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The stability of internal gravity waves in a bounded atmospheric shear layer in the presence of moisture

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सार—संतृप्त परिमित की उपस्थिति में समतापी सीमित स्पर्श रेखा वेग के पार्श्वचित्र द्वारा उत्पन्न आन्तरिक गुरुत्व तरंगों की स्थिरता की विशिष्टताओं का अध्ययन किया गया है। आर्द्र परत की अविरल मोटाई का आर्द्रता की मात्रा में उतार-चढ़ाव के विन्दु और परिवर्तनों के संबंध में विभिन्न स्तरों का परिचय दिया गया है तथा मूल से दूरी की जांच की गई है। लालास और इनाउडी (1976) की तकनीक द्वारा अश्यान तथा बूसिने सीमा में गति के पूरे समीकरणों के रैखिकीकृत विवरणों को संख्यात्मकता से हल किए जाने के द्वारा अस्थिर तरंगों की विशिष्टताओं को प्राप्त किया गया है।

यह दर्शाया गया है कि आर्द्र परत की उपस्थिति तरंगों की स्थिरता की विशिष्टताओं को सार्थक ढंग से प्रभावित कर सकती है। यह पाया गया है कि आर्द्रता में वृद्धि तथा उतार-चढ़ाव बिन्दु से परत की दूरी तरंग की अनुक्रिया का विस्तार या ह्रास करती है क्योंकि संतृप्त परत बहाव के लिए दृढ़ सीमा जैसा कार्य करती है। यह दर्शाया गया है कि इस प्रकार के प्रभाव वाली परत लघु तरंग दैर्घ्यों को स्थिर करती है और दीर्घ-तरंग दैर्घ्यों को अस्थिर करती है। अन्त में वास्तविक वायुमंडल से निदेश के परिणामों के अनुप्रयोज्य पर विचार किया गया है।

ABSTRACT. The stability characteristics of internal gravity waves, generated by an isothermal bounded tangent velocity profile in the presence of a saturated finite layer, are studied. The moist layer with constant thickness is introduced at different levels in respect to the point of inflection and the variations of moisture content and distance from the origin are examined. The characteristics of the unstable waves are obtained by solving numerically the linearized versions of the full equations of motion, in the inviscid and Boussinesq limit, through the technique of Lalas and Einaudi (1976).

It is shown that the presence of the moist layer can significantly affect the stability characteristics of the waves. Increases in the moisture and distance of the layer from the inflection point are found to amplify or decay the wave response, because the saturated layer behaves as a solid boundary to the flow. The presence of such effective layer is shown to stabilize short wavelengths and destabilise longer wavelengths. Finally, an application of the model's results to the real atmosphere is discussed.

Key words - Hyperbolic velocity profile, internal gravity modes moisture, shear flow, stablity.

1. Introduction

It becoming apparent that knowledge of the characteristics of internal gravity waves in the atmosphere is necessary to our understanding of a wide variety of tropospheric phenomena, such as momentum and energy transport, boundary layer characteristics, inversions, thunderstorms and severe storm initiation, lee waves, tropospheric jet streams and others (Uccellini and Johnson 1979, Durran and Klemp 1982b, King *et al.* 1987, Finnigan *et al.* 1984, Stobie *et al.* 1983, Wang *et al.* 1983, Testud *et al.* 1980). The advent of the new remote-sensing techniques of satellite and Doppler radar imagery has substantially increased our ability to identify and study such wave motions in the atmosphere.

A full understanding of their generation mechanism, propagation and influence is likely to require a nonlinear treatment which, however, requires as a first step, the extensive study of the linearly unstable gravity modes. Thus the appearance of unstable waves in a dynamically unstable shear flow has been studied by many authors both experimentally and theoretically (Drazin 1958, Miles 1961, Jones 1967, Lindzen and Rosenthal 1976, Davis and Peltier 1976, Pellacani *et al.* 1978, Fritts 1980, Narayanan and Sachdev 1982).

Of direct pertinence is the work of Lalas and Einaudi (1976) who studied the stability properties of a hyperbolic tangent velocity profile in an isothermal atmosphere in the presence of a rigid lower boundary. The unstable waves they found, belong to a multitude of modes. Four modes with the largest growth rates were analysed in detail : Mode I which coincides with the Kelvin-Helmholtz type mode found by Drazin (1958) and Modes II, III and IV which are new modes introduced by the presence of the ground. While Modes I, II and IV are trapped modes (*i.e.,* modes that are rapidly evanescent outside the region of shear), Mode III is propagating in the vertical providing an escape of energy upward, away from the shear zone.

