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Estimation of turbulence parameters for application in air pollution modelling

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सार — दिन के समय के दौरान दिल्ली शहर के लिए उपलब्ध दैनिक मौसम सम्बन्धी प्रेक्षणों की सहायता से भूपृष्ठ पर घर्षण वेग u_* और घर्षण ताप θ_*, नामीय जैसे विक्षोभ प्राचलों की गणना के लिए मौनिन-औबूरखोव समानता सिद्धान्त पर आधार पार्श्विका सम्बन्धों का उपयोग किया गया है। इस विधि को अधिक वास्तविक बनाने के ध्येय से समानता अभिगम के लिए अपनाई गई पुनरावृतीय प्रक्रिया के साथ एक विकिरण मॉडल का भी समावेश किया गया है। संवहनी वेग परिमाप w_* की संगणना, स्थायित्व प्राचल ζ, पवन वेग और भंवरी विसरणता रूपरेखा और विसरित होते हुए मेघ की ऊर्ध्वाधर विसरित ऊँचाई σ_z जैसे बहुत महत्वपूर्ण किन्तु गुणात्मक रूप से ज्ञात नामक विक्षोभ प्राचलों के बहुविध उपयोगिताओं को प्रस्तुत किया गया है और उनका विस्तार से विवेचन किया गया है।

ABSTRACT. Profile relationships based on Monin-Obukhov similarity theory have been used to compute turbulence parameters, *viz.*, friction velocity, u_*, and friction temperature, θ_*, at the surface with available routine meteorological observations for the city of Delhi during day time. In order to make the method more realistic a radiation model has been coupled with the iterative procedure adotped for the similarity approach. The multifarious applications of turbulence parameters, *viz.*, the computation of convective velccity scale w_*, stability parameter ζ, wind velocity and eddy-diffusivity profiles, and the most important though qualitatively known, the vertical dispersion height, σ_z of diffusing cloud have been presented and discussed at length.

1. Introduction

The estimation of turbulence parmeters like friction velocity, u_* and friction temperature, θ_* are very important for air pollution modelling studies. Direct determination of these parameters can be done by extensive and difficult measurements with highly sophisticated instrumentation. In the absence of such measurements, which is very often the case, some alternative means have to be found which could easily be used for the purpose of computation of these parameters using available routine data. Following Berkhowicz and Prahm (1982), the present study deals with the computation of turbulence parameters using wind and temperature measurements in the surface layer based on Monin-Obukhov similarity theory.

2. Procedure

2.1. *List of symbols used*

c_p — Specific heat of air at constant pressure

E — Evaporation rate

f — Coriolis parameter (S^{-1})

G — Soil heat flux

H — Surface sensible heat flux

k — Von-Karman constant (0.4)

$\tilde{k}$ — A constant of value equal to 0.74

k_z — Eddy-diffusivity coefficient (m^2s^{-1})

$\hat{k}(z)$ — Non-dimensional eddy diffusivity coefficient

L' — Latent heat of vaporisation of water

L — Monin-Obukhov length scale (m)

θ' — Fluctuating component of potential temperature over mean

θ_* — Friction temperature at the surface (°C)

$\Delta\theta$ — Potential temperature difference at two levels in the surface layer (°C)

R — Net radiation

u_* — Friction velocity at the surface (ms^{-1})

u or Δu — Wind speed at anemometer height (ms^{-1})

u_g — Geostrophic wind speed (ms^{-1})

w_* — Convective velocity scale (ms^{-1})

w' — Fluctuating component of wind velocity over mean in the vertical direction

$\bar{\theta}'$ — Kinematic heat flux

z — Height at which u is measured

z_0 — Surface roughness length (m)

$\hat{Z}$ — Non-dimensional height

z_i — Mixing height (m)

x — Downwind distance (m)
X — Non-dimensional travel distance
ρ — Density of air
σ_y — Plume dispersion in the y direction (m)
σ_z — Vertical dispersion height of the plume (m)
η — Stability parameter for neutral boundary layer
ζ — Stability parameter (z_i/L)

