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A mechanism of stress accumulation near a strike-slip fault

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ABSTRACT. A vertical strike-slip fault, situated in visco-elastic layer, representing the lithosphere, is considered. The upper part of the fault is assumed to remain locked, except during an earthquake, when it slips. It is assumed that the lower part of the fault slips freely, so that the parts of the lithosphere on opposite sides of the lower part of the fault slip past each other. It is also assumed that tectonic forces maintain a shear stress far away from the fault. Exact solutions are obtained for the displacements and stresses in the system and it is shown that gradual accumulation of shear stress would occur in the neighbourhood of the fault. There would be considerable amplification of shear stress on the locked part of the fault, leading finally to a sudden slip of the upper part of the fault under suitable circumstances. The mathematical results are compared with some relevant observations on the surface deformations in the neighbourhood of strike-slip faults. It is shown that such comparison can be used to obtain estimates of the probable time of sudden fault slip, if sufficient data are available, and would also lead to estimates of the ratio of the effective viscosity of the lithosphere in the neighbourhood of the fault and the shear stress maintained by tectonic forces far away from the fault. It is also shown that the results are likely to be useful in obtaining greater insight into the problem of earthquake prediction and in estimating the changes of seismic risk with time near an active fault.

1. Introduction

In recent years the problem of earthquake prediction has attracted widespread attention among the seismologists and it is hoped that effective programme of earthquake prediction would become feasible in the near future. In this connection it would be useful to have a better understanding of the mechanism of stress accumulation in the neighbourhood of the faults, which may lead eventually to a sudden fault slip generating an earthquake. If it is possible to devise suitable theoretical models which incorporate the essential features of the mechanism of stress accumulation, then an effective quantitative analysis of the stress accumulation and fault slip would be possible. For strike-slip faults, some theoretical models have been developed by Turcotte and Spence (1974), Savage (1975), Spence and Turcotte (1976) and Budiansky and Amazigo (1976). Turcotte and Spence (1974), Savage (1975) and Spence and Turcotte (1976) consider models in which the stress accumulation near locked faults is taken to be due to relative motion of the parts of the lithosphere on the two sides of the fault. The mechanism of this relative motion is not considered in such models. In the model considered by Budiansky and Amazigo (1976) a locked vertical strike slip fault situated in a visco-elastic layer which is free to slide over the material below it is considered. A constant shear stress is taken to be maintained in the layer far away from the fault. It is shown that accumulation of shear stress would occur in the layer due to the creep of the material. But the accumulation of shear stress does not exceed the shear stress maintained far away from the fault. We note, however, that fault slip would normally require fairly high shear stress and hence a large shear stress has to be maintained far away from the fault to cause sufficient stress accumulation for fault slip. The evidence for the existence of tectonic forces capable of maintaining such large shear stresses does not appear to be adequate. Moreover the observed coseismic fault slip on shallow strike slip faults, such as the San Andreas fault, is often found to extend to depths of 10 km or 15 km only. One possible explanation of this phenomenon would be that although the actual fault extends from the surface to the boundary between the lithosphere and the asthenosphere, only the upper part of the fault (to a depth of 10 km to 15 km) remains locked and the lower part of the fault slips freely as stress accumulates. Such an assumption

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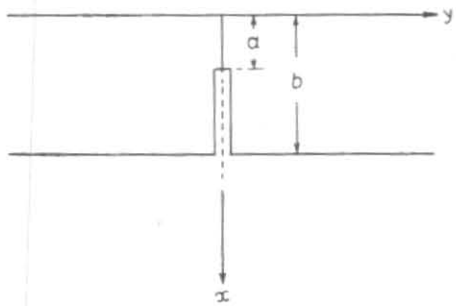

Fig. 1. Section of the lithospheric layer containing the fault by the plane $z=0$

was made by Turcotte and Spence (1974). However they considered an elastic lithosphere. In order to explain the accumulation of large shear stresses on the fault and the fact that the coseismic fault slip does not extend to depths greater than 10 km to 15 km, we consider a visco-elastic model of the lithosphere, free to slide over the asthenosphere below it. We consider a vertical strike slip fault, the upper part of which remains locked while the lower part slips continuously. The upper part slips suddenly, generating an earthquake if and only if shear stress across it becomes sufficiently large. We suppose that tectonic forces maintain a constant shear stress far away from the fault and study the accumulation of shear stress in the system.

