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Excessive overshooting of cumulonimbus

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ABSTRACT. It is generally accepted that the classical parcel method gives the correct estimate of the height of cumulonimbus clouds. Formerly it was thought that a parcel of air rising convectively could rise from convective condensational level upto a level where the temperatures of the parcel and the environment become equal. We may call it as equilibrium level or Z_e. Between CCL and Z_e the parcel rises since the temperature of the parcel is higher and the density is always lower than the environment. This Z_e is also the anvil level. It has been found that the cumulonimbus often crosses the anvil level and grows higher. This phenomenon is termed as overshooting. Based on this classical parcel method theories for overshooting have been proposed by Newton (1966) and Roach (1967). They postulated that the rising parcel would cross Z_e because of the inertia and could go to a level Z_p so that the energy acquired by the parcel during its journey from CCL to Z_e is spent up.

On some occasions it has been found that heights of cumulonimbus have exceeded Z_e. This paper deals with such cases. This phenomenon has been termed as excessive overshooting. Fujita (1974) presented a model of overshooting cumulonimbus. With the help of this and the model presented by Roach a mechanism of cooling of the immediate environment of the overshooting clouds have been proposed. This modified environment then allows further growth of the cloud. Based on the mixing of cold air due to overshooting and surrounding air in proportion to their speeds, a quantitative estimation of the cooling of the environment has been possible. From the considerations of inertia again it has been proposed that the rising parcel can go beyond Z_p and to height $Z_{p'}$ which can be easily calculated from thermodynamic diagrams. The calculated values have been compared with observed heights of excessive overshooting and the agreements have been quite good.

1. Introduction

Growth of cumulonimbus clouds is an old problem. Barnes (1976) has given a nice review. It is seen from his article that models of these clouds produced before the Second World War concluded that the visible anvil is the top of the cumulonimbus. Some, of course, visualised from theoretical grounds that these clouds may grow above the anvil. The growth of cumulonimbus clouds above anvil is termed as "overshooting".

Newton (1963, 1966) produced an overshooting model with updraft extending upto overshooting top. These papers presented a possibility of measuring sub-anvil updraft from anvil top topography. Roach (1967) from horizon to horizon pictures taken from U2 aircraft combined with radiometric observations produced a model of cloud overshooting dome assumed to be supported by quasi-steady updraft. Mukherjee *et al.* (1972) from analysis of radar data of Calcutta and Conford and Spavins (1973) from measurements from Royal Air Force aircraft over northeast India showed that overshooting of cumulonimbus is a rather common phenomenon in this area during the pre-monsoon season.

Mukherjee and Chaudhury (1971) in another paper pointed out, from analysis of radar observations, that sometimes the overshooting exceeds the theoretical value. They postulated a mechanism of cooling of the air above the cumulonimbus overshooting and explained the phenomenon. The fact that cooling actually takes place has been proved by observations taken by Fujita (1974) from Learjet aircraft during "Cloud Truth" experiments in U.S.A. Mukherjee and Chaudhury (1971) introduced a degree of arbitrariness in calculating the excessive overshooting. In the present paper an attempt has been made to calculate from theoretical considerations the height of the cloud top after the excessive overshooting. For this, the motion of overshooting top has been calculated first.

2. Theory of overshooting

Let us take the case of actual sounding of 1200 GMT of 12 May 1969 for Calcutta (Fig. 1). The curve drawn with thick line is the actual ascent from radiosonde data. Convective Condensation Level (CCL) is marked on the figure; dashed line is ascent curve for a parcel of air rising adiabatically. These two curves intersect at a level which we may term as Z_e. At this level environmental air and

