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SH waves across a column sandwiched in an inhomogeneous medium

UMESH KUMARI

Kurukshetra University, Kurukshetra

ABSTRACT. The propagation of love waves across a vertical column sandwiched in a medium of different properties has been studied. Three different models are considered : (*i*) an infinite column in a semi-infinite medium, (*ii*) a column in a layer resting over a rigid bottom and (*iii*) a column in a layer resting over a semi-infinite elastic medium in welded contact with it. In each case the elastic properties vary vertically but this variation is the same both inside and outside the column. Frequency equations have been obtained for the regions inside and outside the column and dispersion curves drawn. As expected the wave periods change as the waves pass into and out of the column. Displacements in these regions have been obtained and presented graphically for points in the free surface. Reflection and transmission coefficients for normal incidence at vertical discontinuities have also been obtained.

2. Introduction

Theoretical studies of love waves in a vertically inhomogeneous medium have already been discussed by many investigators. However, the effect of lateral inhomgeneity on surface wave dispersion has not been studied in sufficient details. Lateral inhomogeneity may also be discussed by considering vertical discontinuities in the earth. Sinha (1964), Alsop (1966) discussed reflection and transmission of *SH* waves incident normally at vertical discontinuity.

In this paper we discuss the propagation of love waves in the following thee models:

(*i*) An infinite column sandwiched in a half-space of different elastic properties, the vertical variation in the column and the half space being the same.

(*ii*) An infinite column in a layer overlying a rigid bottom, the vertical variation being the same in the column and the layer.

(*iii*) The same as (*ii*) with the half space elastic and inhomogeneous.

Reflection and transmission coefficients for normal incidence at vertical discontinuities have been obtained and numerical calculations made for models (*ii*) and (*iii*). Dispersion curves have been drawn for the regions inside as well as outside the column. Displacements in these regions have also been obtained and graphically presented for points in the free surface.

It may be observed at the onset that, in a similar problem Alsop (1966) has taken a number of modes in the transmitted and reflected waves and has tried to obtain an approximate solution of the problem by satisfying the boundary conditions at vertical discontinuity as far as possible in the least square sense. This is not necessary here as ours is an exact solution which satisfies the appropriate wave equations in the various regions and the boundary conditions at both the free surface and the vertical interfaces.

2. Equation of motion and its solution.

The geometry of the problem under consideration is shown in Fig. 1. The configuration is referred to a rectangular co-ordinate system with z-axis directed downward and xy plane coinciding with the horizontal free surface. We specialize to the two dimensional model in which the paramters λ, μ, ρ and the displacements are independent of y-co-ordinate.

The equation of small motion of the *SH*-type is

$$\frac{\partial}{\partial x}\left(\mu\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial V}{\partial z}\right)=\rho\frac{\partial^2 V}{\partial t^2} \quad (2.1)$$

If we put

$$V=\mu^{-1/2}\,U \quad (2\cdot2)$$

Eqn. (2.1) reduces to :

$$\mu\nabla^2 U+\tfrac{1}{2}\left[\frac{1}{2\mu}(\mu_x^2+\mu_z^2)-\mu_{xx}-\mu_{zz}\right]U=\rho\,U_{tt} \quad (2\cdot3)$$

where, $\mu_x=\dfrac{\partial\mu}{\partial x}$, $\mu_{xx}=\dfrac{\partial^2\mu}{\partial x^2}$ etc.

Let the elastic parameters of the medium be specified by :

$$\mu(x, z) = \mu_0\, q(z)\, h(x), \quad \rho(x, z) = \rho_0\, r(z)\, p(x) \tag{2.4}$$

with

$$h(x) = h_0 + (h_1 - h_0)\, B(x), \quad p(x) = p_0 + (p_1 - p_0)\, B(x) \tag{2.5}$$

where, $\mu_0, \rho_0, h_0, h_1, p_0, p_1$ are constants, $B(x)$ is the unit Box function for the interval $-a \leqslant x \leqslant a$, *i.e.*,

$$B(x) = \begin{cases} 1 & \text{for } -a \leqslant x \leqslant a \\ 0 & \text{for all other } x. \end{cases}$$