2. Formulations

We take b to be the thickness of the lithospheric layer. Let a ($< b$) be the depth of the locked upper part of the vertical strike slip fault. We consider a fault whose length is large compare to its depth. We introduce cartesian co-ordinates (x, y, z) with the free surface as the plane $x=0$, the plane of the fault as the plane $y=0$ and the z—axis along the trace of the fault on the free surface. For a long fault we assume that the displacement and stress are independent of z. We take $w\ (x, y, t)$ to be the displacement parallel to the z—axis, associated with strike slip faulting. We represent the relevant stress components by τ_{xz} and τ_{yz}. We assume that the lithospheric layer behaves like a linearly visco-elastic material of the Maxwell type for the process of stress accumulation we are considering. The constitutive equations may then be written as:

$$\left.\begin{aligned} &\frac{1}{\mu}\frac{\partial}{\partial t}(\tau_{xz}) + \frac{\tau_{xz}}{\eta} = \frac{\partial^2 \omega}{\partial x \partial t} \\ \text{and}\quad &\frac{1}{\mu}\frac{\partial}{\partial t}(\tau_{yz}) + \frac{\tau_{yz}}{\eta} = \frac{\partial^2 \omega}{\partial y \partial t} \end{aligned}\right\} \qquad (1)$$

as in Budiansky and Amazigo (1976). We note in this connection that the process of the long term creep of the lithosphere is likely to be characterised by secondary creep, for which we may assume the Nabarro-Herring creep mechanism (Heard 1976). For such a mechanism, the material would have Newtonian viscosity, and the constitutive Eqn. (1) would be reasonable.

We consider steady accumulation of shear stress in the model, leaving out the period just following fault slip when the elastic waves generated by faulting are present near the fault. Both before fault slip and after the elastic waves have propagated far away, the inertial forces would be very small and can be neglected. This assumption was made by Budiansky and Amazigo (1976) and also by Braslau and Lieber (1968), Rosenman and Singh (1973a, 1973b), Singh and Rosenman (1974), Nur and Mavko (1974), Barker (1976) and Rundle and Jackson (1977), who studied post-seismic stress relaxation following fault slip. For such quasi-steady deformations, the stresses satisfying the relation :

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0 \qquad (2)$$

We assume that the shear stress τ_{yz} has a constant value τ_∞ far away from the fault. We note that for the system we are considering the boundary conditions given by :

$$\left.\begin{aligned} &\tau_{yz} \longrightarrow \tau_\infty && \text{as } y \longrightarrow \infty \\ &\tau_{xz} = 0 && \text{for } x = 0,\ b \\ &\omega = 0 && \text{on } y = 0,\ 0 < x < a \\ &\tau_{yz} = 0 && \text{for } y = 0,\ a < x < b \end{aligned}\right\} \qquad (3)$$

We also assume that at the time $t=0$ from which we study the deformation of the system, there is an initial stress and displacement field $\omega_0, (\tau_{xz})_0, (\tau_{yz})_0$ satisfying the conditions (1), (2), (3). Fig. 1 shows the section of the lithospheric layer (with the fault) by the plane $z=0$.

3. Displacements and stresses in the lithospheric layer

To obtain the displacements and stresses, we take Laplace transforms of both sides of the equations (1)—(3) with respect to t. The resulting boundary value problem can be solved, as explained in the Appendix, using the results given by Turcotte and Spence (1974). On inverting the

Laplace transforms, we finally obtain exact solutions in closed form for the displacements and stresses given by, using the notation $z_1 = x + iy$,

$$\left.\begin{aligned}
\omega(x,y,t) &= \frac{2b\tau_\infty}{\pi\eta}\, t\, Re \left\{ \log_e \left[\frac{\sin\left(\frac{\pi z_1}{2b}\right) + \sqrt{\sin^2\left(\frac{\pi z_1}{2b}\right) - \sin^2\left(\frac{\pi a}{2b}\right)}}{\sin\left(\frac{\pi a}{2b}\right)} \right] \right\} \\
&\quad + \omega_0(x,y,0), \\
\tau_{xz}(x,y,t) &= \tau_\infty \left(1 - e^{-\frac{\mu}{\eta}t}\right) . Re \left[\frac{\cos\left(\frac{\pi z_1}{2b}\right)}{\sqrt{\sin^2\left(\frac{\pi z_1}{2b}\right) - \sin^2\left(\frac{\pi a}{2b}\right)}} \right] \\
&\quad + e^{-\frac{\mu}{\eta}t} . \tau_{xz}(x,y,0), \\
\tau_{yz}(x,y,t) &= -\tau_\infty \left(1 - e^{-\frac{\mu}{\eta}t}\right) . Im \left[\frac{\cos\left(\frac{\pi z_1}{2b}\right)}{\sqrt{\sin^2\left(\frac{\pi z_1}{2b}\right) - \sin^2\left(\frac{\pi a}{2b}\right)}} \right] \\
&\quad + e^{-\frac{\mu}{\eta}t} . \tau_{yz}(x,y,0)
\end{aligned}\right\} \quad (4)$$