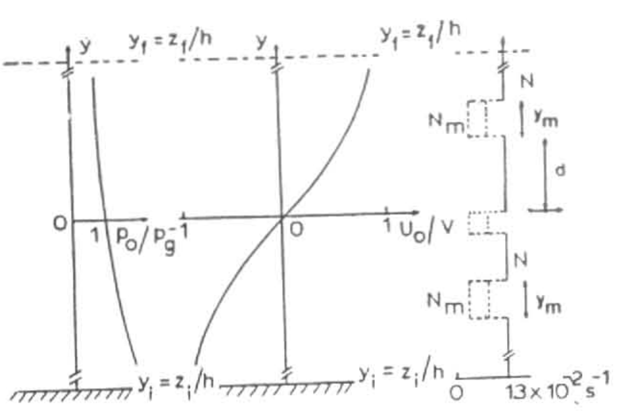

Fig. 1. The normalized density and velocity profiles and the (BV) frequency distribution of the basic flow. The dashed line indicates the moist layer locations

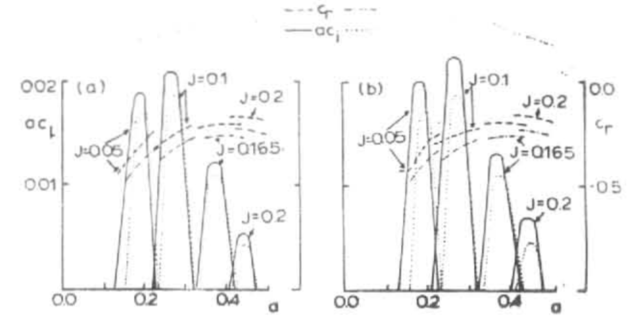

Fig. 2. Normalized growth rates aC_i (———) and phase velocities C_r (– – –) versus $a=k_xh$, of Mode II for different values of J and a saturated layer of normalized thickness $y_m=3.0$, located at a normalized distance from the inflection point $d=5.0$ (a)$\delta=0.46$ and (b)$\delta=0.24$.

The corresponding curves for a dry atmosphere are also shown with dotted (growth rates) and dot-dashed (phase speeds) lines

Although the above mentioned authors interpreted their results in terms of tropospheric phenomena, they neglected moisture process in their models. Since moisture is an important process in the tropospheric dynamics, and since it is generally a stabilising or destabilising property of the flow, the slowly growing resonant modes should not be assigned a major role in determining the structure of the various tropospheric phenomena until the effects of moisture upon them have been assessed. It is the main purpose of the present model to do this. We will show that the unstable waves of the resonant modes continue to exist, with somewhat smaller or greater growth rates, depending on their propagation characteristics, when moisture has been incorporated into the model.

The present work differs from the work of Lalas and Einaudi (1976) in the use of linearized versions of the full equations of motion, so that more complete analysis is performed here. Also, the present model differs from similar works of Pellacani *et al.* (1978) and Narayanan and Sachdev (1982) in the use of continuous profiles for density and velocity.

2. The governing equations

Lalas and Einaudi (1976) restricted attention to a "shallow convection" version of the Boussinesq approximation. In the present study the equations to be solved are the linearised versions of the full equations of motion, thermodynamics and continuity for adiabatic stratified flow, instead. In the absence of coriolis effects, these equations can be written (in the inviscid Boussinesq limit) as follows :

$$\rho_0\left(\frac{\partial u}{\partial t}+U_0\frac{\partial u}{\partial x}\right)+w\rho_0\frac{dU_0}{dz}=-\frac{\partial p}{\partial x} \quad (1)$$

$$\rho_0\left(\frac{\partial w}{\partial t}+U_0\frac{\partial w}{\partial x}\right)+\frac{\partial p}{\partial z}=-\rho g \quad (2)$$

$$\frac{\partial \rho}{\partial t}+U_0\frac{\partial \rho}{\partial x}+w\frac{\partial \rho_0}{\partial z}=-\rho_0\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right) \quad (3)$$

$$\frac{\partial p}{\partial t}+U_0\frac{\partial p}{\partial x}+w\frac{\partial p_0}{\partial z}=-p_0\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right) \quad (4)$$

where, g is the acceleration of gravity. In Eqns. (1) - (4), u, w, p, ρ denote the horizontal velocity, vertical velocity, pressure and density respectively, of the disturbance, U_0 is the basic state velocity in the x direction, ρ_0 is the basic state density and p_0 is that of pressure. The velocity profile in non-dimensional form is given as :

$$u_0(y)=\frac{U_0}{V}=\tan h(y) \quad (5)$$

where $y=z/h$ is the non-dimensional height, h is the characteristic length scale and V is the velocity scale. The background density field ρ_0 is of the form,

$$\rho_0(z)=\rho_g\exp(-z/H) \quad (6)$$

ρ_g is the density at the level of maximum shear, and H the scale height of the atmosphere.

