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Model evaluation of solar radiation on an inclined plane

H. P. GARG and SAT NARAIN GARG

Centre of Energy Studies,

Indian Institute of Technology, New Delhi

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सार – इस शोध पत्र में सौर ऊर्जा उपयोगिता की दृष्टि से ढालू तल पर भूमण्डलीय सौर विकिरण के आकलन के लिए उपलब्ध कुछ भिन्न-भिन्न विधियों का मूल्यांकन करने का प्रयास किया गया है । तीन सामान्य विधियों जैसे बीम की विधि, ल्यू एवं जोरडन विधि (दैनिक मानों और घंटावारी मानों का उपयोग करते हुए) क्लुचर विधि का वर्णन किया गया है । यह देखा गया है कि जब बीम विधि से विकिरण के आकलन का मान अधिक आता है तब ल्यू और जोरडन दोनों विधियों से मान हमेशा कम आकलित होता है । क्लुचर विधि में सभी मौसम की दशाओं में कम से कम त्रुटि आती है । इसलिये उष्णकटिबंधीय देशों में इसके उपयोग की संस्तुति की गई है ।

ABSTRACT. In this paper an attempt has been made to evaluate some of the different methods available for estimating global solar radiation on an inclined plane, from the solar energy utilization point of view. Three commonly used methods like the Beam method, Liu and Jordan's method (using daily values and hourly values), and the Klucher method are discussed. It is observed that while the beam method overestimates the radiation, Liu & Jordan's both methods, always underestimate it. The Klucher method gives the least error under all weather conditions and is hence recommended for use in tropical countries.

1. Introduction

Most solar energy appliances use flat-plate collectors to absorb radiant energy, tilted towards the equator at an angle depending on the latitude of the place. At most places, solar radiation is generally measured on a horizontal surface. To determine it on a tilted surface, one employs generally either the beam method or the Liu and Jordan method. In this paper in addition to the beam method (1958) and Liu & Jordan's (1961, 1969) two methods, we have considered a third method by Klucher (1979) which is an improvement on the Temps and Coulson's (1977) method. We have made comparative study of all these methods which are commonly used, to test their validity under the semi-arid conditions prevalent at New Delhi and where the percentage of diffuse radiation is greater than that at other continental locations.

2. Theory

Nomenclature

H = Daily summation of global solar radiation on the horizontal surface.

H_t = Daily summation of global solar radiation on the tilted surface.

D = Daily summation of diffuse radiation on the horizontal surface.

I_{Th} = Global solar radiation received by the horizontal surface per unit area per unit time.

I_{dh} = Diffuse radiation received by the horizontal surface per unit area per unit time.

R_D = Conversion factor for daily direct radiation

$$= \frac{\text{Daily direct radiation incident on the tilted surface}}{\text{Daily direct radiation incident on the horizontal surface}}$$

R_d = Conversion factor for daily diffuse radiation

$$= \frac{\text{Daily diffuse radiation incident on the tilted surface}}{\text{Daily diffuse radiation incident on the horizontal surface}}$$

R = Conversion factor for ground reflected radiation

$$R = \frac{\text{Daily ground reflected radiation incident on the tilted surface}}{\text{Daily total radiation reflected by the horizontal surface}}$$

ρ = Albedo of the ground near the tilted surface which is the reflectivity of the ground and is given by

$$\rho = \frac{\text{Daily global solar radiation reflected diffusely by the ground}}{\text{Daily global solar radiation incident upon the ground}}$$

ρ = 0.2 (assumed)

W_s = Sunset hour angle

W_s' = Hour angle when the direct sun rays just start falling upon the tilted surface

$F = 1 - I_{dh} / I_{Th}$

θ_h = Angle of incidence of the direct sun rays on the horizontal surface. This is same as the zenith angle

θ_t = Angle of incidence of the direct sun rays on the tilted surface

$\alpha = (90 - \theta_h)$, is the altitude of the sun

L = Latitude of the place under observation, for Delhi $L = 28° 38'$

A_θ = Azimuth angle of the sun

n = Number of day of the year

δ = Declination of the sun and is given by

$$\delta = 23.45 \sin \frac{(284+n)\ 360}{365}$$

β = Angle of inclination of the surface with the horizontal ($\beta = 45°$ in these observations)

An inclined surface receives direct, diffuse and reflected components of solar radiation and for modelling convenience these will be treated separately.

