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The effect of changing-elasticity on the stress concentration near a semi-infinite crack tip

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ABSTRACT One of the very convincing premonitory changes in earthquake predictions concerns the variation of the velocity ratio (v_p/v_s) preceding the occurrence of the event. The aim of the present investigation is to construct a the retical basis to evaluate the consequences of such changes on, say, the stress concentration near an already existing fault with a given static initial field. The change in elasticity is assumed over a limited part of the medium near the fault.

1. Introduction

Starting with the article on earthquake prediction by Aggarwal *et ai.* (1973) there has been considerable attention in the search for identifying the measurable parameters that may form the basis for predictions in future [*See*, for instance, Brady (1974, 1975, 1976 a, b), Gupta (1975), Steppe *et al.* (1977) and Wyss (1977).] Among the various causes one notices the possibility of elastic changes in the values of v_p and v_s as one of the easier measurable pre-effects. In the present paper we set out to look into a simple model of *SH*-motion where a static stress field in a medium containing a semi-infinite crack is perturbed by the changes in the shear wave velocity in the neighbourhood of the crack tip. The analysis aims at calculating the increase in the stress concentration near the tip.

2. Statement of the problem

Consider the anti-plane motion and assume that a crack occupies the region $y=0$, $x<0$ of the $x=y$ plane. The displacement W in the anti-plane direction satisfies the equation of motion:

$$\rho \frac{\partial^2 W}{\partial t^2} = \frac{\partial}{\partial x}\left(\mu \frac{\partial W}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial W}{\partial y}\right) \tag{1}$$

Assume that

$$\begin{aligned} \mu &= \mu_0 + \mu_1(x, y, t) \\ \rho &= \rho_0 + \rho_1(x, y, t) \end{aligned} \tag{2}$$

where μ_0, ρ_0 are the initial values of shear modulus and density while μ_1, ρ_1 are their perturbed values for $t>0$. Similarly, let the initial static field be $W_0(x, y)$ and the perturbed field be $W_1(x, y, t)$ so that $W = W_0 + W_1$. It is assumed for our analysis that all the perturbations are small so that their products and squares can be ignored. The equation of motion (1) can then be written as :

$$(\mu_0/\rho_0) \nabla^2 W_1 = \frac{\partial^2 W_1}{\partial t^2} - S(x, y, t) \tag{3}$$

where,

$$S(x, y, t) = \left\{\frac{\partial \mu_1}{\partial x}\frac{\partial W_0}{\partial x} + \frac{\partial \mu_1}{\partial y}\frac{\partial W_1}{\partial y}\right\}\frac{1}{\mu_0} \tag{4}$$

Apply Fourier transform-exponential-in x and t with parametrs ξ and ω to get :

$$\frac{d^2 \bar{W}_1}{dy^2} - \nu_0^2 \bar{W}_1 = -\bar{S}(\xi, y, \omega) \tag{5}$$

$$\nu_0 = (\xi^2 - k_0^2)^{1/2},\ k_0 = \omega/c_0,\ c_0 = (\mu_0/\rho_0)^{1/2}$$

Taking Re. $\nu_0 \geqslant 0$, the solution for $y \geqslant 0$ can be taken as :

$$\bar{W}_1 = A(\xi)\, e^{-\nu_0 y} + \frac{1}{2\nu_0}\left[e^{\nu_0 y}\int_y^\infty e^{-\nu_0 y'}\, \bar{S}(\xi, y'\, \omega)\, dy' + e^{-\nu_0 y}\int_0^y e^{-\nu_0 y}\, \bar{S}(\xi, y', \omega)\, dy' \right] \tag{6}$$

Note that we implicitly assume that $S(x, y, t)$ is assymmetric about $y=0$ which means that W_0 is assymmetric but μ_1 is symmetric in y. Other situations can be handled on the same approach without special difficulty.

Next apply the boundary conditions that :

(*i*) the shear stress $\tau_{zy}=0$ on $y=0$, $x<0$;

(*ii*) the displacement $W_1=0$ on $y=0$, $x>0$ due to assymmetry.

These conditions lead to the following relations:

$$A(\xi)\,\nu_0 - \frac{1}{2}\int_0^\infty e^{-\nu_0 y'}\,\bar{S}(\xi, y', \omega)\,dy' = -\,q_1^+(\xi)$$

$$A(\xi) + \frac{1}{2\nu_0}\int_0^\infty e^{-\nu_0 y'}\,\bar{S}(\xi, y', \omega)\,dy' = p_1^-(\xi) \tag{7}$$

where $q_1^+(\xi)$ and $p_1^-(\xi)$ are typical Wiener-Hopf functions analytic in the regions Im. $\xi \gtrless 0$ respectively.

Elimination of $A(\xi)$ from the above equations leads to the relation

$$\nu_0 p_1^-(\xi) + q_1^+(\xi) = \int_0^\infty e^{-\nu_0 y'}\,\bar{S}(\xi, y', \omega)\,dy' \tag{8}$$

This can be re-written in the form :

$$\sqrt{\xi-k_0}\;p_1^-(\xi) - m^-(\xi) = -\frac{q_1^+(\xi)}{\sqrt{\xi+k_0}} + m^+(\xi) = 0 \text{ (say)} \tag{9}$$

where,

$$m(\xi) = m^+(\xi) + m^-(\xi) = \frac{1}{\sqrt{\xi+k_0}}\int_0^\infty e^{-\nu_0 y'}\,\bar{S}(\xi, y', \omega)\,dy' \tag{10}$$

3. Expression for the stress concentration

The function $S(x, y, t)$ which depends on the assumed static field and assumed variations in the shear modulus, is supposed to be known. Even viscoelastic case may be assumed without any analytical complexity.

