Mausam, (1982), 33, 2, 159-164

551.515.3(545:547)

A theoretical approach to the problem of visibility associated with *Andhi* — Part 1 : Settling of raised dust due to gravity

A. K. MUKHERJEE and B. K. BASU

Meteorological Office, Pune

(*Received 14 November 1980*)

सार — जोसफ एवं अन्य (1980) ने उत्तर-पश्चिम भारत के संवहनी धूल भरे तूफान की आंधी से संबद्ध दृश्यता में परिवर्तनों को उपकरण द्वारा लिए गए प्रेक्षणों से दर्शाया है। इस शोध पत्र में समीप आते हुए चंडवात के झोंकाग्र पर अल्पावधि स्थिर उद्वाह के साधारण प्रदर्श का उपयोग करके सैद्धांतिक रूप से विचार करके उपरोक्त को ज्ञात करने का प्रयास किया गया है। केवल गुरुत्व के कारण धूल के नीचे बैठने से दृश्यता में सुधार हो जाने के बारे में विचार किया गया है। कुछ वास्तविक मानों को लेकर यह दिखाया गया है कि दृश्यता विचरण प्रेक्षणों से बहुत अच्छी प्रकार मेल खाते हैं।

ABSTRACT. Joseph *et al.* (1980) have shown from instrumental observations the changes in visibility associated with *Andhi*, the convective duststorm of northwest India. In the present paper an attempt has been made to deduce the above from theoretical consideration using a simple model of short period constant updraft at the gust front of approaching squall. The improvement of visibility by settling of dust due to gravity alone has been considered. Taking some realistic values the visibility variations have been shown to agree very well with the observed ones.

1. Introduction

April, May and June are months of *Andhi*, the convective duststorms of northwest India. Joseph *et al.* (1980) have reviewed the existing knowledge of *Andhi* and have compared them with *Haboob* of Sudan (Lawson 1971) and similar duststorms over U.S.A. (Idso *et al.* 1972). Whereas all previous work on duststorms concentrated on the description and climatology of the storm itself with some details about the extent, in vertical, of these storms, none have described the variation of visibility associated with these storms. Main difference of duststorms and thundersquall lies in the deterioration of visibility due to raised dust. In this matter attempt by Joseph *et al.* is the first of its kind. In the present work a theoretical study has been made about the improvement of visibility after the passage of the squall. After the dust is raised in a duststorm the improvement of visibility will be due settling by action of gravity *plus* any modification due to the physical and dynamical condition of the environment. The present paper gives the computation of improvement of visibility by gravity settling.

A simple model of storm has been assumed. From the data of different parameters obtained by Lawson (1971) and Idso *et al.* (1972) some realistic values of speed of gust front and thickness of dust wall were taken. From these a computation has been done to find out the variation of visibility in *Andhi*. The computed values were found to agree well with those observed by Joseph *et al.* (1980).

2. Theory

A model of the gust front has been assumed first. It has been found by previous workers that the duststorm occurs due to the gust front of the spreading downdraft from a cumulonimbus. It has been found by Charba (1974) and Goff (1976) that a fairly strong updraft is associated with the gust front. Though they are short lived, the value of updraft may be about 10 mps. It has been assumed that the dust is raised by updraft and then settles by itself due to action of gravity. Thus even in moderate wind following the passage of the gust front the updraft associated with turbulence has been taken to be negligible. The steps followed in the development of the model have been given below.