2.2. Following non-dimensional wind shear and potential temperature gradient equations are used for the computation of turbulence paramenters :

$$\frac{kz}{u_*}\ \frac{\partial u}{\partial z} = \phi_m(\zeta) \tag{1}$$

$$\frac{kz}{\theta_*}\ \frac{\partial \theta}{\partial z} = \phi_h(\zeta) \tag{2}$$

The empirical analytical formlae for flux profile relationships ϕ_m and ϕ_h are those given by Businger *et al.* (1971) :

$$\phi_m(\zeta) = \begin{cases} (1-\gamma_m\,\zeta)^{-1/4} & \text{(unstable conditions)} \quad \text{(3a)} \\ 1+\beta\zeta & \text{(stable conditions)} \quad \text{(3b)} \end{cases}$$

$$\phi_h(\zeta) = \begin{cases} \widetilde{k}\,(1-\gamma_h\,\zeta)^{-1/2} & \text{(unstable conditions)} \quad \text{(4a)} \\ \widetilde{k}\,(1+\dfrac{\beta}{\widetilde{k}}\,\zeta) & \text{(stable conditions)} \quad \text{(4b)} \end{cases}$$

where, $\gamma_m = 15$, $\gamma_h = 9$, $\beta = 4.7$ and $\widetilde{k} = 0.74$.

The integration of Eqns. (1) and (2) leads to the following expressions (Paulson 1970) :

$$u_* = \triangle uk/[\ln\,(z/z_0) - \psi_m(z/L) + \psi_m(z_0/L)] \tag{5}$$

$$\theta_* = k\,(\triangle\theta)/\widetilde{k}\,[\ln\,(z/z_0) - \psi_h(z/L) + \psi_h(z_0/L)] \tag{6}$$

where L is defined as :

$$L = \frac{\theta\, u_*^2}{kg\theta_*} \tag{7}$$

where z is the height at which u is measured (19.8 m) and z_0 is the surface roughness length (1 m). $\widetilde{k}$ is a constant taken equal to 0.74.

Upon integrating Eqns. (1) and (2), following forms of ψ functions are obtained for unstable atmosphere (Paulson 1970) :

$$\phi_m(\zeta) = \int_0^{\zeta} \frac{1-\phi_m(\zeta)}{\zeta}\, d\zeta \tag{8}$$

$$= \ln\left[\left(\frac{1+X}{2}\right)^2\left(\frac{1+X^2}{2}\right) - 2\arctan X + \frac{\pi}{2}\right] \tag{9}$$

with

$$X = (1-\gamma_m\,\zeta)^{1/4} \tag{10}$$

and

$$\psi_h(\zeta) = \int_0^{\zeta} \frac{1-\dfrac{1}{\widetilde{k}}\,\phi_h(\zeta)}{\zeta}\, d\zeta \tag{11}$$

$$= \ln\left|\left(\frac{1+y}{2}\right)^2\right| \tag{12}$$

with

$$y = (1-\gamma_h\,\zeta)^{1/2} \tag{13}$$

Though, the terms $\psi_m(z_0/L)$ and $\psi_h(z_0/L)$ in Eqns. (5) and (6) are very small and may be neglected, they are indeed important for highly convective conditions. Hourly wind, temperature and other data required for this study have been obtained (Mani 1980).

In order to avoid the use of temperature at different levels we have estimated surface sensible heat flux from a radiation model which could be coupled with the similarity theory for the estimation of u_* and θ_*.

2.3. *Coupling of radiation model with the Monin-Obukhov similarity theory*

The energy budget at the surface may be represented as :

$$R = H + LE + G \tag{14}$$

where R is net radiation, LE is latent heat flux, G is soil heat flux and H is surface sensible heat flux. Each of these fluxes can be parameterised in terms of routine meteorological surface observations in a manner described by Manju Kumari and Sharma (1987). The surface sensible heat flux is related to the kinematic heat flux as :

$$H = -\,\rho\, c_p\, u_*\, \theta_* \tag{15}$$

In view of the above relation we may write,

$$\theta_* = -\,H/\rho\, c_p\, u_* \tag{16}$$

where ρ is the density of air and c_p is is the specific heat of air at constant pressure which is taken equal to 1005 $\mathrm{JK^{-1}\,kg^{-1}}$.