To obtain the stress concentration let us then assume that the function $m^+(\xi)$ can be found by the standard method as an integral in the lower half of the complex ξ-plane. Further, assume that this integral can be asymptotically expanded in the form :

$$m^+(\xi) \approx \frac{a_0}{\xi} + \frac{a_1}{\xi^2} + \ldots, \quad (\xi \to +\infty) \tag{11}$$

with the coefficients $a_0, a_1, \ldots\ldots$ known for each model tried.

Then from (9) it can be shown that :

$$q_1^+(\xi) \approx \frac{a_0}{\xi^{1/2}}, \quad \xi \to +\infty \tag{12}$$

which is the same as the expression for the stress concentration near the crack tip given by

$$\tau_{zy}(x, 0) \approx \frac{\mu a_0 b_0}{x^{1/2}}, \; x \to 0^+, \tag{13}$$

$$\left\{ b_0 = \frac{1}{(\pi i)^{1/2}} \right\}$$

Thus the required stress concentration can be achieved only when the Wiener-Hopf function $m^+(\xi)$ is first worked out. In the following section a very simple case is worked out for illustration.

4. An illustrative example

Consider a very simple example with the assymmetric distribution of the static field W_0 as

$$W_0 = Ay \tag{14}$$

and take the symmetric change in shear modulus to be :

$$\mu_1 = \frac{1}{2}\,\mu_{00}\,y^2 e^{-|x|} \left\{ \mu_{00} = \mu_{00}(t) \right\} \tag{15}$$

where, for simplicity, we have not restricted the region of elasticity-change. For this choice we get :

$$S(x, y, t) = \frac{\mu_{00} Ay}{\mu_0}\,.\,e^{-|x|} \tag{16}$$

$$\bar{S}(\xi, y, \omega) = \frac{\bar{\mu}_{00}(\omega)}{\mu_0 \pi}\;Ay\,.\,\frac{1}{\xi^2+1}$$

so that

$$m(\xi) = \frac{\bar{\mu}_{00}(\omega)}{\mu_0 \pi}\,A\left\{\frac{1}{\nu_0^2\sqrt{\xi+k_0}\,(\xi^2+1)}\right\} \tag{17}$$

It is easily shown that for this case

$$m^+(\xi) = \frac{1}{\sqrt{\xi+k_0}}\left[\frac{1}{i(\xi+i)} - \frac{1}{k_0(\xi+k_0)}\right] \times \frac{A\{\bar{\mu}_{00}(\omega)/\mu_0\pi\}}{2(1+k_0^2)} + \frac{1}{2k_0(1+k_0^2)} \times \frac{1}{(\xi-k_0)}\left\{\frac{1}{\sqrt{\xi+k_0}} - \frac{1}{\sqrt{2k_0}}\right\}\frac{A\bar{\mu}_{00}(\omega)}{\mu_0\pi} - \frac{1}{2i(1+k_0^2)}\cdot\frac{1}{(\xi-i)}\left\{\frac{1}{\sqrt{\xi+k_0}} - \frac{1}{\sqrt{i+k_0}}\right\}\frac{A\bar{\mu}_{00}(\omega)}{\mu_0\pi} \tag{18}$$

so that for $\xi \rightarrow +\infty$

$$m^{+}(\xi) \approx \frac{a_0}{\xi}, \tag{19}$$

with

$$a_0 = \frac{A\,\bar{\mu}_{00}(\omega)}{2\mu_0\,\pi\,(1+k_0^2)} \left\{\frac{1}{i\sqrt{i+k_0}} - \frac{1}{\sqrt{2}k_0^{3/2}}\right\} \tag{20}$$

The stress concentration becomes

$$\tau_{zy}(x,0) \approx \frac{A\bar{\mu}_{00}(\omega)}{2\pi\,(1+k_0^2)\,(\pi i)^{1/2}} \left\{\frac{1}{i\sqrt{i+k_0}} - \frac{1}{\sqrt{2}\,k_0^{3/2}}\right\} \frac{1}{x^{1/2}},\ (x \rightarrow 0^{+}) \tag{21}$$

The inversion w.r.t. time can be carried out by the usual methods both for elastic and visco-elastic cases.

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References

Aggarwal, Y. P. *et al.*, 1973, *Nature,* **241,** pp. 101-104

Brady, B. T., 1974, *Pure and Applied Geophys.*, **112,** pp. 701-726.
1975, *Ibid.*, **113**, pp. 149-169.
1976a, b, *Ibid.*, **114,** pp. 119-139 and pp. 1031-1082.

Gupta, I. N., 1975, *Bull. seismol. Soc. Amer.*, **65,** pp. 425-437.

Steppe, A. *et al.*, 1977, *Bull. seismol. Soc. Amer.*, **67,** pp 1075-1090.

Wyss, M., 1977, *Bull. seismol. Soc. Amer.*, **67,** pp. 1091 1098.