Hence the rate of accumulation of shear strain on the free surface is given by :

$$\left[\frac{\partial}{\partial t}\left(\frac{\partial \omega}{\partial y}\right) \right]_{x=0} = \frac{\tau_\infty}{\eta} \left[\frac{\cosh\left(\frac{\pi y}{2b}\right)}{\sqrt{\sinh^2\left(\frac{\pi y}{4b}\right) + \sin^2\left(\frac{\pi a}{2b}\right)}} \right] \quad (5)$$

On the free surface near the fault the rate of accumulation of shear strain is

$$\left[\frac{\partial}{\partial t}\left(\frac{\partial \omega}{\partial y}\right) \right]_{\substack{x=0 \\ y=0}} = \frac{\tau_\infty}{\eta} \cdot \frac{1}{\sin^2\left(\frac{\pi a}{2b}\right)} \quad (6)$$

From the solution (4) we find that the initial stresses relax completely with time and tend to zero as $t \to \infty$. The initial displacement remains in the system. Under the applied stress τ_∞ the material of the layer creeps as indicated by the monotonically increasing displacement w. This leads to an increasing accumulation of shear stress. The stress τ_{yz} finally tends to

$$\tau_\infty \; I_m \left[\cos\left(\frac{\pi z}{2b}\right) \Big/ \sqrt{\sin^2\left(\frac{\pi z}{2b}\right) - \sin^2\left(\frac{\pi a}{2b}\right)} \right] \quad (7)$$

If this shear stress exceeds the critical shear stress required for fault slip at any point of the locked part of the fault then the locked part would slip when τ_{yz} reaches this critical value.

The shear stress τ_{yz} across the locked part of the fault is given by

$$\left[\tau_{yz}\right]_{y=0} = \tau_\infty \left(1 - e^{-\frac{\mu}{\eta}t}\right) \left[\frac{\cos\left(\frac{\pi x}{2b}\right)}{\sqrt{\sin^2\left(\frac{\pi a}{2b}\right) - \sin^2\left(\frac{\pi x}{2b}\right)}} \right] + \tau_{yz}(x,0,0)\, e^{-\frac{\mu}{\eta}t},$$
$$(0 \leqslant x < a) \quad (8)$$

4. Discussion of the results and applications

We now try to apply these results to observe strike slip faults. For the San Andreas fault we take $a=10$ km and $b=100$ km, where the value of b is taken to be equal to the commonly used value of the thickness of the lithosphere. We note that it has been reported by Savage and Burford (1973) that the average rate of the strain accumulation near a locked section of the San Andreas fault near Ross Mountain has been found to be 0.55μ strain/year. On equating this to the expression for strain accumulation on the surface given by (6) we obtain the value of τ_∞/η. We note in this connection that if sufficient data are available on the rate of strain accumulation on the free surface at different distances from the fault we may use the equation (5) to obtain a more reliable estimate of τ_∞/η. To choose a suitable value of η for our model we note that Cathles III (1975) has considered theoretical models for the post-glacial uplift in Fennoscandia and Canada, and has compared the theoretically calculated results with obsevational data. He estimates that the effective viscosity of the lower lithosphere for slow deformations is of the order of 10^{21} poise. Budiansky and Amazigo (1976) also estimate a similar value of η. We therefore use the estimate $\eta=10^{21}$ poise. From the estimate of τ_∞/η obtained earlier, we can obtain an estimate for τ_∞.

