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Measurement of rainfall with the aid of weather radar

A. B. BHATTACHARYA

Department of Meteorology,
Massachusetts Institute of Technology, Cambridge, U. S. A.
and
R. BHATTACHARYA

Institute of Radio Physics and Electronics,
University of Calcutta, Calcutta

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सार — रेडार परावर्तकता घटक सिद्धांत और रेडार प्राचलों को कैसे निश्चित किया जाता है, यह मौसम रेडार की सहायता से वर्षा के सही मापों के लिए स्पष्ट रूप से ज्ञात हो जाना चाहिए । इन लक्ष्यों को प्राप्त करने के लिए, प्रक्रियाओं को पहले प्रस्तुत किया जाता है । Z-R संबंधों पर शुद्ध गति की और सूक्ष्म भौतिकी प्रभावों और वर्षा प्राक्कलनों के प्रभाव को बताया गया है । अंशदान के लिए प्रमापियों का प्रयोग करते हुए वर्षा के रेडार क्षेत्रीय प्राक्कलनों पर विचार करने से उस समय प्रतिशत त्रुटि में महत्वपूर्ण कमी होने का पता चलता है जब तूफान के आधार पर समायोजन किया जाता है और इस प्रकार इससे उन्नत तकनीक का संकेत मिलता है । ऐसे मापों को प्रभावित करने वाले घटकों को अंतिम रूप से प्रस्तुत किया गया है ।

ABSTRACT. The theory of the radar reflectivity factor and how to determine the radar parameters must be clearly known for an accurate measurement of rainfall with the help of a weather radar. Procedures for achieving these ends are presented first. Kinematic and microphysical influences on Z-R relationships and the effect on the rainfall estimates are pointed out. Consideration of radar areal estimates of rainfall utilizing gauges for calibration indicates a significant reduction in per cent error when adjustments are made on a storm basis and thus suggesting an improved technique. Factors affecting such measurements are finally outlined.

1. Introduction

The difficulties of rainfall measurement over a wide area are well recognised. Due to the small scale spatial variability of rain, a network of gauges has severe limitations in sampling representatively. On the other hand, a radar suffers from some basic uncertainties primarily due to the fact that the measured quantity, *viz.*, the equivalent radar reflectivity factor is not uniquely related to the rainfall rate. Although radar has been used experimentally for over 30 years for measuring precipitation, operational implementation has been slow. Now we find that the data available are under utilized and both misunderstanding and confusion exist about the accuracy of the results. However, implementation of digital recording and processing of weather radar data, in recent years, has largely removed this obstacle. For operational forecasting of river flow and flash floods, dense raingauge observations are desirable, no doubt, but their installation has not been practical. Thus, there has been a considerable interest to use weather radar as it provides spatially and temporally continuous measurements available immediately in one location. Both scatter and attenuation of microwaves are based on precipitation estimated. Presently reflectivity data are considered practical for operational measurement of rainfall over large areas.

2. Theoretical considerations

The backscattered radar power due to precipitation particles is directly proportional to the sixth power of particle diameters (D_i^6) per unit volume illuminated by radar beam. If N_i denotes the number of drops in a unit volume of air with diameter D_i then the radar reflectivity factor Z can be mathematically expressed as:

$$Z = \sum_i N_i D_i^6 = \int_0^\infty N(D) D^6 dD \qquad (1)$$

where $N(D)$ is the number of drops with diameters between D and $(D+dD)$ in a unit volume of air. If the vertical air motions are absent then the rainfall rate R is related to D by an equation,

$$R = \frac{\pi}{6} \int_0^\infty N(D) D^3 V_t(D) dD \qquad (2)$$

where $V_t(D)$ represents the terminal velocity (in cm s^{-1}) of a drop of diameter D that is approximated by:

$$V_t = 1400 D^{1/2} \text{ (Spilhaus 1948)} \qquad (3)$$

Substituting the Marshall-Palmer (1948) exponential drop size distribution into Eqns. (1) and (2) and using the empirical relation between V_t and D, one may get a relation between Z and R which is of the form :