To analyse the stability of the above configurations, we follow closely the approach and notation of Lalas and Einaudi (1976). Thus, in Eqns. (1)-(4) a solution is sought in the form :

$$(u,w,p,\rho)=\rho_0^{-1/2}\,\mathrm{Re}\,\{\hat{u}(z),\Psi(z),\hat{p}(z),\hat{\rho}(z)\exp[i(k_xx-\omega t)]\} \quad (7)$$

where, Re denotes the real part. Ψ is an unknown function of height (the eigenfunction), and k_x, $\lambda_x=2\pi/k_x$ and $\omega=\omega_r+i\omega_i$ are the horizontal wave-number, wave length and complex frequency respectively. Substitution of (7) into Eqns. (1)-(4) and subsequent elimination of $\hat{u}, \hat{p}, \hat{\rho}$ results after much algebra, in a single equation of Ψ,

$$\frac{d^2\Psi}{dy^2}-\Lambda(y)\Psi=0 \quad (8)$$

where,

$$\wedge(y) = a^2 - \frac{N^2h^2}{V^2\Omega^2} - \frac{1}{\Omega}\left(\frac{1}{\rho_0}\frac{d\rho_0}{dy}\frac{du_0}{dy}\right) + \frac{1}{2\rho_0}\left(\frac{d^2\rho_0}{dy^2}\right) - \frac{1}{4\rho_0^2}\left(\frac{d\rho_0}{dy}\right)^2 - \frac{1}{\Omega}\frac{d^2u_0}{dy^2} \tag{9}$$

Here $a = k_x h$, $C_r = \omega_r / k_x V$, $C_i = \omega_i / k_x V$ and $\Omega = C_r - u_0 + iC_i$ are the normalized horizontal wavenumber, normalized phase velocity of the wave, normalized imaginary part of the frequency of oscillation and Doppler frequency respectively. N^2 is the square of the Brunt-Vaisala (BV) frequency to be defined later. Eqn. (9) is quite different from that of Lalas and Einaudi (1976), since the present analysis deals with the full hydrodynamic equations of motion. This difference resulted on the propagation characteristics of the unstable modes, especially on the phase velocities as well as the growth rates.

A rigid boundary condition is imposed on Ψ :

$$\Psi(y_i) = 0 \quad \text{at } y = y_i = z_i/h \tag{10}$$

where, y_i is the height of the lower boundary from the point of inflection. At large heights $y \gg y_f$, $\wedge(y)$ is essential constant and equal to $\wedge(y_f)$, and the solution to Eqn. (8) in this region takes the form :

$$\Psi \sim \exp(iKy),$$

where, K is the normalized vertical wavenumber given by:

$$K = \pm[-\wedge(y=y_f)]^{1/2} = K_r + iK_i \tag{11}$$

If $K_i \neq 0$ then the sign in (11) is chosen so that $K_i > 0$, implying exponential decay of the wave amplitude above $y = y_f$. If $K_i = 0$, then the sign in (11) is chosen so that the group velocity has a positive vertical component. The upper boundary condition is :

$$\frac{d\Psi}{dy} = iK\Psi \qquad \text{at } y = y_f \tag{12}$$

Following Thorpe (1969), we choose J, the minimum value of the Richardson number in the flow, as the main stability parameter, so that,

$$J = \frac{N^2h^2}{V^2} \tag{13}$$

where, N^2 is the BV frequency, which is given as,

$$N^2 = \frac{g}{T_0}\left(\frac{dT}{dz} + \Gamma_d\right) \tag{14}$$

where, T_0 is the temperature of the atmosphere, and Γ_d the dry adiabatic lapse rate. It is a well known fact that unstable solutions will exist only in the range $0 < J \leqslant 1/4$, as required by the Miles-Howard (1961) theorem and its extension to compressible fluids of Chimonas (1970). If $J > 1/4$ the flow is stable.