In all the models so far developed, direct radiation is treated as a vector quantity and calculated for a tilted surface from trignometrical relations. The main difficulty lies in the prediction of diffuse radiation on a tilted surface. Different authors have made different assumptions in its computation. Morse and Czarnecki treats diffuse radiation also as a part of the beam radiation. Their argument is that on a clear and bright day, most of of the diffuse radiation comes from the direction of the sun (so-called circumsolar diffuse radiation) and so it may be treated as the beam radiation. On cloudy days, diffuse radiation as well as direct radiation are very small and so this assumption will not affect the results very seriously. Liu and Jordan (1961, 1969) assume isotropic distribution of the diffuse radiation. Kiucher (1979) has introduced several factors with the diffuse component to account for the circumsolar radiation and the brightening of the sky near the horizon. These methods in short are discussed below :

(*a*) *Beam method*

To compute hourly values of the global solar radiation on a tilted surface, first of all we calculate the angle of incidence (also called zenith angle of the sun) on the horizontal surface using the following equation :

$$\cos \theta_h = \cos L \cos \delta \cos W + \sin L \sin \delta \quad (1)$$

where δ is calculated for each day using the equation :

$$\delta = 23.45 \sin [(n+284)\ 360/365] \quad (2)$$

where, n is the number of the day of the year.

The azimuth angle for a particular hour can be calculated by the following equation :

$$\cos A_\theta = \frac{(\sin L \cos \delta \cos W - \cos L \sin \delta)}{\sin \theta_h} \quad (3)$$

The angle of incidence, θ_t, on a tilted surface facing south can be computed from the following equation :

$$\cos \theta_t = \cos \theta_h \cos \beta + \sin \theta_h \sin \beta \cos A_\theta \quad (4)$$

After this $\cos \theta_t / \cos \theta_h$ is calculated, which is the hourly conversion factor for the given tilted surface. These hourly values are summed to give the global radiation for the whole day.

(*b*) *Liu & Jordan's method* (*using total values for the whole day*)

Liu and Jordan have assumed that the conversion factor (ratio of direct radiation on the tilted surface to that on the horizontal surface) for direct radiation is the same on the ground as at the top of the earth's atmosphere. For the diffuse component also, they assume that it is isotropic over the whole sky dome. Also the ground reflected radiation is assumed to be diffuse in nature. The expression for global solar radiation on an inclined plane, H_t, is given as :

$$H_t = (H - D)\, R_D + D R_d + H \rho R_\rho \quad (5)$$

where,

$$R_D = \frac{\cos(L-\beta)}{\cos L}\left(\frac{\sin W_s - W_s \cos W_s'}{\sin W_s - W_s \cos W_s}\right) \text{if } W_s \leqslant W_s' \quad (6)$$

$$R_D = \frac{\cos(L-\beta)}{\cos L}\left(\frac{\sin W_s' - W_s' \cos W_s'}{\sin W_s - W_s \cos W_s}\right) \text{if } W_s' \leqslant W_s \quad (7)$$

and

$$R_d = (1 + \cos \beta)/2 = (1 + \cos 45°)/2 = 0.8536 \text{ for } \beta = 45° \quad (8)$$

and lastly

$$R_\rho = (1 - \cos \beta)/2 = 0.1465 \quad (9)$$

TABLE 1

Comparison of daily totals of measured and computed radiation at 45° angle with the horizontal at New Delhi (India)
(Unit is : MJ/M^2 day)