A numerical iterative procedure has been followed for the computation of u_*, θ_* and L in the following manner :

Step (*a*) — In the beginning a guess value for u_* is given as :

$$u_* = k\triangle u/[\ln\,(z/z_0)] \tag{17}$$

with $L = \infty$ in Eqn. (5). $\triangle u$ is the difference between the wind speed at the ground and anemometer level. θ_* is estimated from Eqn. (16).

Step (*b*) — Now L is computed by means of Eqn. (7).

Step (*c*) — The new value of L is then substituted in Eqn. (5) and u_* is again computed. θ_* is evaluated by means of Eqn. (16).

Step (d) — Step (b) and Step (d) are repeated till we get the convergence for L to the desired accuracy. It takes 5-6 iteration to achieve the accuracy of order 10^{-5}.

3. Application of turbulence parameters

Surface fluxes and turbulence parameters have numerous applications which we now describe in detail.

(*i*) *Non-dimensional height and stability parameter*— As pointed out by Arya (1975), the similarity theory which was first proposed by Kazanski and Monin (1960) and later on refined by others, refers that velocity and scalar fields in the PBL should be some universal functions of the following non-dimensional height, μ and stability parameter, η :

$$\mu = |f|\, z/u_*; \;\; \eta = ku_* \,/ |f| L| \tag{18}$$

where f is the coriolis parameter. The particular choice of the above similarity parameters is rather controversial as it is based on the doubtful assumption that under all stability conditions, the boundary layer height (z_i) is proportional to the dynamic height scale u_*/f. On the other hand, recent numerical studies have suggested that for the unstable boundary layer, its height is only fixed by the lowest inversion base (z_i), and u_*/f is not a relevent height scale (Deardorff 1972), which implies that one would have in place of Eqn. (18) :

$$\mu = z/z_i \,; \; \zeta = z_i/L \tag{19}$$

as the appropriate similarity parameters for convective conditions. Present study computes the parameter ζ using mixed layer height z_i (Manju Kumari 1985). z_i has been computed using Holzworth's (1967) technique.

Following Holzworth (1967) the height of intersection where the dry adiabat from surface temperature cuts the morning radiosonde (RS) temperature profile is known as mixing height. Based on five years data (1968-1972) mean monthly diurnal variation of the mixing height has been computed. The morning temperature profiles from RS observations have been obtained from *Aerological data of India,* published by India Meteorological Department, Delhi and surface temperature data were used from solar radiation data (Mani 1980).

(*ii*) *Wind speed profiles in the atmospheric boundary layer* — Generally, a constant value of geostrophic wind u_g, needs to be prescribed to evaluate u profile from various formulae. However, when the model is applied to real situation, u_g has to be computed from observed velocity profiles in the PBL. Problem arises due to lack of observations throughout the day; moreover, it is likely that these may not cover entire range of stabilities. We have overcome this difficulty, which arises when u_g is used by employing the formula given by Smith and Blackall (1979) :

$$u = \frac{u_*}{k}\left[\ln \frac{y-1}{y+1} + 2 \tan^{-1} y + \right.$$
$$\left. + \left\{ \ln \frac{|L|}{2z_0} - \frac{\pi}{2} \right\} \right]^{1/2} \tag{20}$$

where,

$$y = \left(1 + 16 \; \frac{z}{L} \right)^{1/4}$$

This has also been derived using similarity theory extended to the whole boundary layer during convective conditions.

(*iii*) *Evaluation of eddy-diffusivity profiles* — Schayes (1982) has recommended Wippermann's (1972) stationary barotropic formulation following the Rossby similarity relationship in Ekman layer. On comparing with the K_z profiles obtained from Wippermann's profile and from a one-dimensional 1.5 order closure PBL model, he found that this form agrees reasonably well with the 1.5 order closure model results. Accordingly, the non-dimensional K_z profile is given by a simple relation like :

$$\hat{K}(z) = \hat{z} \; \exp\,(- \overline{C}\, z^{0\cdot 764}) \tag{21}$$

where all symbols are dimensionless quantities given by

$$\hat{K} = \frac{K_z f}{k^2 {u_*}^2} \text{ and } \hat{z} = \frac{zf}{ku_*}$$

The parameter $\overline{C}$ is dependent on ζ and is given as :

$$\overline{C} = \exp\,(0.264 + 0.0162\, \zeta' + .000396\, \zeta'^2) \tag{22}$$

where, $\zeta' = \zeta + 50$.