We now study the changes in the shear stress $[\tau_{yz}]_{y=0}$ across the fault with time. We take $\eta=10^{21}$ poise, $\mu=3.78\times10^{11}$ dynes/cm² following Aki (1967), $a=10$ km and $b=100$ km. We study the change in the ratio :

$$R = \frac{[\tau_{yz}(x,\,y,\,t)]_{y=0} - [\tau_{yz}\,(x,\,0,\,0)]}{\tau_\infty} \tag{9}$$

With time, at different points on the locked part of the fault at different depths x. Here R gives the ratio of the stress accumulation on the fault to the stress τ_∞. In Fig. 2, we show the variations of this ratio R with time, assuming that the initial stress $\tau_{yz}\,(x,0,0)$ is small. The time t is in years and the numbers against the different curves give the depths of the points of the fault in km. It is found that there is considerable amplification of stress on the locked part of the fault. The amplification becomes greater with increase in depth and is very large near the lower end of the locked part. Even close to the free surface where the amplification is a minimum, τ_{yz} approaches a value which is greater than $6\,\tau_\infty$. Thus even if τ_∞ is small, the shear stress can become large on the locked part of the fault due to this amplification. Hence after a sufficient time the shear stress τ_{yz} on the locked fault may become sufficiently large to cause a sudden fault slip, resulting in an earthquake.

We now study the changes in the average value $\bar{R}$ of the ratio R on the fault, given by

$$\bar{R} = \frac{1}{a}\int_0^a R\,dx \tag{10}$$

which is easily obtained on using Eqn. (8).

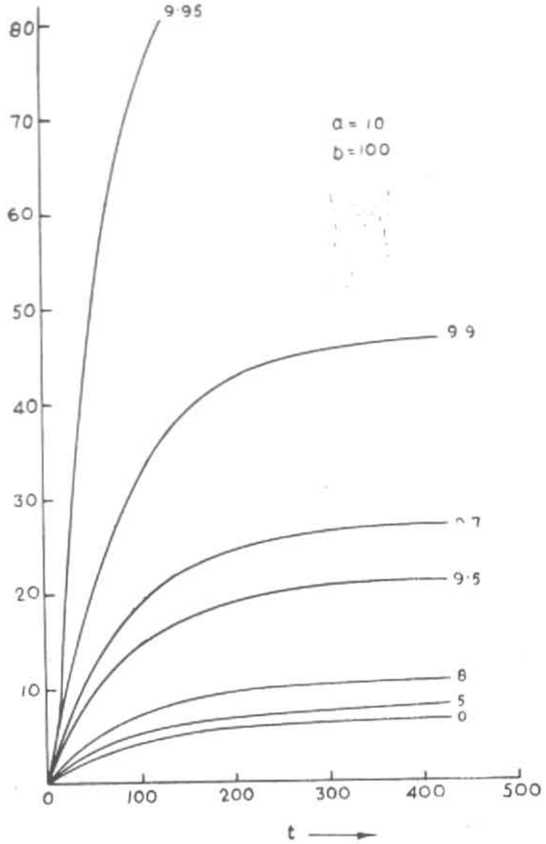

Fig. 2. Variation of (R) with increase in time. The time t is in years.

We find that $\bar{R}$ approaches a value greater than 10. If the average stress accumulation in the critical configuration for fault slip is less than $10\,\tau_\infty$ the fault would slip after a sufficient time, which can be determined from Fig. 3, if the average stress accumulation in the critical configuration is known. For example, if the average stress accumulation is $7\,\tau_\infty$ in the critical state, the fault would be expected to slip after 100 years. But if the average shear stress in the critical state is $9\,\tau_\infty$, the fault would slip after nearly 200 years. Thus we can obtain some estimates of the return times for slip on the fault and we can also estimate the changes in seismic risk with time.

The uncertainty about the average stress in the critical configuration can be reduced to some extent if we assume that fault slip would occur when the post-seismic stress accumulation becomes equal to the co-seismic stress drop. To estimate the co-seismic stress drop on the fault, we may use the result given by Chinnery (1964). However, in our model, the coseismic dislocation over the fault may not be uniform as in Chinnery (1964), since there may be some coseismic slip in the lower part of the fault, and it may be necessary to take into account the case of a non-uniform dislocation, using the results given by Maruyama

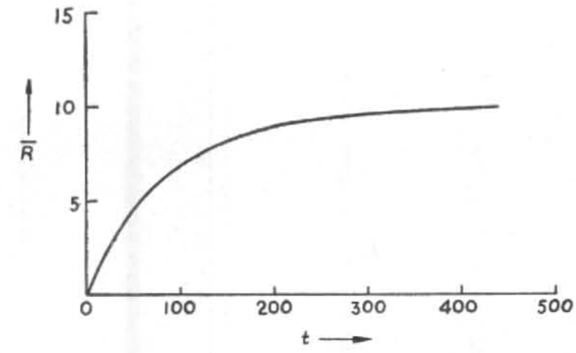

Fig. 3. Increase in R with time The time t is in years.