$$Z = AR^b \qquad (4)$$

Hence, if the drop size distributions are exponential and also the vertical air motions are small relative to the drop terminal velocities, then no fundamental limitation arises for the accurate estimates of rainfall using radar. But in practice, the drop size distribution is rarely known and it varies in time and space. Furthermore, the vertical air motions are frequently of the same magnitude as the terminal velocities, particularly in cases of thunderstorms. Thus, the Z-R relation is not a unique one and we are compelled to rely on average empirical relations. Battan (1973) has presented a comprehensive list of Z-R relationships as established by different investigators. A frequently used relation is of the form :

$$Z = 200 \, R^{1 \cdot 6} \qquad (5)$$

2.1. Determination of radar parameters

The average power received $\bar{P}_r$ at range r can be expressed as :

$$\bar{P}_r = \frac{P_t \, G^2 \, \lambda^2 \, h \, \theta^2}{512 \, \pi^2 \, r^2} \sum_{\text{Vol}} \sigma_i \qquad (6)$$

where,

P_t = peak transmitted power in the pulser
G = antenna gain
h = pulse length in space
λ = wave length
θ = beam width between half power points
r = range

Again,

$$\sigma_i = \frac{\pi^5}{\lambda^4} \left| k \right|^2 D_i^6 \qquad (7)$$

where $|k|^2$ is a function of the dielectric constant of the targets and is approximately 0.93 for water and 0.179 for ice in the microwave band. By Eqns. (6) and (7),

$$\bar{P}_r = \frac{P_t \, G^2 \, h \, \theta^2}{512 \, r^2} \cdot \frac{\pi^3}{\lambda^2} \left| k \right|^2 \sum_{\text{Vol}} D_i^6$$

$$= \frac{P_t \, G^2 \, h \, \theta^2 \, \pi^3 |k|^2}{512 \, r^2 \, \lambda^2} Z \qquad (8)$$

The quantity $\sum_{\text{Vol}} D_i^6$ commonly designated by Z, is known as reflectivity factor.

In practice the gain G is realized only at the centre of the beam and the intensity tapers off with increasing axial angle. If the antenna is a circular paraboloid, the distribution of intensity in the main beam is closely approximated by Gaussian function of the axial angle α and is expressed as :

$$G(\alpha) = G \exp - [4 (\ln z) \, \alpha^2/\theta^2] \qquad (9)$$

An effective beam width θ_e can be found by integrating the actual beam intensity pattern for two-way transmission out to some axial angle α_1 which is considered large enough to include all the scatterers which contribute to the signal. Then,

$$G^2 \left[\frac{\theta_e}{2} \right]^2 = \int_0^{\alpha_1} G^2 (\alpha) \, 2\alpha \, d\alpha \qquad (10)$$

Integration of Eqn. (10) yields :

$$\theta_e^2 = 0.69 \, \theta^2 \quad \text{for} \quad \alpha_1 = \tfrac{3}{4}\theta$$

and

$$= 0.72 \, \theta^2 \quad \text{for} \quad \alpha_1 \geqslant \theta$$

So the choice of α_1 is not critical and can be set equal to 0.7 θ^2 with very little error. For a conical beam Eqn. (8) then becomes :

$$\bar{P}_r = \frac{P_t G^2 h \theta^2 \, \pi^3 \, |k|^2}{730 \, r^2 \, \lambda^2} Z \qquad (11)$$

In log form the above equation reduces to,

$$Z_e \, (\text{dB} \, z) = \bar{P}_r \, (\text{dB}m) - 10 \log P_t + 20 \log r + C \qquad (12)$$

where,

$$Z_e \, (\text{dB} \, z) = 10 \log Z_e \, (\text{mm}^6 \, \text{m}^{-3}) \qquad (13)$$

and

$$C = \text{radar constant.}$$

The power transmitted P_t is not included in the constant as it varies somewhat with transmitter age etc and so to be monitored regularly. In Eqn. (11) h and λ can be accurately determined with standard test instruments. Beam patterns at the desired frequencies are usually manufactured at the factory and the beam width can be accurately obtained directly from them. By comparing with a standard horn, the gain is also manufactured in the factory but the value is not adequate for the field. Two-way losses like waveguide, rotary joint and radome losses must also be taken into consideration.