Substituting for μ and ρ from (2.4) in (2·3), and assuming the solution of this equation in the form :

$$U(x,z,t) = \begin{cases} Z_1(z)\,[Ae^{i(kx-\omega t)} + Be^{-i(kx+\omega t)}], & \text{for } x < -a, \\ Z_2(z)\,[Ce^{i(k_1 x-\omega t)} + De^{-i(k_1 x+\omega t)}], & \text{for } -a \leqslant x \leqslant a, \\ Z_3(z)\, Fe^{i(kx-\omega t)}, & \text{for } x > a \end{cases} \tag{2.6}$$

we see that Z_1, Z_2, Z_3, satisfy the equations

$$\frac{d^2 Z_{1,3}}{dz^2} + \left[\frac{\omega^2 p_0\, r(z)}{\beta_0^2 h_0\, q(z)} - k^2 + \frac{1}{4}\left(\frac{q_z}{q}\right)^2 - \frac{1}{2}\frac{q_{zz}}{q} \right] Z_{1,3} = 0, \text{ for } x>a, x<-a, \tag{2.7a}$$

$$\frac{d^2 Z_2}{dz^2} + \left[\frac{\omega^2 p_1\, r(z)}{\beta_0^2 h_1\, q(z)} - k_1^2 + \frac{1}{4}\left(\frac{q_z}{q}\right)^2 - \frac{1}{2}\frac{q_{zz}}{q} \right] Z_2 = 0, \text{ for } -a\leqslant x\leqslant a, \tag{2.7b}$$

where $\beta_0 = (\mu_0/\rho_0)^{1/2}$, A, B, C, D, F are the amplitudes of the wave motion, ω is the angular frequency, k is the wave number same for all z, and all x in $x > a$, $x < -a$ and k_1 is the wave number same for all z, and all x in $-a \leqslant x \leqslant a$.

The only non-zero component of stress acting at vertical discontinuities is p_{xy} and the boundary conditions on these surfaces of discontinuity are

$$[p_{xy}] = 0 \text{ at } x = \pm a \text{ for all } z \text{ and } t, \quad [V] = 0 \text{ at } x = \pm a \text{ for all } z \text{ and } t. \tag{2·8}$$

The square brackets in (2.8) denote the change in the value of a quantity across the surface of discontinuity. The continuity of stress and displacement for all z requires that $Z_1(z)$, $Z_2(z)$, $Z_3(z)$ must be the same for the regions $x<-a$, $-a\leqslant x\leqslant a$ and $x > a$. These will be the sme if the differential equations satisfied by them, *i.e.* (2·7a) (2·7b) will be the same and this requirement leads

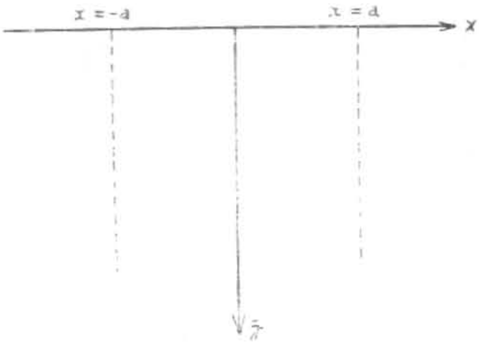

Fig. 1.

us to the condition :

$$\frac{\omega^2 p_0\, r(z)}{\beta_0^2 h_0\, q(z)} - k^2 = \frac{\omega^2 p_1\, r(z)}{\beta_0^2 h_1\, q(z)} - k_1^2 \tag{2.9}$$

Since $\omega = kc = k_1 c_1$, where c is the phase velocity for the regions $x>a$, $x<-a$ and c_1 for the region $-a\leqslant x\leqslant a$, Eqn. (2.9) can be written as :

$$k_1^2 = k^2 \left[1 + \frac{r(z)}{q(z)} \frac{c^2}{\beta_0^2} \left(\frac{p_1}{h_1} - \frac{p_0}{h_0} \right) \right] \tag{2.10}$$

This equation gives k_1 as a function of z, but k_1 is assumed to be independent of z and this condition will be met only if $r(z)=q(z)$. Therefore this analysis is valid only if shear wave velocity is independent of z. Eqn. (2·10) will now give the change in wave number of the transmitted wave in the column $-a\leqslant x \leqslant a$.