(1966). However, the detailed observational data needed to distinguish between different possible distributions of the coseismic dislocation on the fault do not appear to be available. When such data are available, it would be possible to obtain definite estimates for the average co-seismic stress drop, and this would enable us to estimate more definitely the time of slip on the fault, using the results given in Fig. 3. Thus, if sufficient observational data on ground deformation are available, the results we have obtained may be used for long-term earthquake prediction and for the estimation of the time-dependence of seismic risk.

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Appendix

Taking Laplace transform of (1), (2) and (3) with respect to t we obtain the relations

$$\left.\begin{aligned} \bar{\tau}_{xz} &= \bar{\mu}\frac{\partial \bar{\omega}}{\partial x} + \frac{(\tau_{xz})_0 - \mu\dfrac{\partial \omega_0}{\partial x}}{p + \dfrac{\mu}{\eta}} \\ \bar{\tau}_{yz} &= \bar{\mu}\frac{\partial \bar{\omega}}{\partial y} + \frac{(\tau_{yz})_0 - \mu\dfrac{\partial \omega_0}{\partial y}}{p + \dfrac{\mu}{\eta}} \end{aligned}\right\} \quad (1a)$$

$$\frac{\partial \bar{\tau}_{xz}}{\partial x} + \frac{\partial \bar{\tau}_{yz}}{\partial y} = 0 \quad (2a)$$

$$\left.\begin{aligned} \bar{\tau}_{xz} &= 0 \text{ for } x = 0, b \\ \bar{\omega} &= 0 \text{ on } y = 0, 0 < x < a \\ \bar{\tau}_{yz} &= 0 \text{ for } y = 0, a < x < b \end{aligned}\right\} \quad (3a)$$

$$\bar{\tau}_{yz} \longrightarrow \frac{\tau_\infty}{p} \text{ as } y \longrightarrow \infty \quad (4a)$$

where p is the Laplace transform variable and

$$\bar{\mu} = \frac{p}{\dfrac{p}{\mu} + \dfrac{1}{\eta}} \quad (5a)$$

The conditions (1a) to (4a) would be satisfied if:

$$\bar{\omega} = \bar{\omega}_1 + \bar{\omega}_2$$

$$\bar{\tau}_{xz} = (\bar{\tau}_{xz})_1 + (\bar{\tau}_{xz})_2$$

$$\bar{\tau}_{yz} = (\bar{\tau}_{yz})_1 + (\bar{\tau}_{yz})_2$$

where $\bar{\omega}_1$, $(\bar{\tau}_{xz})_1$, $(\bar{\tau}_{yz})_1$ satisfy the relations (1a) — (3a) and the condition :

$$\bar{\tau}_{yz} \longrightarrow 0 \text{ as } y \longrightarrow \infty \quad (4b)$$

which replaces (4a) and $\bar{\omega}_2$, $(\bar{\tau}_{xz})_2$, $(\bar{\tau}_{yz})_2$ satisfy the relation (2a), (3a) and (4a) and the following relations which replace (1a), :

$$\left.\begin{aligned} \bar{\tau}_{xz} &= \bar{\mu}\frac{\partial \omega}{\partial x} \\ \bar{\tau}_{yz} &= \bar{\mu}\frac{\partial \omega}{\partial y} \end{aligned}\right\} \quad (1b)$$

The boundray value problem for $\bar{\omega}_2$, $(\bar{\tau}_{xz})_2$, $(\bar{\tau}_{yz})_2$ can be solved on using the results given by Turcotte and Spence (1974) for a fault in an elastic lithosphere. The boundary value problem for $\bar{\omega}_1$, $(\bar{\tau}_{xz})_1$, $(\bar{\tau}_{yz})_1$ can also be obtained without much difficulty in the fault :

$$\left.\begin{aligned} \bar{\omega}_1 &= \frac{\omega}{p} \\ (\bar{\tau}_{xz})_1 &= \frac{(\tau_{xz})_0}{p + \dfrac{\mu}{\eta}} \\ (\bar{\tau}_{yz})_1 &= \frac{(\tau_{yz})_0}{p + \dfrac{\mu}{\eta}} \end{aligned}\right\} \quad (2b)$$

Finally on inverting the Laplace transform we obtain the exact solution of the displacements and stresses.