2.1.1. Reflectivity factor

The most important parameter that is measured by using meteorological radar is the reflectivity of the the scattering volume. From a knowledge of reflectivity by using suitable empirical relations one may deduce useful meteorological quantities like rainfall rate and liquid water content. Moreover the severe storms, hail storms, in particular, can often be identified by their high reflectivities (Mazur 1986).

In order to determine reflectivity, the quantity which is to be measured is the power received. From the average power received $\bar{P}_r$ and the radar equation the volume reflectivity can be calculated. If it is then normalized for wavelength we can get reflectivity factor Z_e. By definition Z is equal to the summation over the sixth power of the drop diameters but when it is obtained from radar measurements with the radar equation it is called equivalent Z and denoted as Z_e. The accuracy of Z_e depends on how accurately one can measure $\bar{P}_r$ and also how well we know the parameters in the radar equation.

2.1.2. Signal averaging

The power received from a radar volume at any instant (P_r) depends mainly on the configuration of

TABLE 1

Kinematic and microphysical influences on Z-R relationships and the effect on rainfall estimates

Process	Change in $Z(=AR^b)$		Probable effect on radar rainfall (when Z-R is not adjusted)	Probable region of maximum influence
	A	b		
Kinematic				
Size sorting	Increase	Decrease	Tendency to overestimate	Regions of strong inflow and outflow
Vertical motion				
Updraft	Increase	Decrease	Overestimate	
Downdraft	Decrease	Increase	Underestimate	
Microphysical				
Evaporation	Increase	Decrease	Overestimate	Inflow regions, fringe areas
Accretion of cloud particles	Decrease	Increase	Underestimate	Downdraft
Collision, coalescence	Increase	Decrease	Overestimate	Reflectivity core
Break up	Decrease	Decrease	Underestimate	Reflectivity core

scatters at that moment. By averaging P_r either in time or in range or both, the desired quantity $\bar{P}_r$ can be achieved. In general, the time integration is obtained by digital sweep integrators. The signal from the receiver (a log receiver usually, for accommodating the large dynamic range involved) is digitized in a number of range bins, a few hundred generally, and finally summed over a number of transmitted pulses of the order of 16 or 32. Range averaging is obtained either digitally by combining range bins or analogwise by some sort of smearing (e.g., RC or tapped delay line). Each integration cycle should be completed in about a beam width but it is not desired to turn the antenna too slowly. In practice 30 or 40 samples are averaged which results in a standard deviation of less than a dB in the fluctuations of the signal. But for an unaveraged signal the standard deviation is about ± 6 dB. The performance of the receiver also affect the measurement of $\bar{P}_r$. One source of variation is drift in the D.C. level. It is important to note that this either be held to a few millivolts or to be compensated by some means.

2.1.3. Antenna gain

There are two ways of measuring antenna gain in the field : (i) to measure the backscatter from a standard target and (ii) to measure the power received by a standard gain horn in the far field. The 'horn' method is superior to the standard target one due to its steadiness and reliability of measurement and also its error-cancelling characteristics (Smith 1974). If P_{ff} is the power received by an antenna of gain G_r then in the far field of an antenna of gain G_t and transmitting a power P_t, one can write,

$$P_{ff} = \frac{P_t\, G_t\, G_r\, \lambda^2}{(4\pi r)^2} \qquad (14)$$

P_t is measured through a directional coupler at the same place where the receiver is calibrated by signal generator; G_r is precisely known as the receiving antenna in a calibrated standard gain horn, P_{ff} is measured with the same power meter used to measure P_t. From a knowledge of wavelength and range accurately, the effective gain G_t can be computed.