Substituting for V from (2·2) and (2·6) in (2·8) we get four equations and solving these equations for B, C, D and F we get :

$$B = A\, \frac{2i(h_1^2 k_1^2 - h_0^2 k^2)}{\triangle} \sin(2k_1 a)\, e^{-2ika},$$

$$C = A\, \frac{2k(h_0 h_1)^{1/2}(h_0 k + h_1 k_1)}{\triangle}\, e^{-i(k+k_1)a} \tag{2.11}$$

$$D = A\, \frac{2k(h_0 h_1)^{1/2}(h_1 k_1 - h_0 k)}{\triangle}\, e^{-i(k-k_1)a},$$

$$F = A\, \frac{4kk_1 h_0 h_1}{\triangle}\, e^{-2ika}$$

where,

$$\triangle = 4kk_1 h_0 h_1 \cos 2k_1 a - 2i(h_0^2 k^2 + h_1^2 k_1^2) \sin 2k_1 a,$$

Writing

$$\frac{B}{A} = Re^{i\theta}, \quad \frac{F}{A} = R_1 e^{i\phi} \tag{2.12}$$

We can verify that the relation :

$$R^2 + R_1^2 = 1, \qquad (2\cdot13)$$

is satisfied which shows that there is no accumulation of energy in the column. Further, we can see that perfect transmission is possible when

$$\sin\ 2k_1 a = 0$$

$$i.e.,\quad k_1 a = \frac{n\pi}{2},\ n = 0, 1, 2, \qquad (2\cdot14)$$

Using the relation $\omega = k_1 c_1$, we find that perfect transmission is possible at the frequencies:

$$\omega = \frac{n\pi}{2a}\, c_1 \qquad (2.15)$$

In such a case there is no reflected wave and amplitude of transmitted wave is maximum being equal to the amplitude of the incident wave.

In any particular problem, μ is a given function of z. Substituting its value in (2.7*a*), it may be possible to solve the resulting differential equation in terms of known functions. The solution of (2·7b) can then be obtained from the solution of (2.7a) merely by replacing the constants h_0, p_0, k by h_1, p_1 and k_1.

For mathematical simplification of the problem we now assume that $q(z)$ is such that

$$\tfrac{1}{4}\left(\frac{q_z}{q}\right)^2 - \tfrac{1}{2}\,\frac{q_{zz}}{q} = -\,b_0^2 \qquad (2.16)$$

where b_0 is a constant. The solution of (2·7*a*) and (2·7b) is

$$Z = A_1 e^{s_1 z} + B_1 e^{-s_1 z} \qquad (2\cdot17)$$

where A_1, B_1 are arbitrary constants, s_1 for the region $x < -a$, $x > a$ is given by

$$s_1 = \begin{cases} \left(k^2 + b_0^2 - \dfrac{\omega^2 p_0}{\beta_0^2 h_0}\right)^{1/2}, \\ \quad\text{when}\quad (k^2 + b_0^2) > \dfrac{\omega^2 p_0}{\beta_0^2 h_0}, \\ -i\left(\dfrac{\omega^2 p_0}{\beta_0^2 h_0} - k^2 - b_0^2\right)^{\frac{1}{2}}, \\ \quad\text{when}\quad (k^2 + b_0^2) < \dfrac{\omega^2 p_0}{\beta_0^2 h_0}, \end{cases} \qquad (2\cdot18)$$

and for $-a<x<a$ is the same as (2.18) where p_0, h_0, k have been replaced by p_1, h_1, k_1.