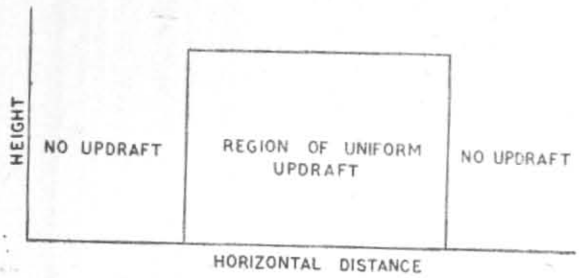

Fig. 1. Schematic diagram of gust front

2.1. *A model of the gust front*

In the present model it is envisaged that a region of uniform strong updraft exists within the advancing gust front. As this region of updraft passes over a stretch of open surface, dust and loose sand are raised from ground and forced upward due to viscous drag. Assuming an updraft profile as shown in Fig. 1, any dust particle raised from ground will experience a constant updraft as long as it is within the region of updraft and zero updraft outside this region. Further, if it is assumed that the region of updraft is deep in the vertical, and the gust front moves swiftly, then the dust particle will be within the region of updraft for the entire period of time during which the region of updraft passes over the point on earth's surface from where the dust particle was initially lifted up. Thus, a dust or sand particle lifted up from the ground at any instant of time after the onset of the region of updraft will experience a uniform updraft, till the region of updraft passes over this point on earth's surface, continue to move upward and finally exhausting its upward momentum will start falling towards the surface of earth.

2.2. *Motion of individual particles*

The equation of motion of the dust particle can be written as :

$$m \frac{d^2z}{dt^2} = -mg + \eta A \frac{dv}{dr} \qquad (1)$$

where η is the coefficient of viscosity, A is the effective area of interception and dv/dr is the horizontal gradient in the velocity field. Assuming the shape of the particle to be cubic and substituting $A = ka^2$ and $dv/dr = (w' - w)/k'a$, where k and k' are constants of proportionality, w' is the constant updraft and w, the vertical velocity of the particle, we can rewrite Eqn. (1) in the form :

$$\frac{d^2z}{dt^2} = -g + \frac{K}{a^2}(w' - w) \qquad (2)$$

where,

$$K = \eta\, k/\rho_s k'$$

Integrating Eqn. (2) we get :

$$w = \frac{dz}{dt} = (w' - ga^2/K)(1 - e^{-Kt/a^2}) \qquad (3)$$

and

$$z = (w' - \frac{ga^2}{K}) \left\{ t - \frac{a^2}{K}\left(1 - e^{-Kt/a^2}\right)\right\} \qquad (4)$$

where the constants of integration have been evaluated using the boundary conditions $w = 0$ and $z = 0$ at $t = 0$. It is seen from Eqn. (3) that the maximum velocity reached by particles at $t \rightarrow \infty$ is given by :

$$w_0 = (w' - ga^2/K) \qquad (5)$$

This is similar to the terminal velocity of particles falling under gravity through a fluid. If T be the duration of the region of updraft over any point on the surface of earth, then t_0 is less than or at most equal to T. The highest point z_h in the trajectory of the particle is reached at a time t_h such that

$$w_h = 0,$$

$$t_h = t_0 + \frac{a^2}{K} \ln \frac{Kw'}{ga^2}$$

$$\text{and} \quad z_h = w_0 t_0 - \frac{ga^4}{K^2} \ln \frac{Kw'}{ga^2} \qquad (6)$$

Evidently no particle of size 'a' can be lifted beyond the height :

$$z_{max} = w_0 T - \frac{ga^4}{K^2} \ln \frac{Kw'}{ga^2} \qquad (7)$$

2.3. *Dust content and particle size distribution in duststorms*

In a duststorm, the region of updraft due to the gust front passes over any fixed point on ground for a short period of time. Joseph *et al.* (1980) calculated from the time difference in the onset of *Andhi* at two neighbouring locations that the gust front moves with a speed of about 60 kmph. From Lawson (1971) and Idso *et al.* (1972) we know that the thickness of the layer of raised dust is about 2 km. From the work of Charba (1974) it is further known that the magnitude of updraft in gust fronts is about 10 m/sec. Hence, the region of updraft in the present model will be over any location for about three and a half minutes. It is extremely difficult to carry