3. Z-R relationships

Measuring the drop size distributions in many types of rains, the characteristic Z-R relationships have been reported by many investigators (Srivastava 1971, Wilson and Brandes 1979). With increasing convective intensity the coefficient of Eqn. (4) increases and the exponent decreases, in general. The variations in the reported Z-R relationship are thought to reflect the predominance of one or another physical process that influences the drop size distribution. In Table 1, the physical mechanisms that may alter the drop size distributions are listed. The table also indicates the probable influence on radar-estimated rainfall and the storm region where the effect is probably at a maximum.

Some earlier results taken by the MIT (Cambridge) group are cited here. The rainfall reflectivity points are plotted in Fig. 1 (Austin and Geotis 1979). The overall relation between Z (mm^{6}m^{-3}) and R(mm h^{-1}), as indicated by a line in the figure is given by :

$$Z = 180\, R^{1\cdot35} \qquad (15)$$

In a separate observation Garrison (1972) by considering three thunderstorms, found it as :

$$Z = 430\, R^{1\cdot44} \qquad (16)$$

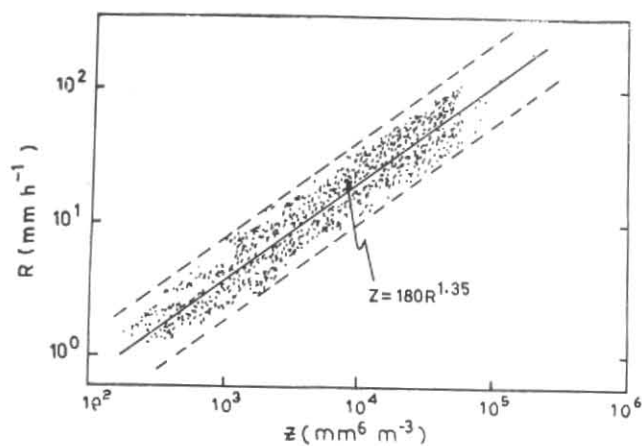and for two storms containing non-cellular rain,

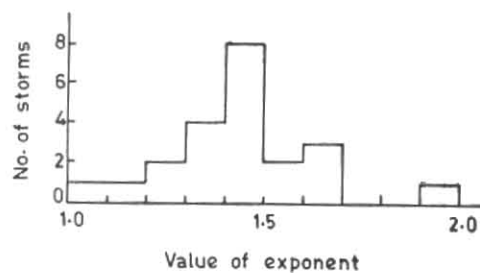Fig. 1. Rainfall and reflectivity relationship (*After* Austin & Geotis 1979)

Fig. 2. Distribution of exponents for 22 storms

TABLE 2

Radar estimates of rainfall using gauges for calibration

Rain type	$Z\text{-}R$ relation	Radar			Area size (km^2)	Per cent error		Reference
		$\lambda(cm)/$ θ (deg)	Obser- ving fre- quency (min)	Range (km)		Before adjust- ment	After adjust- ment	
Showers	$200R^{1.6}$	3/1.8	5	18-28	180	43	23	Jatila & Puhakka (1973)
Showers	$300R^{1.4}$	10/2	5	85-115	570	43	30	Woodley *et al.* (1974)
Showers	$200R^{1.6}$	5/1.7	10	95-112	170	49	22	Wilson (1975)
Thunder- showers	$200R^{1.6}$	10/2	5	45-100	3000	52	13	Brandes (1975)
Showers	$300R$	10/1	3	10-100	5300	55	27	Huff & Towery (1978)

$$Z = 110 \, R^{1.44} \tag{17}$$

The distribution of exponents for 22 storms was as shown in Fig. 2.