Date (1980)	Radiation on horizontal surface		Global radiation at 45° angle	Beam method		Liu & Jordan's method of daily conversion factor		Liu & Jordan's method of hourly value		Klucher's method	
	Global	Diffuse		Computed	Error (%)	Computed	Error (%)	Computed	Error (%)	Computed	Error (%)
26 Oct	15.64	6.06	21.03	22.67	+7.8	19.98	—5.0	19.17	—8.2	20.22	—3.8
27 Oct	15.29	4.82	20.57	21.65	+5.3	20.25	—1.5	19.21	—6.6	20.27	—1.5
28 Oct	16.91	5.10	23.60	25.01	+6.0	22.55	—4.4	21.98	—6.7	22.82	—3.3
29 Oct	16.85	4.62	23.38	24.30	+3.9	22.76	—2.6	21.97	—6.0	23.05	—1.4
30 Oct	15.35	6.16	20.42	22.32	+9.3	19.48	—4.6	18.94	—7.2	20.15	—1.3
1 Nov	8.40	5.88	10.19	13.01	+27.6	9.30	—8.8	9.15	—10.2	9.74	—4.5
4 Nov	12.75	7.61	16.99	19.75	+16.2	15.09	—11.2	14.76	—13.1	15.85	—6.7
5 Nov	15.22	6.12	21.37	23.46	+9.7	20.22	—5.4	19.45	—9.0	20.78	—2.8
6 Nov	17.34	3.17	25.96	26.68	+3.1	25.89	0.0	24.84	—4.3	25.75	—0.8

The sun set hour angle, W_s, is calculated using the equation :

$$\cos W_s = -\tan L \tan \delta \qquad (10)$$

W_s', which is the hour angle when the direct sun's rays are just grazing the sloping surface, is calculated using the equation :

$$\cos W_s' = -\tan (L-\beta) \tan \delta \qquad (11)$$

For the calculation of R_D, we use either Eqn. (6) or Eqn. (7) depending on whether W_s or W_s' is smaller.

After substituting values of R_d, ρ and that of R_ρ Eqn. (5) becomes:

$$H_t = (H-D)\, R_D + D \times 0.8536 + H \times 0.0293 \qquad (12)$$

Values of H and D for the whole day are substituted in Eqn. (12) and H_t is calculated and the same are shown in Table 1.

(c) Liu & Jordan method (using hourly values)

This method differs from the preceding Liu & Jordan method in that here direct radiation on a tilted surface is calculated as follows :

Hourly Direct Component

$$\text{on tilted surface} = (I_{Th}-I_{dh}) \frac{\cos \theta_t}{\cos \theta_h} \qquad (13)$$

Hourly values are calculated and then summed to give the direct radiation on the tilted surface for the whole day. Diffuse and ground reflected components are calculated in the same way as in the preceding method.

(d) Klucher method

Liu and Jordan assumed that diffuse radiation and the ground reflected radiation are distributed isotropically. But the non-isotropic nature of these radiations including a maximum in the direction of the direct radiation and near the horizon, a minimum in a direction perpendicular to the direction of the sun, introduces a significant error in the calculation of radiation on a sloping surface. From Table 1 and Fig. 1 we see that the percentage error by the Liu & Jordan method (both the methods) is very large on cloudy days which is as high as 13 per cent for 60 per cent cloudy days (cloudiness is defined as D/H, D and H values are for the whole day). This method works well on clear days when the diffuse part is very small.

To improve the results, Temps and Coulson (1977) introduced another model, an 'anisotropic-clear-sky model' in which they modified the diffuse part by associating several factors to account for various observed facts.

According to them, the diffuse part is given by:

$$I_{dh}\, [(1 + \cos \beta)/2]\, (1 + \sin^3 \beta/2) \times \times (1 + \cos^2 \theta_t \cos^3 \alpha) \qquad (14)$$

where the factor $(1+\sin^3 \beta/2)$ stands for the increase in skylight near the horizon on clear days. The factor $(1 + \cos^2 \theta_t \cos^3 \alpha)$ accounts for circumsolar diffuse radiation. But this model was also limited to clear days only. On cloudy days it gave higher values. In this effort, another model was developed by Klucher who modified the Temps and Coulson model by introducing a term F, defined as :