Now, if a region of the atmosphere is saturated, its effective stability may be lowered because of the release of latent heat associated with condensation process. Therefore, in the presence of moisture, we choose as the main stability parameter, the modified effective (BV) frequency N_m^2, as this is given by Lalas and Einaudi (1974) and Durran and Klemp (1982a) :

$$N_m^2 = \frac{g}{T_0}\left(\frac{dT}{dz} + \Gamma_m\right)\left(1 + \frac{Lq_s}{RT_0}\right) - \frac{g}{1+q_w}\frac{dq_w}{dz} \tag{15}$$

where, q_s is the saturation mixing ratio, q_w the total water mixing ratio which is the sum of q_s and the liquid water mixing ratio q_L ($q_w = q_s + q_L$). L is the latent heat of vaporization and R is the gas constant for dry air. Γ_m is the saturated adiabatic lapse rate. In fact two dynamical systems, with identical values of N and N_m, will have an identical linear behaviour. The nonlinear behaviours of the moist and dry systems differ primary because the saturated adiabatic lapse rate Γ_m is not independent of height, and nonlinear effects act to increase the buoyancy restoring force in the wave crests beyond that predicted by linear theory, while decreasing it in the troughs. Durran and Klemp (1982b) have demonstrated that the difference in the behaviour of linearly equivalent wet and dry systems becomes significant in moderately strong waves. In any case, since we demand that the fluid is stably-stratified both N^2 and N_m^2 will be taken as positive constants in the present treatment. Fig. 1 depicts the normalized density and velocity profiles and the dry BV frequency distribution of the basic flow.

In our modelling we assume that the moist region is uniform , so that N_m^2 is constant; this region contains sufficient moisture so that it always remains saturated. This can be the case when the saturated layer contains liquid water of the order of about 0.2 gm/kg to 0.4gm/kg. Fig. 1 shows the positions of the moist layer which indicated by the BV frequency distribution. According to Lalas and Einaudi (1974) in a saturated environment, the linear wave equation has the same form as that for a dry atmosphere, if the stability parameter is appropriately altered to include the influence of moist process. In this way, N_m has replaced N (*see* Bretherton 1966; Durran and Klemp 1982a). Thus, the two dimensional, linearized hydrodynamic equations of motion lead to the same second order differential equation as Eqn. (8), except that the dispersion relation includes the stability reduction, *i.e.*, the new effective BV frequency N_m, so that we can write for $\wedge(y)$:

$$\wedge(y) = a^2 - \frac{N_m^2h^2}{V^2\Omega^2} - \frac{1}{\Omega}\left(\frac{1}{\rho_0}\frac{d\rho_0}{dy}\frac{du_0}{dy}\right) + \frac{1}{2\rho_0}\left(\frac{d^2\rho_0}{dy^2}\right) - \frac{1}{4\rho_0^2}\left(\frac{d\rho_0}{dy}\right)^2 - \frac{1}{\Omega}\frac{d^2u_0}{dy^2} \tag{16}$$

Eqn. (8) with boundary conditions (10) and (12) is an eigenvalue problem for the complex frequency. It is solved iteratively using the algorithms described by Lalas and Einaudi 1976). In the presence of the moist layer Eqn. (8) is integrated from $y = y_f$ to $y = y_u$ (y_u is the height of the upper edge of the layer) employing Eqn. (9) as the dispersion relation. From $y = y_u$ to $y = y_d$ (y_d is the lower edge of the layer) the modified dispersion relation Eqn. (16) is used; and from $y = y_d$ to $y = y_i$, again Eqn. (9) is used in the procedure.