Eqn. (2·16) may be integrated to give $q(z)$as $e^{\pm\delta z}$, $(1+\delta z.)^2$, $\sinh^2(\delta_1 z+\delta_2)$ (*see* Singh *et al.* 1976)

From Eqns. (2·2), (2·6) and (2·17) we obtain

$$V(x,z,t) = \frac{1}{[\mu_0 h_0 q(z)]^{1/2}}\,[A_1 e^{s_1 z} + B_1 e^{-s_1 z}]\times [Ae^{i(kx-\omega t)} + Be^{-i(kx+\omega t)}], \quad \text{for } x < -a,$$

$$= \frac{1}{[\mu_0 h_1 q(z)]^{1/2}}\,[A_1 e^{s_1 z} + B_1 e^{-s_1 z}]\times [Ce^{i(k_1 x-\omega t)} + De^{-i(k_1 x+\omega t)}] \quad \text{for } -a \leqslant x \leqslant a,$$

$$= \frac{1}{[\mu_0 h_0 q(z)]^{1/2}}\,[A_1 e^{s_1 z} + B_1 e^{-s_1 z}]\times F\, e^{i(kx-\omega t)},\ \text{for } x > a \qquad (2.19)$$

3. Love waves across a vertical column in an inhomogeneous half-space

In order that $V\to 0$ as $z\to\infty$, s_1 in Eqn (2·19) must be real and positive and $A_1=0$. The codition of stress free boundary leads to the equation :

$$\tfrac{1}{2}\left[\frac{1}{q(z)}\,\frac{dq}{dz}\right]_{z=0} = -\,s_1 \qquad (3.1)$$

where,

$$s_1 = \begin{cases} \left(k^2+b_0^2 - \dfrac{\omega^2 p_0}{\beta_0^2 h_0}\right)^{1/2}, \text{for } x<-a,\ x>a \\ \left(k_1^2+b_0^2 - \dfrac{\omega^2 p_1}{\beta_0^2 h_1}\right)^{1/2}, \text{for } -a \leqslant x \leqslant a \end{cases} \qquad (3.2)$$

The requirement of s_1 being real imposes restrictions on the phase velocities c and c_1, namely,

$$c < \beta_0\left(1+\frac{b_0^2}{k^2}\right)^{1/2}\left(\frac{h_0}{p_0}\right)^{1/2}, \text{for } x<-a,\ x>a, \qquad (3.3)$$

$$c_1 < \beta_0\left(1+\frac{b_0^2}{k_1^2}\right)^{1/2}\left(\frac{h_1}{p_1}\right)^{1/2}, \text{for } -a\leqslant x\leqslant a.$$

Since s_1 is positive, Eqn. (3·1) will be satisfied only if L.H.S. is negative. This implies that for such a medium, density and rigidity must decrease downwards in the neighbourhood of the free surface.

If we assume that there are no vertical discontinuities, then $p_0=p_1(=1,\text{ say})$, $h_0=h_1\ (=1,\text{ say})$. In this case we will have $c=c_1$, $k=k_1$, $B=D=0$ and $A=C=F$. This corresponds to the laterally homogeneous and vertically inhomogeneous medium and the frequency equation for this case will be the same as (3·1), where now

$$s_1 = \left(k^2 + b_0^2 - \frac{\omega^2}{\beta_0^2}\right)^{1/2}, \text{for all } x. \qquad (3.4)$$

4. Love waves across a vertical column in an inhomogeneous layar overlying a rigid bottom

We consider a layer of thickness H overlying a rigid bottom. The interface is taken at $z=H$. The boundary conditions are :

$$p_{yz} = 0 \quad\text{at}\quad z = 0$$
$$V = 0 \quad\text{at}\quad z = H \qquad (4.1)$$

These will lead to the frequency equation

$$s_1 \coth s_1 H + \frac{1}{2}\left[\frac{1}{q(z)}\ \frac{dq}{dz}\right]_{z=0} = 0 \quad (4.2)$$

If $\left[\frac{1}{q(z)}\ \frac{dq}{dz}\right]_{z=0} < 0$, Eqn. (4.2) can be satisfied for real as well as imaginary value of s_1. However, if $\left[\frac{1}{q(z)}\ \frac{dq}{dz}\right]_{z=0} > 0$ this equation will have solution only if s_1 is imaginary. Therefore we assume that $s_1 = is$ where s is real.

