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Dry spell probability by Markov chain model and its application to crop developmental stages

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ABSTRACT. A first order Markov chain model has been fitted to daily rainfall data of the monsoon months in the Delhi region. Conditional probabilities, length of dry spells for different cumulative probabilities with particular reference to developmental stages of bajra crop in the kharif season have been computed and illustrated by means of nomograms.

1. Introduction

The success or failure of crops particularly under rainfed conditions is closely linked with the rainfall patterns. Simple criteria related to sequential phenomena like dry and wet spells could be used for analysing rainfall data to obtain specific information needed for crop planning and for carrying out agricultural operations (Sastry 1976). A method to evaluate frequencies of continuous days with rainfall above or below any chosen threshold value has been reported earlier (Kapoor and Sastry 1970). Studies on persistency of dry spells on a regional scale have been made by Ramabhadran (1954), and Raman and Krishnan (1960). As synoptic systems inducing rainfall or dry spells have been found to persist for a few days over a region, it is useful to ascertain the probability of sequential events like a wet day following another wet day or a dry day following a wet or dry day during the crop growing season. Markov chain probability model has been found suitable to describe the long term frequency behaviour of wet or dry weather spells (Gabriel and Neuman 1962, Casky 1963, Weiss 1964, Sundararaj and Ramachandra 1975, Medhi 1976 and Bhargava *et al.* 1977). It has found wide application in studies on daily rainfall distribution (Hopkins and Robillard 1964, Pattison 1965 and Maunder *et al.* 1971). However, this has not so far been applied to different crop developmental stages. In this investigation, a first order Markov chain probability model has been applied to rainfall data of the IARI farm area to evaluate conditional probabilities for monsoon months and durations of dry spells at different cumulative probability levels during the growth stages of bajra crop in the Delhi region.

2. Methods and Materials

Markov chain probability model assumes that the probability of rainfall occurring on any day depends on whether the previous day was wet or dry. Rainfall amount is involved only in the definition of occurrence or non-occurrence of rain. In the first order Markov chain the probability of an event that would occur on any single day depends only on the conditions during the preceding day and is independent of events of further preceding days. The parameters used in this model (Weiss 1964) are the two conditional probabilities P_0 and P_1 where,

$$P_1 = P_R\,(\mathrm{W/W}) \qquad (1)$$

is the probability of occurrence (P_R) of a wet day (W) when the preceding day also is wet;

$$P_0 = P_R\,(\mathrm{W/D}) \qquad (2)$$

is the probability of occurrence of a wet day when the previous day is dry (D). The probability of a dry spell of length n days is given by

$$P_0(1-P_0)^{n-1} \qquad (3)$$

The cumulative frequency distribution through n for dry sequences is obtained using the expression :

$$1-(1-P_0)^n \qquad (4)$$

and the probability for dry sequences $>n$ is

$$(1-P_0)^n \qquad (5)$$

Daily rainfall data for the monsoon months June to September recorded at the IARI crop weather observatory, New Delhi (Lat. 28° 35′ N, Long. 77° 10′ E) for 30 years from 1941 to 1970 have been utilized. Days were classified as either 'wet' or 'dry' according to whether a total rainfall of at least 6 mm had occurred in 24 hours or not, respectively. This limit has been chosen since the average potential evapotranspiration during the rainy season in Delhi region is of the order of 4-5 mm/day and this criterion has been found suitable to describe dry spells in the monsoon season (Sastry 1976).

In cases where a dry spell overlaps between two adjoining months, the days of the spell belonging to the first month are not assigned to the adjoining month, but the spell has been treated as ending on the last day of the first month. The rest of the days of the spell are treated as a spell belonging to the next month. Dry spells merging with the withdrawal of the southwest monsoon which takes place around the 3rd or 4th week of September in the Delhi region have been included in the data for September. These have not been treated separately. The conditional probabilities P_0 and P_1 for each month in the monsoon season have been estimated using the corresponding relative frequencies. Only one antecedent day is considered in calculating the conditional probabilities P_0 and P_1 here.