(*iv*) *Vertical dispersion of the diffused cloud* (σ_z) — Increasing air pollution activities all over the globe have resulted in a demand for increased data for plume dispersion parameter that is relevent to the region under study.

A number of studies have pointed out that the amount dispersion undergone by a plume will depend on both the vertical and the stability conditions of the boundary layer. The dispersion coefficients are normally selected from the stability classification used as a substitution for the determination of intensity of turbulence. It was thought that the dispersion of the cloud derived from the turbulence parameters estimated for a particular site under study and its hourly variation is most likely to give a better estimate and understanding of the vertical spread. According to Briggs (1984), following formula has been used for the estimation of vertical dispersion parameter, σ_z, for ground level area sources under convective-neutral conditions :

$$\sigma_z = z_i\, X\, [0.6\, X + (0.64\, u_*/w_*)^2]^{1/2} \tag{23}$$

where X is the non-dimensional travel distance defined a as :

$$X = \frac{x\, w_*}{u z_i} \tag{24}$$

and

$$w_* = \left(\frac{g}{\theta} \, \overline{w'\theta'} \;\; z_i \right)^{1/3} \tag{25}$$

where x is the downwind distance.

4. Results and discussion

Table 1 summarises the hourly variation of friction velocity, convective velocity scale, Monin-Obukhov length scale, stability parameter, wind speed at the top of boundary layer assumed geostrophic wind, u_g and mixing height during daytime for the convective boundary layer. Criterion for selecting the convective state of the atmosphere has been taken from Venkatram (1980). The results are shown for the city of Delhi for the month of October. However, this scheme is general and could be applied under different conditions and for any location. It has been found that u_* and w_* increase with time and show the maximum value at 13 IST and 12 IST respectively and thereafter they decrease. Slight timelag between the two maxima could be explained by the fact that the thermal effects start dominating during daytime, thereby giving an early maxima for w_*. The absolute value of L shows similar trend as in u_* which first increases to a maximum value at 13 IST and then decreases. The absolute value of ζ shows a trend similar to that observed in w_*, *i.e.*, it first increases to a maximum value at 12 IST and decreases later on. The possible reason for this kind of trend between the two variables (w_* and ζ) could be attributed to the greater thermal effects (higher sensible surface heat flux) and higher mixing heights for the former and only high mixing height for the latter. Higher mixing heights during daytime have been explained by Manju Kumari (1985). The table also shows that u_g (taken as wind speed at the top of mixing height) always increases with time.

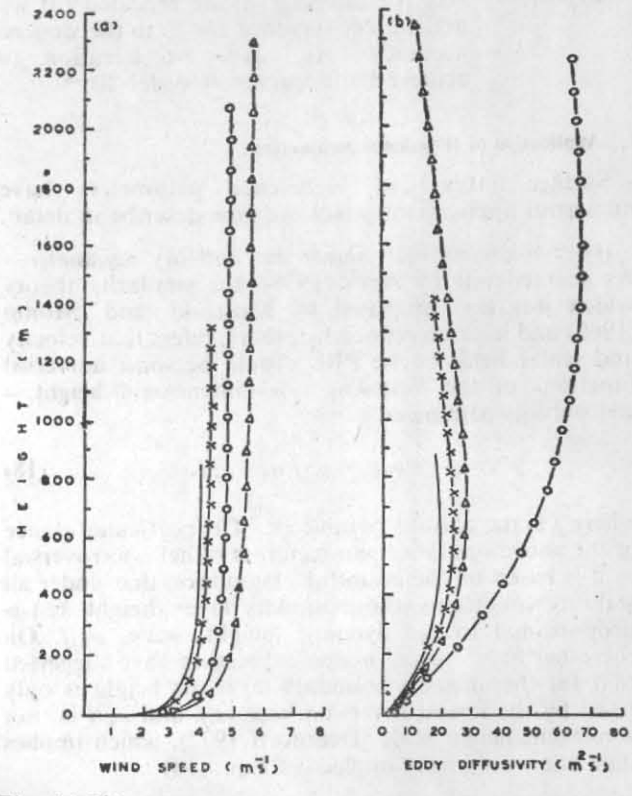
Fig. 1(a) shows the wind speed profile at 9, 10, 12 and 15 IST. It has a sharp gradient in the first few hundred metres and then it shows a steady variation. This kind of profile agrees well with the typical observed profiles.