Eqn. (4·2) now takes the form :

$$\frac{1}{2}\left[\frac{1}{q(z)}\ \frac{dq}{dz}\right]_{z=0} + s \cot sH = 0 \qquad (4.3)$$

This equation will be satisfied provided $\cot sH < 0$. Therefore, for the nth mode, we must have

$$\left(n + \frac{1}{2}\right)\pi < sH < (n+1)\pi,\ n = 0, 1, \ldots \quad (4.4)$$

For the case of a vertically homogeneous and laterally inhomogeneous layer, $q(z)$ is constant and Eqn. (4·3) takes the form :

$$\cot sH = 0 \qquad (4.5)$$

This yields

$$\left(\frac{\omega^2 p_0}{\beta_0^2 h_0} - k^2\right) H^2 = \left(n + \frac{1}{2}\right)^2 \pi^2, \quad \text{for } x < -a,\ x > a, \qquad (4.6)$$

$$\left(\frac{\omega^2 p_1}{\beta_0^2 h_1} - k_1^2\right) H^2 = \left(n + \frac{1}{2}\right)^2 \pi^2, \quad \text{for } -a \leqslant x \leqslant a$$

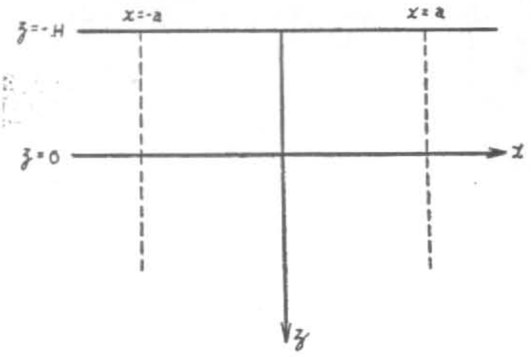

Fig. 2.

5. Love woves across an infinite vertical column, in a layer overlaying an inhomogenous half space

The geometry of the problem is shown in Fig. 2. Let the elastic parameters of the medium be given by (2·4) with $r(z) = q(z)$ and in the half-space by

$$\mu(x, z) = \mu_1\, l(z)\, h(x),$$
$$\rho(x, z) = \rho_1\, l(z)\, p'(x) \qquad (5.1)$$

with

$$p'(x) = p_0' + (p_1' - p_0')\, B(x), \qquad (5.2)$$

where $u_1\ \rho_1\ p_0', p_1'$ are constants, $B(x)$ is the unit Box function as defined earlier. For mathematical simplification, we further assume that

$$\frac{1}{4}\left(\frac{l_z}{l}\right)^2 - \frac{1}{2}\,\frac{l_{zz}}{l} = -b_1^2 \qquad (5.3)$$

where b_1 is a constant.

The displacement in the layer is given by (2.19) and in the halfspace by

$$V(x, z, t) = \frac{B_2 e^{-s_2 z}}{[\mu_1 h_0 l(z)]^{1/2}}\left[A' e^{i(kx-\omega t)} + B' e^{-i(kx+\omega t)}\right] \quad \text{for } x < -a$$
$$= \frac{B_2 e^{-s_2 z}}{[\mu_1 h_1 l(z)]^{1/2}}\left[C' e^{i(k_1' x - \omega t)} + D' e^{-i(k_1' x + \omega t)}\right], \text{ for } -a \leqslant x \leqslant a,$$
$$= \frac{B_2 e^{-s_2 z}}{[\mu_1 h_0 l(z)]^{1/2}} F' e^{i(kx - \omega t)}, \quad \text{for } x > a \qquad (5.4)$$