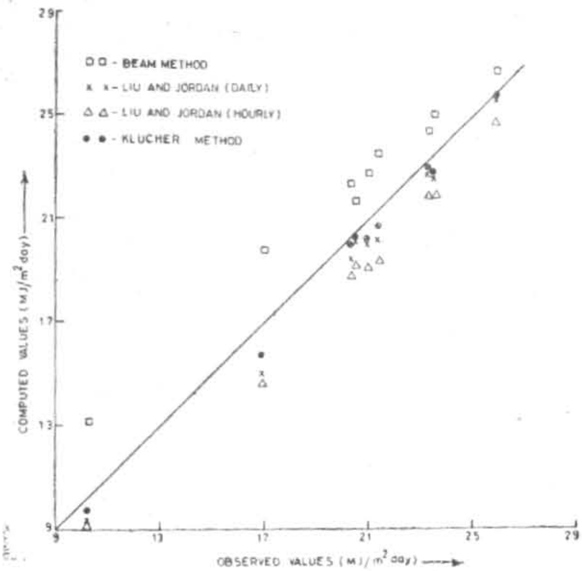

Fig. 1. Daily global radiation at 45° inclination

$$F = (1 - I_{dh}/I_{Th}) \qquad (15)$$

which accounts for all types of days.

Thus the diffuse part on a tilted surface becomes:

$$I_{dh} \; [(1 + \cos\beta) / 2] \times (1 + F \sin^3 \beta/2) \times$$
$$\times (1 + F \cos^2 \theta_t \cos^3 \alpha) \qquad (16)$$

On clear days, I_{dh} is very small compared to I_{Th} and so F is nearly one and then the diffuse part is the same as that given by Temps and Coulson. On cloudy days, I_{dh} is nearly the same as I_{Th} and so F is nearly zero and so the diffuse part becomes the same as that given by Liu and Jordan.

The complete formula can thus be written as :

$$I_{Tt} = (I_{Th} - I_{dh}) \cos\theta_t / \cos\theta_h \; +$$
$$+ I_{dh} [(1 + \cos\beta)/2] (1 + F \sin^3 \beta/2) \times$$
$$\times (1 + F \cos^2 \theta_t \cos^3 \alpha) \; +$$
$$+ I_{Th} \, \rho \, [(1 - \cos\beta) / 2] \qquad (17)$$

The ground reflected component (approx. 3 per cent of global radiation) is very small when compared to the direct or diffuse components and so in the Klucher method, it has been calculated by the Liu and Jordan's method.

3. Method of measurement

Three pyranometers were used for observations. First of all, all the three were calibrated against a fourth pyranometer (used as reference) on a very clear day. One pyranometer (Kipp and Zonen) was set horizontally to measure global radiation on a horizontal surface. Another pyranometer (National Instruments, Calcutta, India) was also set horizontally. It was provided with a shading ring with a diameter of 46.5 cm and width of 5.0 cm to cut off the direct radiation. A correction factor as suggested by Drummond (1964) was applied to account for the diffuse radiation cut-off by the shading ring. The third pyranometer (Eppley) was set at a 45 degree angle facing south which is the optimum tilt for Delhi in winter. All the pyranometers were installed on concrete pillars, six feet above the roof of Indian Institute of Technology, Delhi building and utmost care was taken to avoid any shade or reflection upon these pyranometers from the neighbouring buildings or other objects round the year. To avoid moisture condensation inside the glass dome, silica-gel was replaced from time to time.

The output of all the pyranometers was automatically recorded separately by potentiometric recorders and integrators.

To compare the above discussed models for predicting the solar radiation on an inclined plane we have selected days depending on the precentage of diffuse radiation when compared to the global radiation. In the present study, clear days, partially cloudy days and cloudy days are defined as those days when the diffuse component is less than 30 per cent, between 30 and 40 per cent and above 40 per cent respectively. In the observations of the pyranometer fixed at 45 degree angle, it has been assumed that conversion factor of the pyranometer does not change with the tilt.