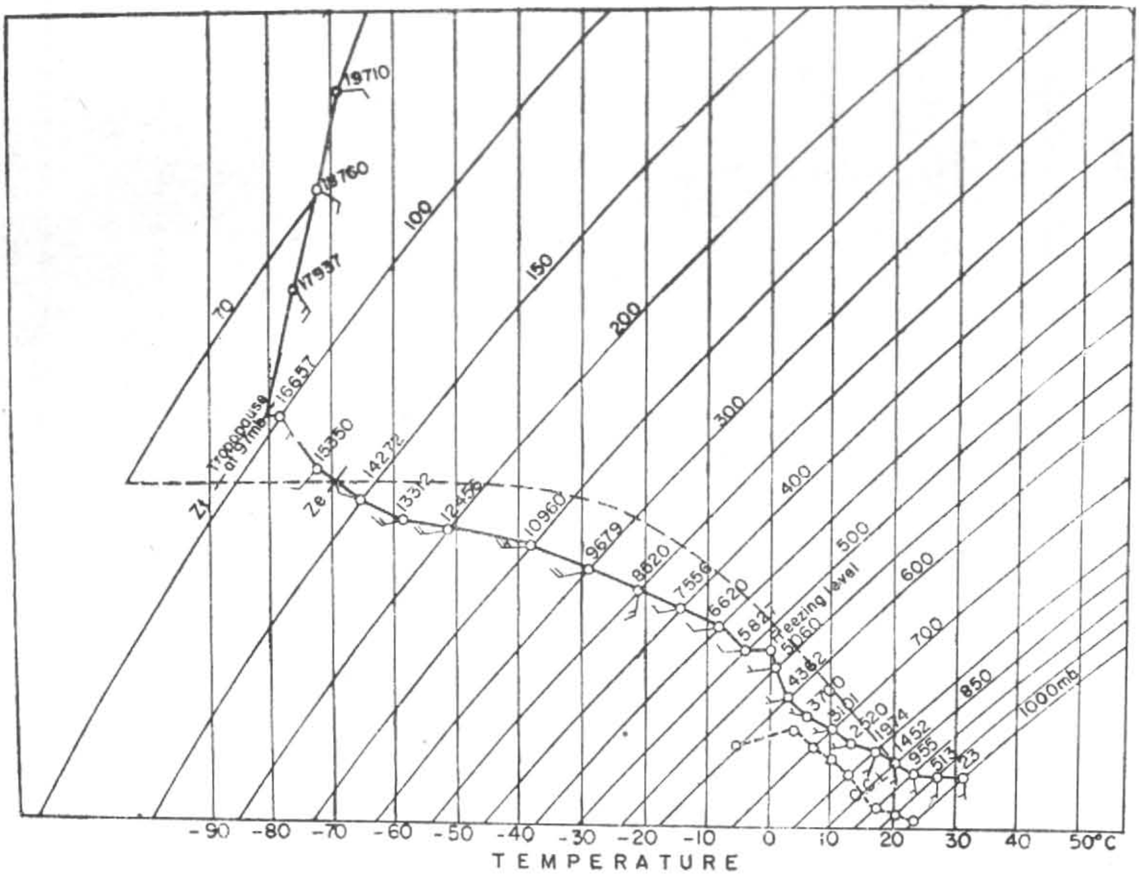

Fig. 1. Tephigram, Calcutta, 12 May 1969

parcel of rising air will have same temperature. It was normally believed that this is the equilibrium level, that is why it is termed Z_e, and parcel will not rise further. At this level, therefore, the parcel will spread out to form anvil.

Let us assume that a cumulonimbus cloud is forming in which the core of updraft contains undiluted parcel of rising air. Under the assumption that vertical pressure gradient inside the clouds equals that outside, the vertical acceleration is given by

$$\frac{dw}{dt} = g\,\frac{\triangle T}{T} \tag{1}$$

or

$$dw = \frac{g}{w}\;\frac{\triangle T}{T}\;dz \tag{2}$$

where, w = vertical velocity

g = Acceleration due to gravity

$\triangle T$ = Excess of virtual temperature of rising parcel due to convection over that of the environment.

T = Virtual temperature of the environment.

From this we get the vertical velocity as :

$$\frac{w^2}{2} = g \int_{\mathrm{CCL}}^{Z_e} \frac{\triangle T}{T}\,dz \tag{3}$$

after neglecting the work done for lifting upto the CCL. Since $\triangle T$ is positive upto Z_e the vertical velocity increases upto Z_e.