The probabilities of different lengths of dry spells (2 day, 3 day,........, n days) have been calculated using Eqn. (3). The expected frequencies of dry spells have been obtained by multiplying these probabilities with the total number of dry days in a month. To test the adequacy of the fit of estimated and observed frequencies, Chi-square test was applied considering the appropriate degrees of freedom, combining the adjoining classes to give an expected frequency of at least 5 within any one class. Length of dry sequence of n days for the cumulative probability levels 98, 90, 80, 50 and 10 per cent were computed.

The other properties of Markov chain probability model describing the number and length of dry and wet spells have also been evaluated. The expected length of dry runs is given by

$$E\text{ (D)} = 1/P_0 \tag{6}$$

Similarly for wet run,

$$E\text{(W)} = 1/(1-P_1) \tag{7}$$

The expected length of a weather cycle, *i.e.*, a dry spell followed by a wet spell or *vice versa*, is then obtained by using

$$E\text{ (C)} = E\text{(W)} + E\text{(D)} = \frac{1}{1-P_1} + \frac{1}{P_0} \tag{8}$$

The stationary probabilities π_1 and π_2 which are independent of the initial conditions are given by

$$\pi_1 = \frac{1-P_1}{1+P_0-P_1} \text{ and } \pi_2 = \frac{P_0}{1+P_0-P_1} \tag{9}$$

For large n, the distribution of the number of wet days (Y_n) tends to normality with mean and variance

$$E(Y_n) \sim \frac{n\ P_0}{1-P_1+P_0}$$

and

$$V(Y_n) \sim \frac{nP_0(1-P_1)\ (1+P_1-P_0)}{(1-P_1+P_0)^3} \tag{10}$$

Conditional probabilities have been computed for the developmental stages of bajra crop (Table 5) taking the second week of July as the normal sowing time in the Delhi region as per the local agronomic practice. Sowing time may vary by ± 1 week in any individual year, depending on the commencement of the monsoon and its activity.

TABLE 1

χ^2 test of significance

Test	Jun	Jul	Aug	Sep
Independence *vs.* Ist order Markov chain (1 *df*)	71.87**	68.75**	35.23**	117.99**
Ist order *vs.* 2nd order Markov chain (2 *df*)	3.65	6.07*	4.56	17.38*

df = degrees of freedom

* = Significant at 5 per cent level

** = Significant at 1 per cent level

3. Results and discussion

3. 1. *Adequacy of the model*

Statistical tests suggested by Anderson and Goodman (1957) have been performed to ensure that the sequences of daily occurrence of precipitation under study does constitute a first order Markov chain. The results of these tests are summarized in Table 1. The test for independence indicates overwhelming evidence that sequence of dry and wet days are dependent. The Chi-square statistic at 0.05 level for the test of first order *versus* second order Markov chain is not significant except for the months of July and September.

The conditional probabilities P_0 and P_1 and simple probability P_d for the monsoon months June to September along with other properties are shown in Table 2. Compared to the probability of a wet day following a dry day (P_0) the probability of a wet day following a wet day (P_1) is higher, thus demonstrating a strong dependence of an event (occurrence of rainfall) on the preceding day's conditions. An important feature is that there is a higher probability (0.450) of a wet day being followed by a wet day (P_1) in the month of September than in August (0.382) even though the latter month receives higher rainfall than September.

On an average, during the active monsoon season (July and August) the dry spell in the Delhi region does not exceed six days whereas the wet run lasts for about two days.

3. 2. *Expected and observed frequencies*

Expected and observed frequencies of sequence of dry days for the months June to September together with Chi-square test for goodness of fit are presented in Table 3. The differences are not significant at 5 per cent level for all the months except for June which is non significant at 1 per cent level.

TABLE 2

Parameters and properties of Markov chain probability model

	Conditional probabilities		Simple probability	Expected length of runs		Dry-wet cycle	Expected No. of days		S.D. of dry or wet days	Stationary probability	
	P_1	P_0	P_d	Dry	Wet		Dry	Wet		π_1	π_2
Jun	0.354	0.055	0.928	18.18	1.55	19.73	27.65	2.35	2.01	0.922	0.078
Jul	0.452	0.172	0.763	5.81	1.83	7.64	23.59	7.41	3.17	0.761	0.239
Aug	0.382	0.185	0.767	5.41	1.62	7.03	23.86	7.14	2.86	0.769	0.230
Sep	0.450	0.088	0.854	11.36	1.82	13.18	25.86	4.14	2.89	0.862	0.138

P_d = Probability of a day being dry (rainfall < 6 mm/day)
P_0 = Probability of wet day preceded by dry day
P_1 = Probability of wet day followed by a wet day.
S.D. = Standard deviation.