4. Discussion of results

(1) *Beam method*

The daily conversion factor, R_D for the direct component is either one or greater than one for a south facing surface ($\beta = 45°$ at Delhi for six months (October to March). It becomes as high as 1.8 in the month of December. During the remaining six months (April to September) R_D is either one or less than one with the minimum value of 0.7 in the month of July. Our observations are for two weeks, in the last week of October and the first week of November. So during this observation period R_D is either one or greater than one. There is no error in the computation of direct component on the tilted surface. But for this period the diffuse component on the tilted surface will be overestimated because the actual conversion factor for diffuse component is always nearly one for 45 deg. tilt. It is never as high as 1.8. So the computed values of global radiation on the tilted surface will be higher than the observed values for these two weeks as is evident from Table 1. On cloudy days the diffuse component is comparatively higher and so more will be the overestimation in global radiation computation. On 1 November 1980 (70 per cent cloudiness) the overestimation is +27.6 per cent while on 6 November 1980 (18.3 per cent cloudiness) it is + 3.1 per cent. So the beam method is more suitable on clear days when the diffuse component is comparatively small. A comparison is shown in Fig. 1. If we apply this method during the period April to September, the computed values of the global radiation will be less than the observed values.

(2) *Liu and Jordan method (using whole day values)*

This method underestimates the diffuse component on tilted surface due to its unsymmetric distribution. On very clear days the distribution is more unsymmetric and hence more is the underestimation in its computation. In the quoted observations, 6 November 1980 is the clearest day when the diffuse component is 18.3 per cent of the global component. On this day this method gives 28 per cent less diffuse component as compared to the Klucher method. But R_D assuming extraterrestrial conditions, is higher than the weighted average value of $\cos\theta_t/\cos\theta_h$ for the same day. So the direct component on the tilted surface is overestimated. But this overestimation in the direct component is very small as compared to the underestimation in the diffuse component. The overall effect is that this method gives less global radiation on the tilted surface than the observed one. On 6 November 1980 (clear day) the error is less than 0.1 per cent. On cloudy days the diffuse component is more but simultaneously it becomes more symmetric and hence less underestimation in its computation on the tilted surface. But due to the high value of diffuse component, the per cent error in global radiation computation is more on cloudy days. The error on 4 November 1980 (70 per cent cloudiness) is —11.2 per cent.

(3) *Liu and Jordan's method (using hourly values)*

This method eliminates the error in the computation of the direct part on the tilted surface by using hourly values of ($\cos\theta_t / \cos\theta_h$) rather than R_D for the whole day. The diffuse part in this method is calculated as in the previous method so that global radiation is underestimated here also. But underestimation in this case is more than that in the previous case, because in the previous case the direct part was slightly overestimated, due to which the error in the previous case were a little less than the errors in this case as can be seen in Table 1.

(4) *Klucher's method*

The direct part is calculated in the same way as in the preceding Liu and Jordan method, *i.e.,* by taking the hourly values of ($\cos\theta_t/\cos\theta_h$) and then multiplying with the corresponding hourly values of (I_{Th}—I_{dh}), finally summing these hourly values of (I_{Th}—I_{dh}) $\cos\theta_t/\cos\theta_h$ for the whole day. For the calculation of the diffuse part on the tilted surface, various correction factors like $(1+F\sin^3\beta/2)$, $(1+F\cos^2\theta_t\cos^3\alpha)$ are applied in addition to the Liu and Jordan conversion factor $(1+\cos\beta)/2$ which is here 0.8536 for $\beta = 45$ deg. The results are compared in Table 1. The estimated and computed values are compared in Fig. 1.

5. Conclusions

(1) The beam method overestimates the radiation on tilted surface and this overestimation is found to be as high as +27.6 per cent depending upon the extent of cloudiness.

(2) Liu and Jordan's first approach (which uses a conversion factor for the whole day) underestimates the global radiation on the tilted surface and shows values as high as —11.2 per cent.

(3) Liu and Jordan's second approach (using $\cos\theta_t/\cos\theta_h$ for direct component calculation) also underestimates the global radiation and this underestimation is more than that of the first approach.

(4) Klucher method gives the best result, among all these four methods. From Table 1, it appears that this method also underestimates the radiation on a sloping surface, but, this underestimation is comparatively insignificant. The maximum error on a 60 per cent cloudy day is only —5.7 per cent.

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