The parcel will rise upto Z_e gathering energy all the way and hence cannot stop at that level and is bound to rise above due to the accumulated energy, of course, in a warmer environment. Next question is how far this parcel can rise.

Again referring to Fig. 1 we find the parcel accumulates energy from CCL to Z_e and this energy is proportional to the area on the thermodynamic diagram bounded by the solid and dashed curves from CCL to Z_e. When this parcel rises it will spend its energy and the maximum amount it can spend is the energy it has gathered. Let us assume it can go upto a level given in Fig. 1 as Z_p where the area between solid and dashed lines

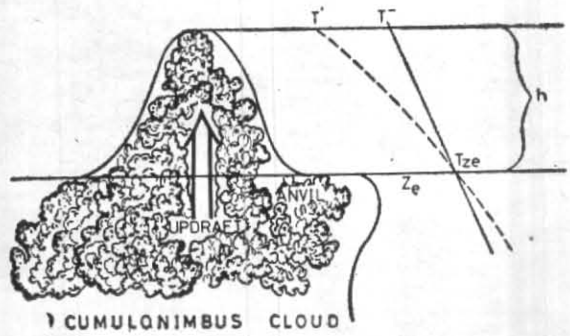

Fig. 2. Schematic diagram of overshooting

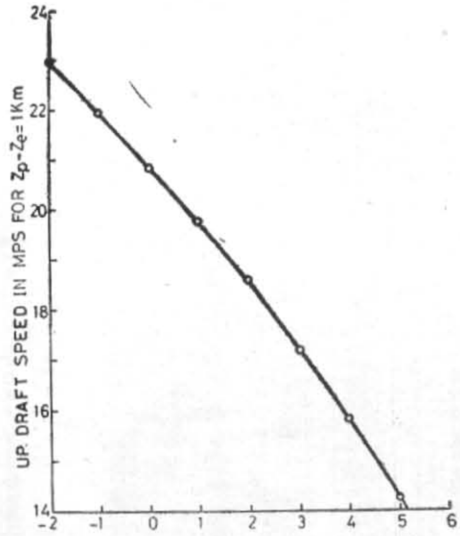

Fig. 3. Environment lapse rate (γ) between Z^e and Z

above $\mathcal{Z}_e$ and bounded by $\mathcal{Z}_p$ becomes equal to the area below $\mathcal{Z}_e$. This way we can find out $\mathcal{Z}_p$, the height of overshooting being $(\mathcal{Z}_p-\mathcal{Z}_e)$.

3. Motion of overshooting parcel (Fujita model)

Let us assume that a parcel has risen to a height h above $\mathcal{Z}_e$. The vertical acceleration is given by (*vide* Fig. 2) :

$$\frac{d^2h}{dt^2} = -g\frac{T-T'}{T} \tag{4}$$

where T refers to actual environmental temperature at h and T' that of the parcel. Negative sign indicates rising parcel is colder than the environment.

Now, $-(T-Tz_e)/h=\gamma$, *i.e.*, the actual lapse rate, and $-(T_e-Tz_e)/h=\gamma'$ the moist adiabatic lapse rate. Hence,

$$\frac{d^2h}{dt^2} = -g\frac{\gamma'-\gamma}{T}\,h \tag{5}$$

or
$$\frac{d^2h}{dt^2}+K^2h=0 \tag{6}$$

where $K^2 = g\,\dfrac{\gamma'-\gamma}{T}$

It is a positive quantity since $\gamma'>\gamma$. Assuming T to be constant (a practical assumption) we get $K^2=$ a constant. This is Brunt-Vaisala wave equation. Solving this we get :

$$h=h_m \sin Kt$$

where h_m is the maximum height the parcel can go above $\mathcal{Z}_e$.

The vertical velocity is given by

$$w=\frac{dh}{dt}=Kh_m\cos Kt \tag{7}$$

where $t=0$, w is maximum and is given as

$$w_m = Kh_m = h_m\sqrt{\frac{g}{T}\,(\gamma'-\gamma)} \tag{8}$$

This value should be same as obtained from Eqn. (3) but here all terms can be readily read off from the tephigram and hence can be used in operational units.