TABLE 3

Expected and observed frequency of dry spells

Run length (days)	June		July		August		September	
	Obs	Exp	Obs	Exp	Obs	Exp	Obs	Exp
1	67	46	143	122	156	132	96	68
2	65	45	105	101	114	108	76	62
3	55	41	81	84	87	88	66	57
4	47	39	69	69	63	72	59	52
5	45	37	53	57	53	58	52	47
6	39	35	44	48	39	48	49	43
7	39	33	36	39	37	39	46	39
8	35	31	26	33	29	32	41	36
9	35	29	22	27	25	26	36	33
10	34	28	19	22	23	21	32	30
11	32	26	17	19	20	17	29	27
12	32	25	15	15	14	14	26	25
13	31	23	11	13	10	11	24	22
14	28	22	8	11	9	9	23	20
15	25	21	6	9	8	8	17	19
16	22	20	6	7	6	6	14	17
17	22	19	5	6	5	5	9	16
18	21	18	5	5	5	4	9	14
19	21	17	5	4	3	3	8	13
20	19	17	4	3	2	3	7	12
21	16	15	4	3	1	2	7	11
22	16	14	4	2	1	2	6	10
23	15	13	3	2	1	2	6	9
24	15	13	3	2	1	1	6	8
25	14	12	3	1	1	1	6	7
26	13	11	3	1	0	1	5	7
27	9	11	3	1	0	1	4	6
28	9	10	3	1	0	1	4	6
29	7	9	2	1	0	0	3	5
30	7	9	1	1	0	0	3	5
31	—	—	1	0	0	0	—	—
χ^2	44.88**		10.89*		9.85*		36.80*	
df	28		17		16		28	

* Non-significant at 5 per cent level. ** Non-significant at the 1 per cent level. Obs — Observed Exp — Expected

df — Degrees of freedom

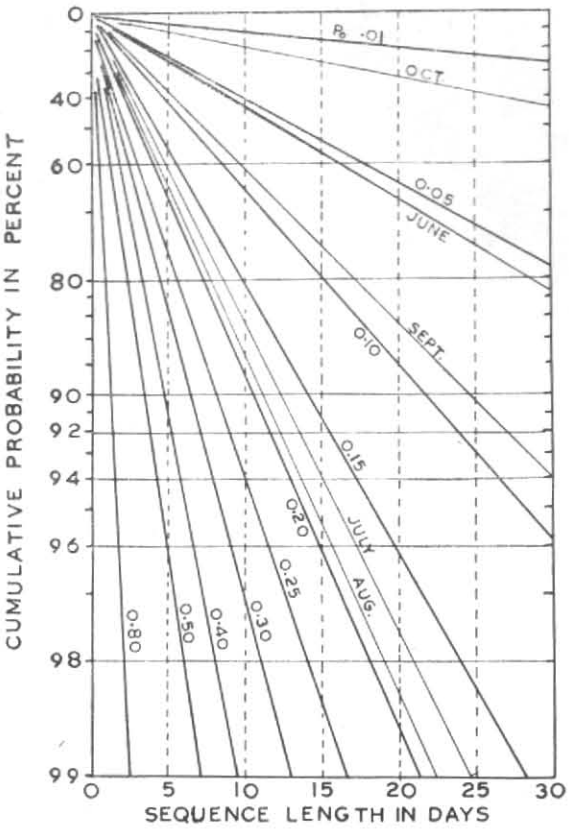

Fig. 1. Cumulative probability nomogram for dry spells during the monsoon months at Delhi

3.3. *Return periods and probabilities*

The probabilities may also be expressed in terms of an average recurrence interval or return period T_d given in years, of sequence length greater than n days (Weiss 1964). For dry day sequence,

$$T_d = \frac{(1-P_1)+P_0}{SP_0(1-P_1)(1-P_0)^n}$$

where S is the total number of days in the month for which the sequences are counted. Dry sequence length (in days) corresponding to a given return period T_d for rainfall < 6 mm/day are given in Table 4. This shows, for example, that only once in 5 years on the average does the Delhi region experience a dry sequence of more than 15 days in August for which precipitation does not exceed 6 mm/day.

