and Rosenman, M., 1974, *Phys. Earth and Planet Interiors*, **8,** 87-101.

Spence, D. A. and Turcotte, D.L., 1976, *Proc. Roy Soc. Lond.*, **A , 349,** 317-341.

Turcotte, D. L. and Spence, D. A., 1974, *J. geophys. Res.* **79,** 4107-4112.

Appendix

On taking Laplace transforms with respect to t of both sides of (1)-(3) we obtain the relations

$$\left.\begin{aligned}\bar{\tau}_{xz} &= \bar{\mu}\frac{\partial \bar{w}}{\partial x} + \frac{(\tau_{xz})_0 - \mu\dfrac{\partial w_0}{\partial x}}{p+\mu/\eta} \\ \bar{\tau}_{yz} &= \bar{\mu}\frac{\partial \bar{w}}{\partial y} + \frac{(\tau_{yz})_0 - \mu\dfrac{\partial w_0}{\partial y}}{p+\mu/\eta}\end{aligned}\right\} \quad (1a)$$

$$\frac{\partial}{\partial x}\bar{\tau}_{xz} + \frac{\partial}{\partial y}\bar{\tau}_{yz} = 0 \quad (2a)$$

$$\left.\begin{aligned}\bar{\tau}_{xz} &= 0 \quad \text{at} \quad x = 0 \\ \bar{\tau}_{xz} &\to 0 \quad \text{as} \quad x \to \infty\end{aligned}\right\} \quad (3a)$$

$$\text{and} \quad \bar{\tau}_{yz} \to \frac{\tau_\infty}{p} \quad \text{as } y \to \infty \quad (4a)$$

where p is the Laplace transform variable,

$$\bar{\mu} = \frac{p}{\dfrac{p}{\mu} + \dfrac{1}{\eta}}$$

and the bars above w, τ_{xz} and τ_{yz} represent the Laplace transform w.r.to t. The solution of the boundary value problem (1a)-(4a) is easily seen to be :

$$\bar{w} = \frac{w_0}{p} + \frac{\tau_\infty y}{\eta p^2}$$

$$\bar{\tau}_{xz} = (\tau_{xz})_0 \Big/ \left(p + \frac{\mu}{\eta}\right)$$

$$\bar{\tau}_{yz} = \frac{(\tau_{yz})_0}{p+\mu/\eta} + \tau_\infty\left[\frac{1}{p} - \frac{1}{p+\mu/\eta}\right]$$

The inversion of the Laplace transforms now gives w, τ_{xz}, τ_{yz} in the absence of fault slip.

After fault slip, we have to satisfy the additional equations :

$$\left.\begin{aligned}&[\bar{w}] = \frac{u}{p} \text{ on } F \\ &\text{and } [\bar{\tau}_{yz}] = 0 = [\bar{\tau}_{xz}] \text{ on } F\end{aligned}\right\} \quad (5a)$$

The conditions (1a)—(4a) would be satisfied if.

$$\bar{w} = \bar{w}_1 + \bar{w}_2$$

$$\bar{\tau}_{xz} = (\bar{\tau}_{xz})_1 + (\bar{\tau}_{xz})_2, \quad (\bar{\tau}_{yz}) = (\bar{\tau}_{yz})_1 + (\bar{\tau}_{yz})_2$$

where $\bar{w}_1$, $(\bar{\tau}_{xz})_1$ and $(\bar{\tau}_{yz})_1$ satisfy the equations (1a) - (4a) as before, where, in (1a) - (4a) w_0, $(\tau_{xz})_0$, $(\tau_{yz})_0$ are to be replaced by w_p, $(\tau_{xz})_p$, $(\tau_{yz})_p$; $\bar{w}_2$, $(\bar{\tau}_{xz})_2$, $(\bar{\tau}_{yz})_2$, satisfy (2a), (3a), (5a) and the following equations (1b) and (4b) which replace (1a) and (4a) :

$$\left.\begin{aligned}\bar{\tau}_{xz} &= \bar{\mu}\frac{\partial \bar{w}}{\partial x} \\ \bar{\tau}_{yz} &= \bar{\mu}\frac{\partial \bar{w}}{\partial y}\end{aligned}\right\} \quad (1b)$$

$$\text{and} \quad \bar{\tau}_{yz} \to 0 \text{ as } y \to \infty \quad (4b)$$

The solution for $\bar{w}_1$, $(\bar{\tau}_{xz})_1$, $(\bar{\tau}_{yz})_1$ has been considered already. The boundary value problem for $\bar{w}_2$, $(\bar{\tau}_{xz})_2$, $(\bar{\tau}_{yz})_2$ is analogous to the the elastostatic problem of determining the displacements and stresses in an elastic half space due to a dislocation on F. The solution to the elastostatic problem has been given by Chinnery (1963). Hence using that solution we obtain after a little simplification the solution for $\bar{w}_2$, $(\bar{\tau}_{xz})_2$,$(\bar{\tau}_{yz})_2$. Finally the inversion of all the Laplace transforms which is fairly straight forward gives us the displacements and the stresses after fault slip.