From the above we find that the motion is a simple harmonic motion where period can be given by

$$P=\frac{2\pi}{K}=2\pi\sqrt{\frac{T}{g(\gamma'-\gamma)}} \tag{9}$$

3.1. *Velocity of overshooting*

Overshooting velocity would depend on the characteristics of rising parcel, but the maximum velocity is given by Eqn. (8). From operational point of view we can find out T (approximately the mean temperature of $\mathcal{Z}_e$ and $\mathcal{Z}_p$), γ, γ', and h_m or $(\mathcal{Z}_p-\mathcal{Z}_e)$ and can calculate w_m for various lapse rates. Assuming $\gamma'=9.3°$ K/km, $T=207°$K, $g=9.8$ m/sec^2, we have calculated w_m for various lapse rates for $h_m=1$ km. This is shown in Fig. 3. For other values of h_m, this can be used as a multiplying factor. Thus for $\gamma=1.5°$ K/km and $h_m=4.660$ km (actual values for mean sounding of May at Calcutta we get the maximum up draft speed $=88.6$ m/sec.

3.2. *Period of overshooting*

One full cycle of overshooting should consist of four distinct parts : (1) rising, (2) descending upto $\mathcal{Z}_e$, (3) movement below $\mathcal{Z}_e$ within the cloud and (4) upward movement upto $\mathcal{Z}_e$. There may be further oscillations, but due to lack of buoyancy above

$\mathcal{Z}_e$ the oscillations may not be important. Of the four motions given above, (1) and (2) are of importance, (3) and (4) will be within the cloud and can not be verified experimentally.

If the overshooting motion is sinusoidal, which, according to the derivation given above, appears to be so, then the time required for rising upto the maximum height and that for returning to the level $\mathcal{Z}_e$ should each be one quarter of overshooting period. The temperature lapse rate for moist adiabat may be approximated for all practical purpose as 9.3°K/km and temperature of the layer of overshooting assumed to be a constant and with value 207° K and g as 9.8 m/sec^2, we have calculated one quarter of period P for different values of γ and that is shown in Fig. 4.

It will be seen that it takes a very short time for the overshooting parcel to rise to a maximum level and then to fall back to $\mathcal{Z}_e$. Calling these as turrets, Fujita (1974) has measured actual time for rise and fall of these turrets. His conclusions are as follows :

(1) The large overshooting turrets 0.5 to 1 mile in diameter overshoot and collapse (*i.e.,* return to $\mathcal{Z}_e$) more or less with Brunt-Vaisala frequency. The rising period is usually 1 to 2 mins.

(2) An overshooting updraft less than 0.5 mile in diameter, reaches its peak height within much shorter time than Brunt-Vaisala frequency implies, *i.e.,* the top does not reach the maximum height.

3.3. *Overshooting domes*

When we see *Cb* in radar we find that the overshot area of part of the cloud remains for a longer time than the period of turret predicted from theoretical consideration. Instead of turrets we may call them as "overshooting domes" or simply "domes". Newton* has called it a "tower". We propose to call it a dome. According to Fujita, these domes are conglomerates of turrets seen at or just above the anvil level, the horizontal dimension of overshooting domes vary between 1 to 10 miles consisting of tens or hundred of turrets of various sizes. The dimensions of some domes are large enough to be detected by ATS sensors with a 2.5 mile resolution at the subpoint. From satellites it is very difficult to distinguish a dome from its environmental cirrus because the brightness contrast is very small.

Overshooting period of a dome shows wide variation with dome height. Shenk's (1973) statistics revealed the average rising period of 5 minutes. This he found from data of 21 domes.