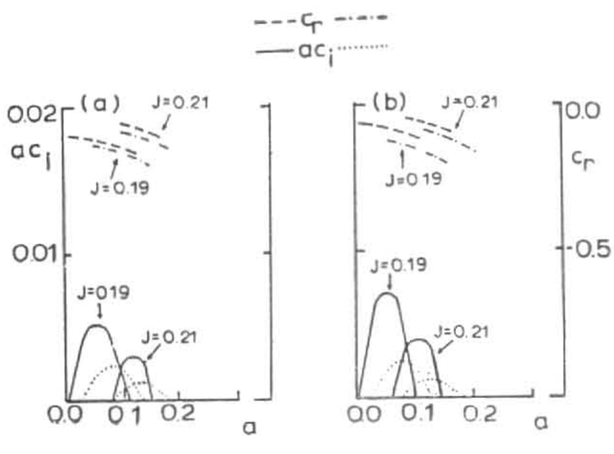

Fig.3. As in Fig. 2 but for Mode IV

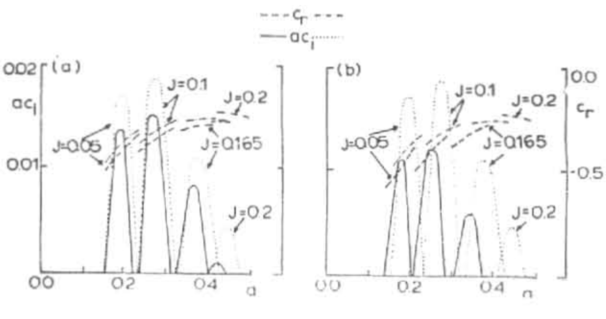

Fig. 4. As in Fig. 2 but for normalized distance from the inflection point $d=-5.0$

3. Numerical results and discussion

The number of the parameters characterizing the problem, *i.e.*, N^2, N_m^2, h, J, V, y_l, y_m, y_d, y_u, etc is so large that we are forced to limit our investigation to two specific parametric studies. The effect of the moist layer location with respect to the ground and the inflection point, and the effect of stability variation, on the waves.

We take the origin to be at the point of inflection. The ground then located at a distance, $y=y_l=-10.0$ and the top of the computational regime above which the velocity is constant and Eqn. (12) holds is at $y=y_f=10.0$. Referring to Fig. 1, we define d, the distance to the beginning of the saturated layer from the inflection, *ie.*, $d=y_d$. In this study the layer thickness is taken to be constant, $y_m=3.0$, while d is taken to be $d=-5.0$, 5.0, and 0.0. In the latter case the moist layer is just below to the inflection point. Here, it must be noted that the choice of the layer positions are the most effective ones. Finally, we define δ as the ratio of N_m to N, *i.e.*, $\delta=N_m/N$, where δ is a measure of the amount of moisture in the saturated layer since it is a function of the specific humidity. Two values of δ have been used, 0.46 ($N_m=0.00623s^{-1}$) and 0.24 ($N_m=0.00322s^{-1}$) which correspond to specific humidity of 0.2 gm/kg and 0.4gm/kg. Although these values could be viewed as large but in the atmosphere are measured routinely instead. In a saturated atmosphere typical values of the moist N_m range around $2\times10^{-3}s^{-1}$.

In presenting the results, normalized growth rates aCi and phase velocities C_r are plotted as functions of the normalized horizontal wave number $a=k_xh$ for different values of the Richardson number. On the same graphs are plotted also, for the sake of comparison the aC_i and C_r for the dry atmosphere.

We will discuss each unstable mode as labelled by Lalas and Einaudi (1976) because each mode is affected differently by the presence of the moist layer.

3.1. *Effect of moist layer position on the unstable Modes*

The unstable waves of Mode I, because of their short wavelengths, have been found not to be affected by the presence or the position of the saturated layer.

3.1.1. *Long wavelengths : Additional Modes II and IV*

The results for waves of Modes II and IV are presented in Figs. 2-6. Figs. 2 and 3 give results with layer thickness $y_m=3.0$ and $d=5.0$, while Fig. 4 represents results with $d=-5.0$; Figs. 5 and 6 depict results with $d=0.0$, but for layer thickness $y_m=2.0$. The presence and position of the saturated layer alter the characteristics of the waves. The position of the critical levels is affected by the different geometry, accompanied by variations in the phase velocities of the disturbances. The growth rates of the unstable waves of both modes are seen to increase substantially and almost three times and more (relative to the dry one) for $d=0.0$. While for $d=5.0$, the growth rates are seen to increase almost double. For $d=-5.0$, different results are obtained : the unstable waves of Mode IV disappear, while substantial reduction of the growth rates of Mode II are observed. The position now of the critical levels y_c is affected drastically and the phase velocities of the disturbances are seen to go to -1, with corresponding levels going toward the lower edge of the shear layer, where the local Richardson number is large.

On the other hand, since the moist layer seems to behaves as a solid boundry (Jacovides 1988) the a_{max} value of the waves is seen to move to the longer wavelengths domain, so that the destabilisation of longer wavelengths is obvious. The case discussed above with $d=0.0$, seems to be in qualitative agreement with the results presented by Lindzen and Tung (1976). In this way, some resonance effect is introduced by the presence of the saturated layer, which in combination with the exponential growth of the waves, drastically alters the characteristics of the disturbances. Perhaps this layer position could be formed a more effective duct together with the ground surface as in the case of Lindzen and Tung (1976).