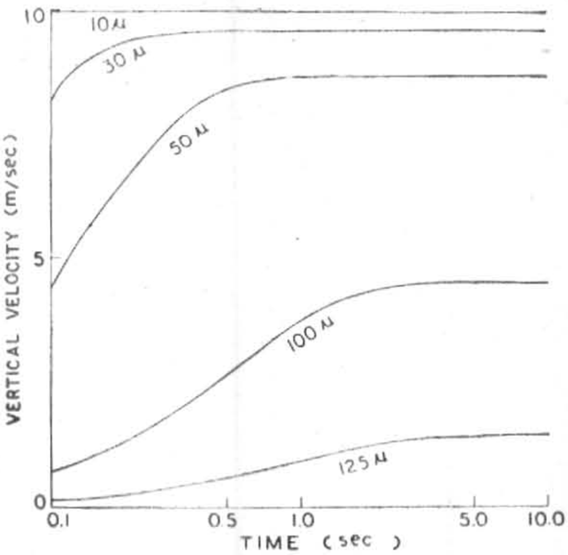

Fig. 2. Variation of vertical velocity with time

out measurements in such a short time to determine the dust content and the distribution of particle size within the region of updraft. However, results of many measurements carried out within the duststorm which continues for a considerable time after the region of updraft has moved away, are available in literature. For *Haboobs* at Khartoum, Lawson (1971) reported a particle concentration not exceeding 40 mg/m^3 and particle size between 10 microns and 50 microns while the fall out time was between 1 and 7 hours. Kharitonova (referred by Lawson) found for duststorms at Uzbekisthan in USSR dust concentrations around 40 mg/m^3 at a height of 3 m and particle sizes between 10 microns and 260 microns. For duststorms in USA Kamra (1972) found, by collecting the particles which are settling at ground, that the particle size varies between 3 microns and 600 microns as in the case of gypsum dunes in New Mexico. Thus, the particle size distribution and the relative abundance of particles of different sizes vary widely from location to location depending on the rate of physical processes which creates, destroys or transports the particles. According to Bagnold (1965), a natural sample of sand has a predominant size usually between 300 microns and 150 microns but in exceptional cases can be as low as 80 microns. In the present work the particles are assumed to be normally distributed at ground with a predominant mean size of 150 microns and a standard deviation of 100 microns. Further, it is assumed that a dust content of 40 mg/m^3 can be obtained with a updraft of 10 m/sec. Under these assumptions the number of particles present in a unit volume of air for a updraft of 10 m/sec and the dust content for other values of updraft can be computed.

The particle size distribution in air is, however, different from that at the ground because of the fact that particles of different sizes move through air with different velocities. To determine the weight-factor let us consider a layer of infinitesimal thickness dz at a height z from ground. If $n\,(a)$ be number of particles per unit volume of size a within this layer, then :

$$n\,(a)\,dz = \frac{dN\,(a)}{dt}\,dt \tag{8}$$

where $dN(a)/dt$ is the rate of depletion of particles of size a from the ground and dt the difference in time taken by the particle to reach from ground to top and the bottom of the layer. If the dust at ground has a homogeneous composition and sufficient depth, the rate of depletion from ground is a function of updraft only. Hence,

$$n\,(a)\,w\,(a) = c\,N\,(a) \tag{9}$$

where $w(a)$ is the velocity of the particle of size a at the height z from ground and c is a constant of proportionality. Since the velocity of particles approach the assymptotic value w_0 within a very short time, we can write :

$$n\,(a) = \frac{(N_0/w_0)\,e^{-(a-a_0)^2/z\,\sigma^2}}{\int_1^{a_m} (1/w_0)\,e^{-(a-a_0)^2/z\,\sigma^2}\,da} \tag{10}$$