4. Excessive overshooting

Upto this point classical parcel theory as modified by Newton can explain the growth of very tall clouds. Roach, Cornford and Spavins, Fujita etc, all workers who investigated the growth of

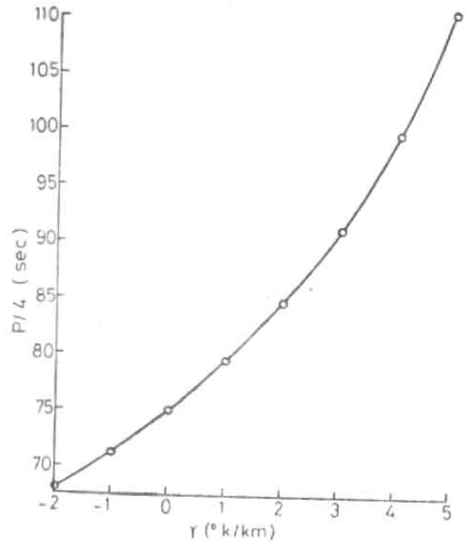

Fig. 4. Time required for overshot parcel to come down to anvil level

tall clouds have been satisfied with this concept. They have all expressed that the concept of rise of undiluted parcel can explain growth of very tall clouds. But a search in literature showed that the data published by these workers contain facts that the growth have sometimes been more than expected. Table 1 contains these data. If the facts are correct that the height of thunderstorm can exceed $\mathcal{Z}_p$, a forecaster has to express liability of such growth and should device some method to determinet his liability in quantitative terms. We have tried to do this in the following paragraphs:

If we look at Fig. 1 and carefully consider the movement of parcel above $\mathcal{Z}_e$ we find that the parcel loses energy due to the movement in the negative buoyancy area. We have, therefore, to find out some reasons for decrease in negative buoyancy of the parcel. Now, the negative buoyancy is due to the difference of temperature of the rising parcel and the environment. There are two possibilities: either the temperature of rising parcel rises or the temperature of the environment falls.

4.1. *Entrainment above anvil*

We may postulate that entrainment of warm air takes place in the rising parcel above $\mathcal{Z}_e$. Assuming such entrainment we may find that the temperature of the parcel rises to some extent.

We come across three difficulties in accepting entrainment as workable hypothesis for our operational requirement:

(1) We have all along assumed that the core of rising parcel was undiluted while rising from CCL to $\mathcal{Z}_e$. It becomes difficult for us to accept idea of entrainment only after the parcel crosses $\mathcal{Z}_e$.

(2) Measurements by Fujita for rising turrets indicate that the overshooting turrets were actually undiluted, *i.e.,* entrainment did not take place.

TABLE 1

Cases of excessive overshooting of cumulonimbus

Date	Place	Z_e	Anvil deck (ft)	Z_t	Z_p	Z_p' Assuming 1:1 mixing	Z_p' Observed top ht.	Z_p' Calculated top ht.
24 May 1962	Oklahoma	40, 400 ft	40,000	38,000 ft	53,300 ft	—	57,000 ft	—
25 May 1962	,,	42,200 ft	42,000	38,000 ft	57,500 ft	65,000 ft	61,000 ft	—
5 June 1962	,,	43,000 ft	44,000	42,000 ft	55,200 ft	—	60,000 ft	—
31 May 1963	,,	38,600 ft	42,000	37,000 ft	48,300 ft	—	51,000 ft	—
30 May 1963	Calcutta	14.0 km	—	16.5 km	18.0 km	19.2 km	19.2 km	19.7 km*
27 May 1966	,,	14.5 km	—	16.5 km	18.3 km	19.5 km	20.0 km	20.4 km
12 May 1969	,,	14.7 km	—	16.5 km	18.7 km	19.4 km	19.3 km	19.7 km

Data for Oklahoma is as per Roach (1967). Units of heights differ due to difference in practices in U.S.A. and in India. Z_t= neight of tropopause. *Taking normal wind at 16 km as actual winds were not available.

(3) Entrainment cannot be estimated beforehand and as such cannot be used to forecast the height attained.

Besides these when we consider the reduction in vertical velocity due to entrainment the height attained by the parcel becomes actually less.

Thus discarding the idea of warming of the rising parcel above Z_e, we have to consider the cooling of the environment above Z_e. Mukherjee and Chaudhury (1971) first postulated a mechanism of cooling of the environment. Fujita (1974) postulated another mechanism. Mukherjee and Chaudhury proposed the mechanism in order to explain excessive overshooting, *i.e.*, growth of cumulonimbus above Z_p. At that time it was not realised that environment above the anvil level actually gets cooled. Confirmation came from the 'Cloud Truth', experiments by Fujita during 1972 and 1973 where he actually observed such cooling. He has, however, proposed another mechanism. In order to bring out the advantage of our method for operational purposes, we shall reverse the chronological order, presenting Fujita's work first.