Fig. 1(b) shows the eddy-diffusivity profiles, again at 9, 10, 12 and 15 IST. It first increases, attains a maximum value at certain height and then decreases as shown by a typical K_z profile. The decrease after attaining a maximum is not signficantly shown in afternoon hours when H is very high and the boundary layer is well mixed. The shape of K_z profiles shows following features which are not well depicted in the empirical K_z profiles used in earlier models (*e.g.*, Ragland 1973) :

(*a*) a smooth and gradual variation of K_z with height.

(*b*) decrease in eddy-diffusivity, once it attains a maximum value.

Fig. 2 shows the relationship between vertical spread σ_z and downwind distance. The values are plotted upto the distance where the vertical spread is less than the mixed layer height since most analytical forms based on Gaussian plume hypothesis, in a way restricts the use of σ_z whenever it exceeds the mixing height. It is noted that this distance increases with time. One of the reasons which may be responsible for this kind of behaviour could be the constant increase of mixed layer height as time progresses.



Figs. 1(a&b). Wind speed and eddy diffusivity profiles at different times of the day. Time symbols used : — : 0900, × : 1000, o : 1200, △ : 1500 IST

TABLE 1

Hourly variation of various physical parameters at Delhi during day time computed from routine observations

Time (IST)	Friction velocity u_* (ms^{-1})	Convective velocity scale, w_* (ms^{-1})	Monin-Obukhov length scale L (m)	Stability parameter (zi/L)	Wind speed at the top of mixing height (ms^{-1})	Mixing height (m)
9	.348	.9	—75.09	—11.55	4.09	867
10	.384	1.23	—61.50	—21.90	4.46	1347
11	.406	1.47	—59.37	—30.18	4.77	1792
12	.428	1.56	—66.30	—31.94	5.17	2118
13	.453	1.54	—86.16	—26.25	5.75	2270
14	.441	1.44	—105.25	—23.01	5.81	2422
15	.420	1.17	—163.63	—14.35	5.94	2348
16	.338	.419	—186.36	—12.60	6.29	2348

In general, the values of σ_z's which have been obtained here are higher than those obtained by Pasquill (1974). It is worth mentioning here that Whaley and Lee (1978) finds that the product of dispersion parameters ($\sigma_y\sigma_z$) consistently reveal higher values than those which would be predicted by Pasquill. This effect is more apparent closer to the source. They also state that in neutral conditions their measured