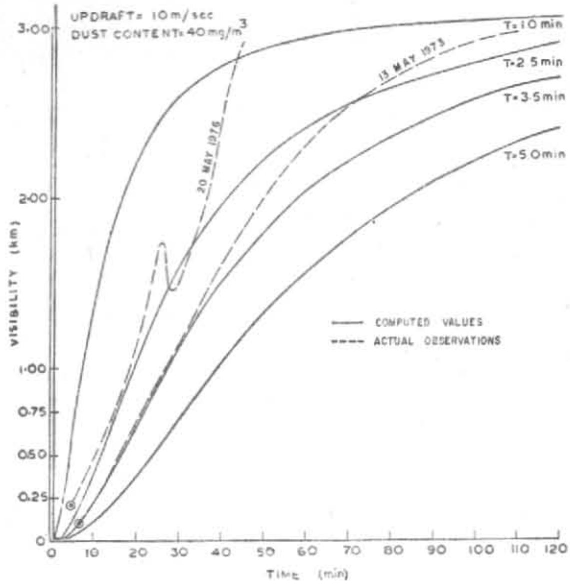

Fig. 3. Variation of visibility with time

TABLE 1

(Updraft = 10 m/sec, T = 3.5 minutes)

	Particle size (micron)								
	1.0	5.0	10.0	20.0	40.0	60.0	80.0	100.0	120.0
w_0 (cm/sec)	999.9	998.6	994.5	978.0	911.9	801.8	647.6	449.4	207.2
T (sec)	.0004	.0097	.0388	.1553	.6211	1.3975	2.4845	3.8220	5.5901
Z_m (m)	2099.92	2097.13	2088.45	2053.71	1914.82	1683.12	1358.76	968.74	435.12

T is the time when vertical velocity acquires a value 99.9% of w_0

TABLE 2

Cut-off size of particles for different updraft value

(K=.000178 cm² /sec)

Updraft (m/sec)	a_m (microns)	Updraft (m/sec)	a_m (microns)	Updraft (m/sec)	a_m (microns)	Updraft (m/sec)	a_m (microns)
0.5	30.13	3.0	73.81	5.5	99.94	8.0	120.54
1.0	42.61	3.5	79.73	6.0	104.39	8.5	124.25
1.5	52.19	4.0	85.23	6.5	108.65	9.0	127.85
2.0	60.27	4.5	90.40	7.0	112.75	9.5	131.35
2.5	67.38	5.0	95.29	7.5	116.71	10.00	134.77

where $a_m = \sqrt{Kw'/g}$ is the upper cut-off in particle size and N_0 is the total number of particles per unit volume of air. It is seen from the equation of motion that particles of size greater than a_m cannot be lifted up from the ground by the updraft as the downward directed force of gravity in this case becomes greater in magnitude than the upward directed viscous drag force. Since, the particles of size less than one micron do not contribute appreciably to the total mass of suspended particles, the arbitrary choice of lower cut-off limit does not lead to any significant loss of accuracy. The total mass of suspended particles per unit volume is now given by :

$$M = \rho_s \int_1^{a_m} a^3 \, n(a) \, da \tag{11}$$

Using a value of 40 mg/m³ for the total mass of suspended particles and 10 m/sec for updraft, it is found by computation that the total number of particles is 2,59097 per c.c.

2.4. *Visibility in duststorms*

In meteorology, visibility is defined as the maximum distance upto which a sufficiently large object can be identified with the help of naked eyes. For a particle of cubic shape, the area of interception $A(a)$ varies between a minimum of a^2 and a maximum a $3a^2/2$ when three sides of the cube make same angle of 30° with the incident light. Assuming all intermediate orientations to be equally probable, the mean value of $5a^2/4$ can be taken as the average area of interception. Hence, we can rewrite the expression for visibility (Johnson 1954) in the form :

$$V_m = 3.13 \Big/ \left(\epsilon + \int_1^{a_m} n(a) \, a^2 \, da \right) \tag{12}$$

where, ϵ stands for the contribution to visibility due to Mie scattering by particles of size less than 10 microns and also due to Rayleigh scattering by air molecules. In northwest India during summer, when most of the incidents of *Andhi* takes place, the normal value of visibility before onset of duststorm is around 3 km. To satisfy this condition an arbitrary value of .00001 has been chosen for ϵ. Substituting for $n(a)$ from Eqn. (10) we can now compute the value of visibility from the above expression. At any particular height z from ground visibility will start decreasing only after particles of size 10 microns and more reach this level; the minimum value of visibility being reached when particles of maximum size that can reach this height has reached this level. At airports visibility values are continuously recorded by the skopograph which is normally placed at a height of 2.8 m. In what follows we shall consider this level only.