4.2. *Mesohigh aloft*

The existence of an outflow field at the anvil level of a thunderstorm was pointed out by a number of authors. McLean (1961) has shown from Doppler winds the existence of significant outflow from the top of a localised squall line. From ATS pictures also outflow has been observed by Sikdar *et al.* (1970) and many others.

From these, the evidence is clear about localised high pressure field at the anvil level. In an attempt to clarify the mechanism of anvil-cloud spreading edges anvils were photographed from Learjet (aircraft used) by Fujita at anvil altitude. He observed that a very thin edge often extends upwind. The edge gives an impression that the uppermost anvil is sliding out with a small curl along the edge. Several times Fujita witnessed cloud vortices located away from the upwind side edge, hanging in free atmosphere. These two views give a definite impression that the clear air above the anvil top is spreading out faster than the underlying anvil materials.

Fujita conceived that due to existence of overshooting parcel which is colder than the surrounding a high pressure area develops similar to the high pressure found below a thunderstorm. He called it "Mesohigh aloft".

According to Fujita (1974) the hydrostatic pressure inside the cold air is higher than in the environment and he assumed it to be maximum at the level Z_e. He states that a field of excess pressure, somewhat like "that of a mesohigh on ground will be found near the anvil top level, *i.e.,* the 'Mesohigh aloft'. The altitude of the mesohigh aloft will be higher than the tropopause at the undisturbed environment. Thus a lifted tropopause forms at the anvil top level. It is the lower stratospheric air which diverges out above the anvil top to descend from around the edge". Chang and Teeson (1974) tried to introduce hydrostatic equation to treat mesohigh aloft on a quantitative basis.

Fujita records one incident when he was flying at 50000 ft to photograph anvil tops. He found outside temperature was as low as $-84°$ C or 189° K. He believes he was flying in a mesohigh aloft.

But for a parcel moving above Z_e, we may assume little difference between dry adiabatic and moist adiabatic lapse rates at such heights. At Z_e the

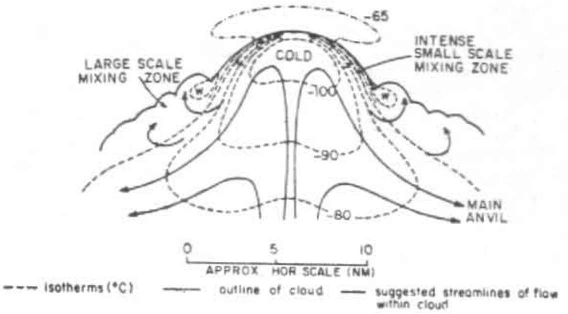

Fig. 5. Tentative model of quasi-steady storm top

air is not cooler than the environment and hence there is no reason to believe that it is cold and will cause the formation of mesohigh there. Only we can imagine that the air moving upwards is colder than the surrounding, but all calculations of adiabatic expansion presuppose the pressure inside the rising parcel and outside it should be same. Thus though there may be outflow at Z_e as observed by Fujita, the physical cause for that does not appear to be the higher hydrostatic pressure inside the rising cold air. Moreover, it is doubtful whether hydrostatic equation can be applied in these types of convective scale motions.

4.3. *Turbulent mixing with environment*

We now present our concept to explain the cooling above anvil. The spreading of cold air above anvil was also proposed by Roach (1967) and we start from that mechanism. But the cooling can also be explained, by assuming turbulent mixing of downward moving air.

Roach (1967) has suggested an airflow in both vertical and horizontal planes to explain his radiometric observations taken from U2 planes flying above anvil as well as above overshooting domes. This is reproduced in Fig. 5. Mukherjee and Chaudhury (1971) have shown that this flow pattern can explain cooling aloft and also the outflow at the anvil level. Since often the spreading air will have greater velocity than the surrounding air, the outflow may be seen upwind also against a strong horizontal wind.