If the layer is laterally and vertically homogeneous then $p_0 = p_1 = 1$, $h_0 = h_1 = 1$, $q(z) =$ const. and Eqn. (4·6) reduces to :

$$\left(\frac{\omega^2}{\beta_0^2} - k^2\right) H^2 = (n + \tfrac{1}{2})^2 \pi^2 \qquad (4.7)$$

This is the well known frequency equation for love waves in a homogeneous layer of depth H overlying a rigid bottom (Hudson 1962).

where B_2, A', B', C', D', F' are constants, k_1' is the wave number in the column $-a \leqslant x \leqslant a$ for $z > 0$, s_2 is real and positive and is given by.

$$s_2 = \begin{cases} \left(k^2 + b_1^2 - \dfrac{\omega^2 p_0'}{\beta_1^2 h_0}\right)^{1/2}, \text{for } x < -a,\ x > a \\[2ex] \left(k_1'^2 + b_1^2 - \dfrac{\omega^2 p_1'}{\beta_1^2 h_1}\right)^{1/2}, \text{ for } -a \leqslant x \leqslant a \end{cases} \qquad (5.5$$

The constants B', C', D', F', may be obtained in terms of A' by applying the boundary conditions at $x = \pm a$ and will be the same as (2.11) with k_1 replaced by k_1' For satisfying the boundary conditions at $x = \pm a$ for all z, the wave numbers k, k_1, k_1' should be related by the following two equations :

$$k_1^2 = k^2 \left[1 + \frac{c^2}{\beta_0^2}\left(\frac{p_1}{h_1} - \frac{p_0}{h_0}\right)\right], \text{ for } -H \leqslant z \leqslant 0$$

$$k_1'^2 = k^2 \left[1 + \frac{c^2}{\beta_1^2}\left(\frac{p_1'}{h_1} - \frac{p_0'}{h_0}\right)\right], \text{ for } z > 0 \quad (5.6)$$

For satisfying the boundary conditions at horizontal discontinuities for all x, k_1 and k_1' should be the same which imposes a restriction on p_1', namely,

$$p_1' = \left[\frac{\beta_1^2}{\beta_0^2}\left(\frac{p_1}{h_1} - \frac{p_0}{h_0}\right) + \frac{p_0'}{h_0}\right] h_1 \quad (5.7)$$

The boundary conditions at the horizontal discontinuities are the continuity of the stress p_{yz} and the displacement at $Z = 0$ and vanishing of the stress p_{yz} at $z = -H$ for all x. These boundary condition will lead to three equations in three constants A_1, B_1, B_2. The condition for the existence of non-zero solution of these equations will give the equation

$$\tanh s_1 H = \frac{s_1\left[\frac{1}{2}\left(\frac{q_z}{q}\right)_{-H} - \frac{1}{2}\left(\frac{q_z}{q}\right)_0 + \frac{\mu_1}{\mu_0}\left(\frac{l}{q}\right)_0\left\{s_2 + \frac{1}{2}\left(\frac{lz}{l}\right)_0\right\}\right]}{\frac{1}{2}\left(\frac{q_z}{q}\right)_{-H}\left[\frac{1}{2}\left(\frac{g_z}{q}\right)_0 - \frac{\mu_1}{\mu_0}\left(\frac{l}{q}\right)_0\left\{s_2 + \frac{1}{2}\left(-\frac{l_z}{l}\right)_0\right\}\right] - s^2_1} \quad (5.8)$$

where $\left(\frac{q_z}{q}\right)_{-H}$ denote that q_z/q is evaluated at $z = -H$.