3.4. *Cumulative probabilities and nomograph*

The lengths of dry spells for cumulative probabilities at 98, 90, 80, 50 and 10 per cent levels for the months June to September have been computed.

TABLE 4

Dry sequence length n (days) corresponding to given return period T_d for rainfall $<$ 6 mm/day

T_d (years)	Jun	Jul	Aug	Sep
100	89	31	29	59
50	77	27	25	52
10	48	19	18	34
5	36	16	15	26
2	20	11	10	16

A nomograph relating conditional probabilities, length of dry sequence and cumulative probability for dry spells for the Delhi region is shown in Fig. 1. For example, the length of dry days in the month of August for 90 per cent probability is 11 and in September, it is 25.

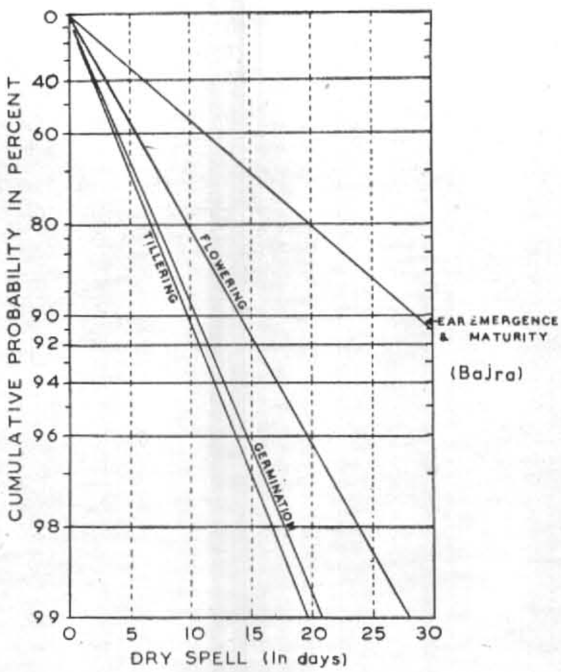

Fig. 2. Cumulative probability nomogram for dry spells during the developmental stages of Bajra crop in the Delhi region

TABLE 5

Rainfall probabilities during development periods of bajra crop (Delhi region)

Probabilities	Sowing and germination (2 weeks)	Tillering (4 weeks)	Flowering (2 weeks)	Ear emergence and maturity (5 weeks)
χ^2 (2 *df*)	2.12	2.74	5.29	33.75*
P_0	0.1980	0.2050	0.1522	0.0786
P_1	0.4871	0.3886	0.4000	0.4626
Pw	0.2785	0.2511	0.2023	0.1276

Pw : Probability of a day being wet (rainfall >6 mm/24 hour). Normal sowing date taken as 9 July to initiate analysis; the crop is harvested after 90-95 days in the second week of October.

df : Degrees of freedom.

* : Denotes rejection of null hypothesis at 0.05 level of significance ($\chi^2 > 5.99$).

3.5. *Probabilities of dry spells in different crop growth stages*

Bajra is one of the main crops grown in the Delhi region under unirrigated conditions. Using the Markov chain model, conditional probabilities together with Chi-square statistics have been computed for the different developmental periods of the bajra crop (Table 5). The first order model is found to be applicable to the different stages of crop growth except during the maturity period. As the season progresses, the probability of occurrence of a wet day decreases. However, during the tillering to maturity period, once a wet day occurs, the probability of the succeeding day being wet becomes higher with the progress of the season. A nomograph giving the cumulative probability of occurrence of dry spells of different durations, for the different crop growth stages of bajra is shown in Fig. 2. For example, during the tillering period, a dry spell of 10 days (rainfall<6 mm/day) can be expected with 90 per cent cumulative probability and during the flowering period, a ten-day dry spell can be expected with a probability of 80 per cent in the Delhi region. The probabilities of occurrence of dry spells during the different stages of crop growth, the conditional probabilities which take into account sequential events and the nomograms provide the basic information on rainfall distribution characteristics necessary for agricultural operations such as irrigation scheduling, choice of transplanting time etc. It also provides information to the plant breeder for developing the crop varieties suited to the local rainfall pattern.

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