The flow pattern given by Roach shows that the rising jet of air above Z_e moves up and the heavier air at Z_p falls toward Z_e by the side of the upward moving parcel — a picture similar to a water fountain. While returning there will be turbulent mixing of air at the edge. This explains the appearance of cirrus clouds at and often above the anvil as observed by Fujita. The downward moving air has to spread at Z_e and while doing so can carry cloud material on all sides. Due to turbulent mixing sometimes cloud materials may be seen to be detached above the anvil level causing curly appearance.

A calculation of downward motion indicates that the speed is so great that immediate environment of the rising parcel can be completely replaced by cold air. In all the above calculations the effect of wind has not been considered. In order to be more realistic we propose to consider the horizontal motion of the surrounding air. We postulate that the air due to two motions — horizontal wind and downward movement of overshoot parcel get mixed to change the temperature of immediate environment of rising air. Let w_d be downward velocity of falling parcel. The amount of air in a certain volume will be proportional to w_d. The change of heat will be proportional to $w_d C_p \triangle T$ where C_p is the specific heat of air and $\triangle T$ difference of temp. between parcel and environmental air. Let u be the horizontal wind speed. Hence the total air entering in the same volume from both above and side taken together in unit time will be proportional to $(u+w_d)$. Hence the change of heat will be proportional to $(u+w_d)\ C_p\ \delta T$, where δT is the change in temperature.

Now, horizontal wind is at the same temperature as the environmental air. Hence, the only heat change is due to falling parcel.

Hence, $w_d\ C_p\ \triangle T = (u+w_d)\, C_p\ \delta T$

$$\delta T = \frac{w_d}{u+w_d} \triangle T$$

It may be mentioned here that the air of immediate environment will be completely displaced by the air due to above motions. Thus if $u=0$, $\delta T = \triangle T$, *i.e.*, the immediate surrounding of the rising overshot parcel will be cooled to that extent. This is, therefore, an idealised assumption — may be taken as justified as the downward moving air is denser and has a fairly great speed.

This can be calculated level by level Z_e to Z_p and a new environment curve can be constructed. For normal tephigram of May at Calcutta this was done and is shown in Fig. 6.