Eqn. (5·8) is the frequency equation for love waves in an inhomogeneous layer overlying an inhomogeneous half-space. The condition for the existence of real roots of this equation is that s_1 is pure imaginary. If we put $s_1 = is$, this equation takes the form :

$$\tan sH = \frac{s\left[\frac{1}{2}\left(\frac{q_z}{q}\right)_{-H} - \frac{1}{2}\left(\frac{q_z}{q}\right)_0 + \frac{\mu_1}{\mu_0}\left(\frac{l}{q}\right)_0\left\{s_2 + \frac{1}{2}\left(\frac{l_z}{l}\right)_0\right\}\right]}{\frac{1}{2}\left(\frac{q_z}{q}\right)_{-H}\left[\frac{1}{2}\left(\frac{q_z}{q}\right) - \frac{\mu_1}{}\left(\frac{l}{}\right)_0\left\{s_2 + \frac{1}{2}\left(\frac{l_z}{l}\right)_0\right\}\right] + s^2} \quad (5.9)$$

The conditions that s_2 is real and s_1 pure imaginary impose restrictions on the phase velocities c and c_1, namely

$$\beta_0\left(\frac{h_0}{p_0}\right)^{1/2}\left[1 + \frac{b_0^2}{k^2}\right]^{1/2} < c < \beta_1\left(\frac{h_0}{p_0}\right)^{1/2}\left[1 + \frac{b_1^2}{k^2}\right]^{1/2}, \text{ for } x < -a,\ x > a,$$

$$\beta_1\left(\frac{h_1}{p_1}\right)^{1/2}\left[1 + \frac{b_0^2}{k^2}\right]^{1/2} < c_1 < \beta_1\left(\frac{h_1}{p_1'}\right)^{1/2}\left[1 + \frac{b_1^2}{k_1^2}\right]^{1/2}, \text{ for } -a \leqslant x \leqslant a$$

For different variations $q(z)$ and $l(z)$ satisfying (2·16) and (5·3), we can obtain the corresponding frequency equation from (5·9).

If we assume that $p_0 = p_1$, $h_0 = h_1$, then, $p_0' = p_1'$, $c = c_1$, $k = k_1$, $B=D=0$ and $A = C = F$. This corresponds to laterally homogeneous medium. If now we take $p_0 = p_0' = 1$, $h_0 = h_1 = 1$, the frequency Eqn. (5·9) will be the same where now

$$s = \left(\frac{\omega^2}{\beta_0^2} - k^2 - b_0^2\right)^{1/2} \text{ for all } x,$$

$$s_2 = \left(k^2 + b_1^2 - \frac{\omega^2}{\beta_1^2}\right)^{1/2} \text{ for all } x, \quad (5.11)$$

For $q(z) = 1$, $l(z) = e^{\delta z}$, $p_0 = p_1 = 1$ and $h_0 = h_1 = 1$, Eqn. (5·9) reduces to the frequency equation derived by Das Gupta (1952). For $q(z) = (1 + \lambda z)^2$ and $q(z) = \cosh^2 \lambda z$, $l(z) = 1$, $p_0 = p_1 = 1$, $h_0 = h_1 = 1$, Eqn. (5·9) reduces to the frequency equation obtained by Sinha (1962). For $q(z) = 1$, $l(z) = 1$, $p_0 = p_1 = 1$, $h_0 = h_1 = 1$, Eqn. (5·9) reduces to the well known frequency equation for love wave propagation in a homogeneous layer overlying a homogeneous half-space.

6. Numerical calculations

Numerical calculations for the model (*iii*) are made assuming :

$$q(z) = e^{d_1 z}, \quad l(z) = e^{d_2 z}$$

where d_1, d_2 are constants.