3.1. Radar rainfall adjustment

Adjustments of the radar estimates can be made either (i) changing the Z-R relationship or (ii) keeping the Z-R relationship fixed and using raingauge observations. In the simplest calibration technique a number of gauges (N) are utilized and a multiplicative adjustment factor (F), the ratio of gauge derived (G) and radar indicated rainfall (R) are computed by :

$$F = \frac{\sum\limits_{i=1}^{N} G_i}{\sum\limits_{i=1}^{N} R_i} \tag{18}$$

or by using :

$$F = \frac{1}{N} \sum_{i=1}^{N} \frac{G_i}{R_i} \tag{19}$$

In Eqn. (18) observations receive a weight proportionate to depth while in Eqn. (19) all gauge-radar comparisons have equal weight. Indeed the relationship selected for converting reflectivity to rainfall is of little importance and has negligible impact on the corrected radar depth estimate (Brandes 1975). Some radar areal estimates of rainfall using gauges for calibration is presented in Table 2 indicating that the error after adjustment were lowered considerably.

The table clearly shows that there is a significant reduction in radar rainfall estimate error when adjustments are made on a storm basis.

4. Factors affecting rainfall measurement

Numerous raindrop size measurements have shown that drop sizes vary significantly from storm to storm and also within storms. This variability introduces uncertainty into the Z-R relation and some meteorologists assume it to be the major cause of uncertainty in radar rainfall measurements. In addition, other factors as discussed below have significant effects, at least as much as those arising from variations in drop sizes (Proctor 1983).

(i) Vertical variations in reflectivity

The spatial averaging at several stages in the recording and processing of radar data tends to depress the peak reflectivity in showers of small dimension. There is evidence also that Z values and the corresponding values of Z_e are sometimes reduced by downdrafts associated with rain shafts.

(ii) Horizontal variation in rainfall

Small-scale variability of precipitation in the horizontal plane creates some complications. Muller and Sims (1975) and Riley and Austin (1976) found that reflectivity gradients as great as 5 dB km^{-1} are not particularly unusual and they often occur in the vicinity of heavy rain.

(iii) Time interval between radar maps

When the time interval between radar maps is larger than the time necessary for small intense echoes to move from one grid point to the next then the time integrations of R will be in error unless the radar patterns are advected along the motion vectors.

(iv) Sensitivity of the radar

A radar may fail to detect light rain at large ranges. For example, it might indicate correctly a 2-hr accumulation of 5 mm due to a short intense shower but fail to detect most of a similar accumulation which occurred as steady light rain. Clearly an adjustment based on either of the above situations would be inappropriate for the other.

5. Conclusions

Measurement of reflectivity is affected by the attenuation of the signal due to precipitation. Attenuation most affects the shorter wavelengths. So a wavelength which is not significantly attenuated by the precipitation to be measured should be chosen. If light rain is involved wavelengths as short as 3 cm may be satisfactory but for the measurement of heavy rain and extended areas, a wavelength of 10 cm is recommended.

Various factors, as discussed here, contribute to some discrepancies in radar measurements. A more comprehensive analysis is desirable to achieve a better physical understanding and a quantitative assessment of the factors.

Since the physical phenomena involved in precipitation are complex and variable, they should be analyzed in depth so that they can properly be taken into account in attempts to develop objective monitoring techniques and useful operational procedures.

In order to adjust the radar, it is desirable that frequent comparison be made between rainfall rates deduced from radar measurements and those measured with raingauges.

Radar meteorology, being largely on observational science, is sensitive to the quantity and cost of the means for making measurements. Recent rapid advances in digital electronics should, therefore, have a significant impact on the field. As part of the weather radar systems, many installations now use digital integrators and some incorporate mini-computers. As a result, the accuracy of measurement has been greatly increased. A computer can also eliminate the necessity of some standard radar components like the specialized displays (e.g., A scope PPI), timing circuitry etc. Its ability to store and integrate the signal over times and areas has obvious hydrological applications. Radar observations from satellite platforms are now considered as further possibilities to establish detailed rainfall patterns in near future, particularly over the tropical oceanic regions and in the maritime continent area.

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