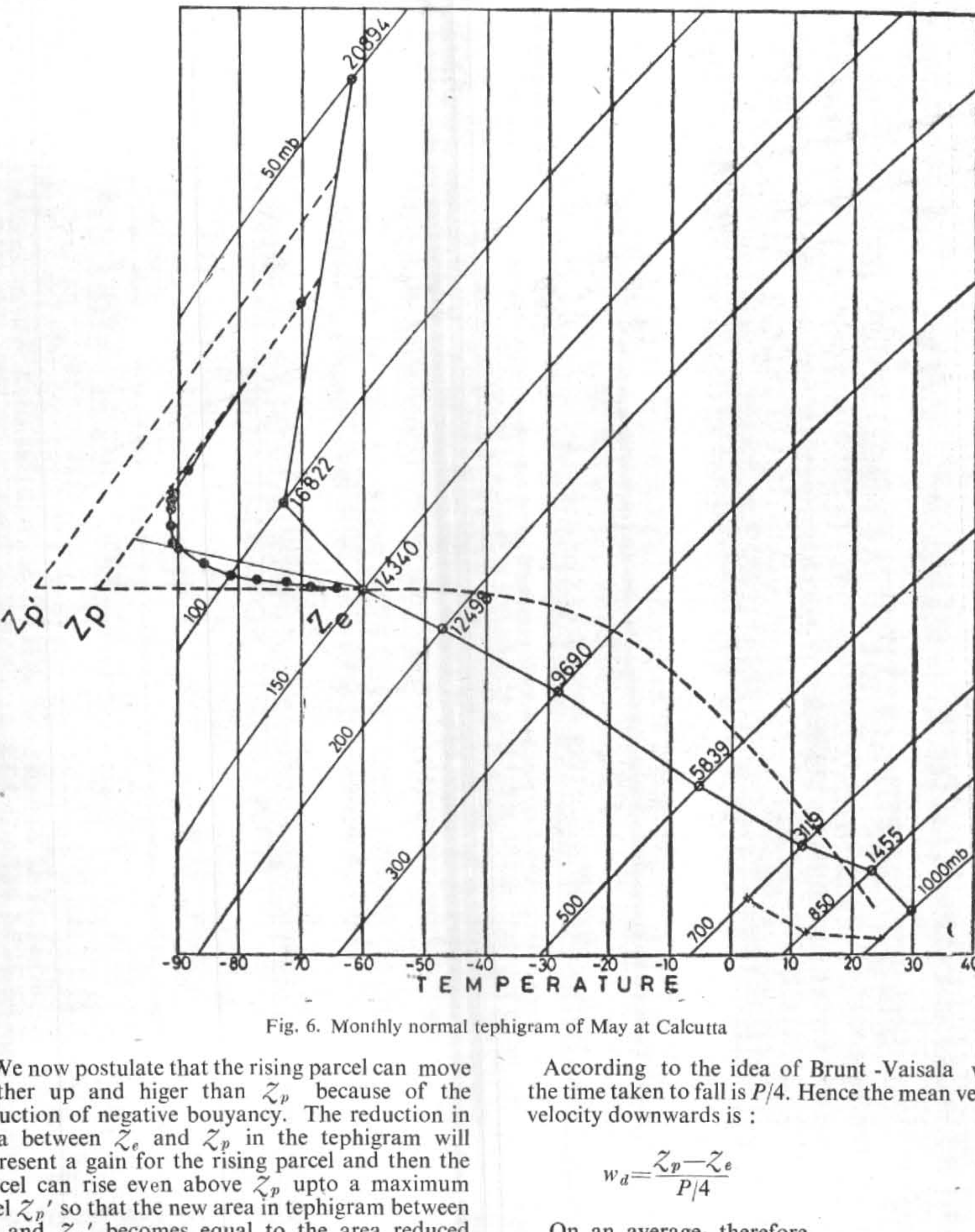

Fig. 6. Monthly normal tephigram of May at Calcutta

We now postulate that the rising parcel can move further up and higer than Z_p because of the reduction of negative bouyancy. The reduction in area between Z_e and Z_p in the tephigram will represent a gain for the rising parcel and then the parcel can rise even above Z_p upto a maximum level Z_p' so that the new area in tephigram between Z_p and Z_p' becomes equal to the area reduced between Z_e and Z_p. The rise above Z_p is termed as excessive overshooting and its height can be upto Z_p'.

We calculated Z_p in this way for the mean tephigram of May at Calcutta. We found that the time required for such calculation is so much that it is not suitable for operational purposes. We, therefore, give in the following paragraph a quick method of calculation suitable for operational use.

According to the idea of Brunt -Vaisala wave, the time taken to fall is $P/4$. Hence the mean vertical velocity downwards is :

$$w_d = \frac{Z_p - Z_e}{P/4}$$

On an average, therefore,

$$\triangle T_e = \frac{w_d}{u_m + w_d} \triangle T$$

where u_m is the mean wind speed between Z_e and Z_p and T_e is the change in environment temperature.

On a tephigram we may draw a new line to express the state of air of immediate environment, this has been drawn in Fig. 6 and is marked Z_eP

The parcel can then move upto Z_p' as per arguments above.

A comparison of the two methods stated just now was made and it was found that the results are very close to each other.

Mukherjee and Chaudhury (1971) could give the reason for cooling of environment as per above picture but could not produce a method of calculating the same. They explained the excessive overshooting on a qualitative reasoning but while attempting to compute Z_p' they assumed that equal amounts of falling air and the environmental air should get mixed. By this method they were able to explain the heights attained by cumulonimbus clouds as observed by them and also by Roach at Oklahoma. Since wind observations from Oklahoma are not available to the authors they have verified the observations for Calcutta for the cases where Z_p was exceeded. The results are given in Table 1.

From this it can be seen that the excessive overshooting can be satisfactorily explained by the present theory, which also gives a quantitative estimate of the cooling. In this calculation we have neglected the friction between the rising parcel and downward moving air. Again there may be some entrainment of environmental air by the downward moving air and hence there may be slight warming of this air.

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