Let $Y = kH$, $Y_1 = k_1 H$, $X = c/\beta_0$, $X_1 = c_1/\beta_0$

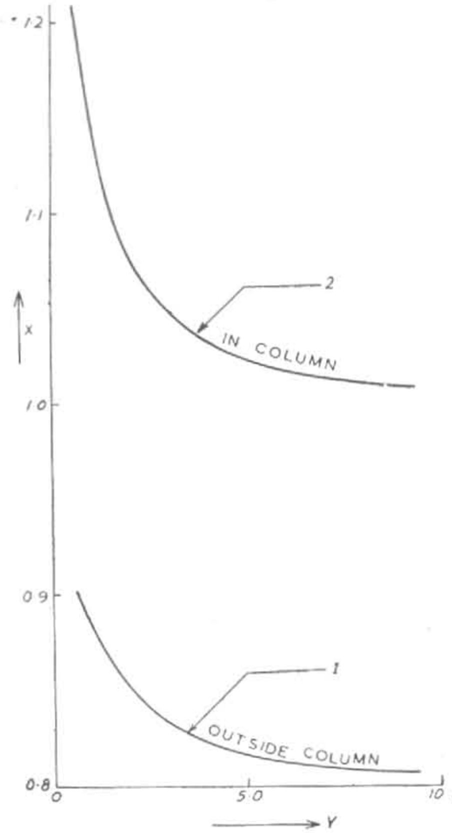

Fig. 3. Phase velocity curves for the fundamental mode for an inhomogeneous elastic layer overlying an inhomogeneous elastic half space (1) Represents curve in the column and (2) outside the column.

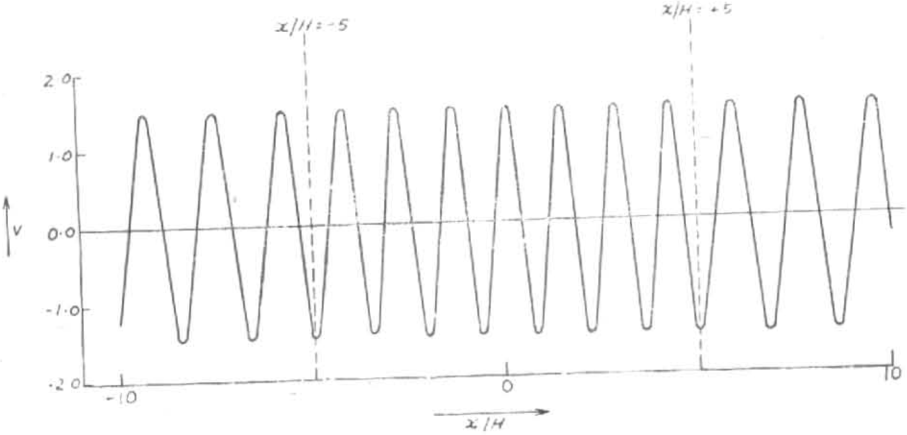

Fig. 4 Displacements at the free surface for an inhomogeneous elastic layer overlying an inhomogeneous elastic half space

From Eqn. (5.9) the values of X and X_1 have been calculated as functions of Υ and Υ_1 respectively assuming the following values of the parameters :

$$p_0=1.0 \quad h_1 = 0\cdot768 \quad \mu_1/\mu_0 = 1\cdot2$$
$$h_0=1\cdot0 \quad d_1H = -0\cdot5 \quad \beta_0 = 3\cdot75 \text{ km/sec.}$$
$$p_1=1\cdot2 \quad d_2H = 0\cdot05 \quad \beta_1 = 4\cdot4 \text{ km/sec.}$$
$$p_0^1=0\cdot9$$

The results are exhibited graphically in Fig. 3.

The displacements at the free surface have been obtained for $\Upsilon=3\cdot5$ $(X=1\cdot04)$ and $a/H=5\cdot0$. These results are exhibited graphically in Fig. 4.

Dispersion curves and displacements at the free surface for the model (*ii*) have also been obtained and are similar to those of model (*iii*).

The decay pattern of displacements with depth has been studied. These are similar to the one's we obtain for a homogeneous layer overlying a homogeneous half-space.

7. Conclusion

It has been observed that the phase velocity in the column decreases or increases with the shear wave velocity in the column. The form of phase velocity curves in the column and the remaining region remains the same.

The figures giving displacements at the free surface show that the wave number of the waves changes (increases in this case) as they enter the column and regain its original values as the waves come out of the column.

It is expected that the results can be generalized to the case when there are $n+1$ vertical discontinuities and thus lateral inhomogeneity can be studied by taking n vertical columns in an inhomogeneous medium. As a next step, the treatment could possibly be extended to include continuous lateral variation by taking a large number of these columns.

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