It is seen from the equations of motion that particles of all sizes acquire the maximum vertical velocity within a very short time after leaving the ground. In other words, the velocity of particles of size a and their population density, remain constant with height except in a shallow layer close to ground. Thus, at the instant of time when the region of updraft moves away from over a fixed point on the surface of earth, all the particles of size a inside a column extending from the top of the shallow layer near the ground to a height determined from Eqn. (4) by substituting for t the duration of the region of updraft, have the same initial upward velocity. Since all the particles of size a will experience the same downward force due to gravity, there will be no relative motion amongst them as the whole column will drift upward till its top touches the height z_{max} as given by Eqn. (7) and then it will start settling downwards to the ground. Hence, the population density of particles of size a and their contribution to visibility will remain same upto the time when the top of the above column will reach a height less than 2.8 m. After this time particles of size equal to and greater than a will have no effect on visibility and the upper limit of integration a_m in Eqn. (12) should be replaced by the value $a - \delta$ where δ is an infinitesimal quantity. The variation of visibility with time, computed as above, is shown in Fig. 3. This agrees very well with the observed variation of visibility as reported by Joseph *et al.* also given in Fig. 3. The agreement is very good upto first 30 minutes after passage of squall. After that the improvement of visibility is faster than the computed one. This may be due to the fact that we have assumed 3 km as background visibility. Some improvement in the agreement is expected if higher background visibility of atmosphere is assumed. The visibility records are accurate for values less than 3 km and hence the improvements upto 3 km are shown. It may be noted that our model 3.5 min curve fits very well with the data for duststorm of 13 May 1973.

3. Conclusions

The simple model gives a satisfactory result. Assuming different values of T we can get different variations of visibility. This is also shown in Fig. 3 and agrees fairly well with some other observations by Joseph *et al.* The case of 20 May 1976 is shown in the figure. Assumption of 10 mps as updraft in a gust front means the cases of severe duststorms have been considered. For moderate or light duststorms the magnitude of updraft should be different. Similarly the concentration of dust should also be different.

The assumption of constant value of updraft is a simplifying assumption. The more realistic assumption, according to present day ideas of gust front would be to have a sudden rise but a gradual decrease of updraft speed. Our next attempt will be assumption of variable rate of ascent. Also the environmental condition should be another influencing factor. Further work in this regard is in progress.

Acknowledgement

Authors' thanks are due to Dr. A.K. Kamra and Mr. P.V. Joseph for valuable discussions.

References

Bangnold, R. A., 1965, *The Phys ics of Blown Sand and Desert Dunes*, Mathew & Co., London, p. 6.

Charba, J., 1974., *Mon. Weath. Rev.*, **102**, pp. 146-156.

Goff, R. C., 1976, *Mon. Weath. Rev.*, **104**, pp. 1429-1440.

Idso, S. B., Ingram, R. S. and Pritchard, J. M., 1972, *Bull. Am. met. Soc.*, **53**, pp. 930-935.

Johnson, J. C., 1954, *Physical Meteorology*, Technological Press & John Wiley & Sons, USA, p. 80.

Joseph, P. V., Raipal, D. K. and Deka, S. N., 1980, *Mausam*, **31**, pp. 431-442.

Kamra, A. K., 1972, *J. geophys. Res.*, **77**, pp. 5856-5869.

Lawson, T. J., 1971, *Weather*, **26**, pp. 105-112.