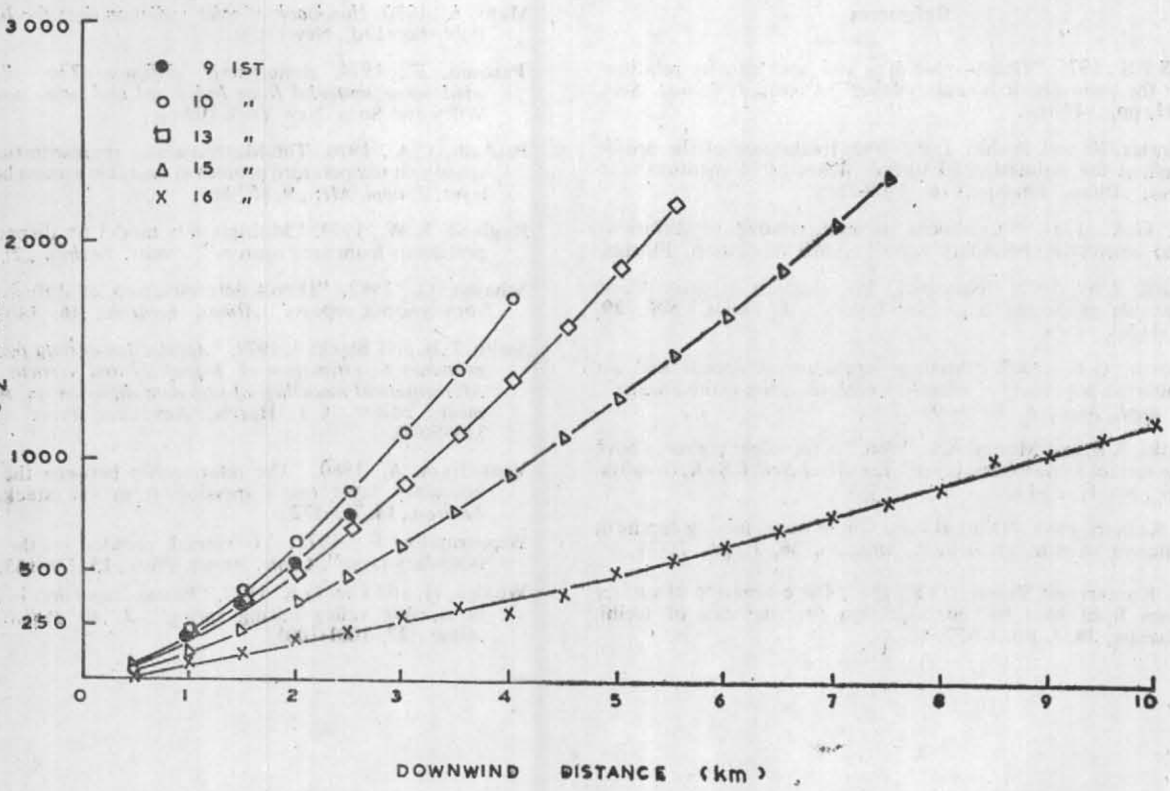


Fig. 2. Vertical dispersion parameters *versus* downwind distance at different times of the day

values were greater than the corresponding Pasquill values by about one order of magnitude close to the source and reducing to half an order of magnitude at 10 km from the source. In stable conditions they found that the measured values were two and a half order of magnitude greater than those obtained by Pasquill (1974). In view of these results we may expect for the convective conditions, an over-estimated σ_z value, though they may not be as large as it has been reported in other studies. These results, however, tell us precisely that experiments for the estimation of σ_z values should certainly provide a better idea regarding the suitability of the existing techniques.

Another interesting feature may also be noted here that σ_z values are higher in the early hours of the day (9 and 10 IST) than those at noon hours (13 and 15 IST) at nearby distances whenever such a comparison is possible. This is due to fact that small scale convective motions prevail during early hours of the day which shall force the plume to mix within this cell completely while the larger convective cells observed at later hours of the day would not do so because the plume size is much smaller than the cell dimensions. Again, there may lie a scope of improvement in the input of Eqn. (24), *viz.*, the value of u_*, w_* and z_i. Efforts are being made in this direction; but emphasis here is mainly on a useful and inexpensive technique which may be quite helpful in estimating many important parameters. Finally, the observations will give a more precise picture regarding the use of qualitatively determined σ_z values or its estimation using the technique presented here.

5. Conclusion

Following are the conclusions of this study :

(*i*) Turbulence parameters have been estimated for the city of Delhi on hourly basis during daytime using routine meteorological observations.

(*ii*) Stability parameter, wind speed and eddy diffusivity profiles, and the vertical dispersion parameter σ_z have been determined using the above mentioned turbulence parameters.

The utmost need now is to have observations regarding the plume spread so that further improvement in the existing technique could be made possible. Though efforts are being made for these improvements on theoretical grounds, however, the observations should merit most in order to ascertain these improvements. Therefore, we may conclude that an easily adaptable technique has been presented here to give useful parameters for the air pollution modelling studies.

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