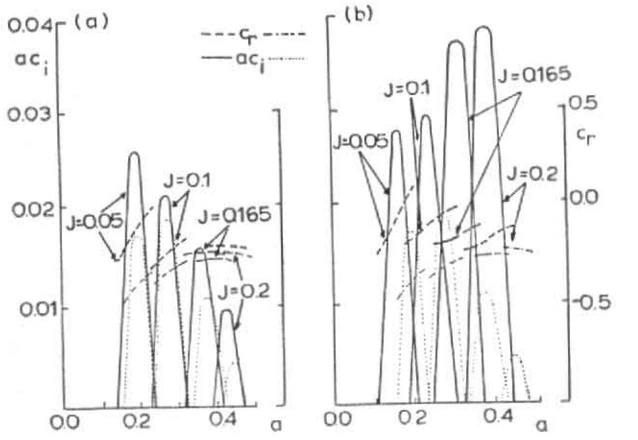

Fig. 5. As in Fig. 2 but for normalized distance from the point of inflection $d=0.0$ and layer thickness $y_m=2.0$

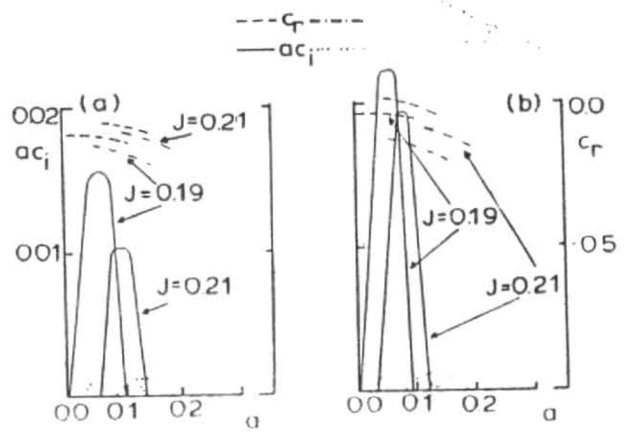

Fig. 6. As in Fig. 3 but for normalized distance from the inflection point $d=0.0$ and layer thickness $y_m=2.0$

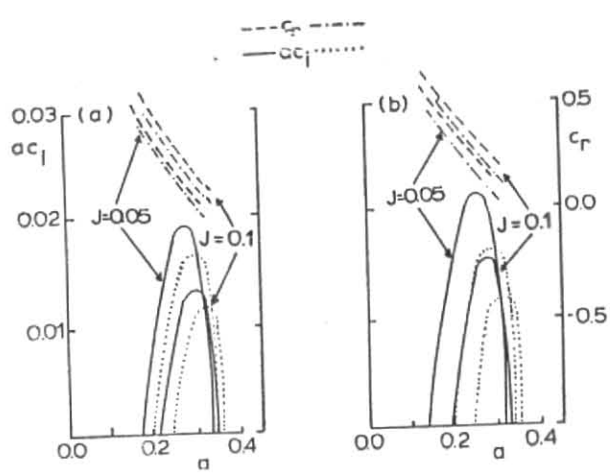

Fig. 7. As in Fig. 4 but for Mode III

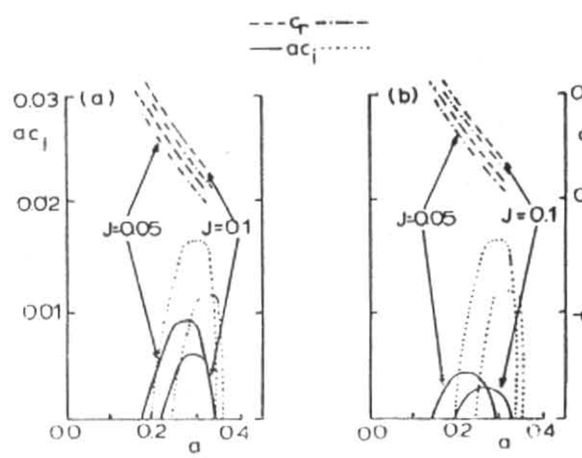

Fig. 8. As in Fig. 7 but for normalized distance from the inflection point $d=5.0$

3.1.2. *Longer wavelengths : Additional Mode III*

The results for the waves of Mode III, are presented in Figs. 7 and 8. Fig. 7 gives results with $d=-5.0$ while Fig. 8 depicts results with $d=5.0$. As we mentioned already, the unstable waves belonging to Mode III are mostly evanescent in the bottom layer and largely propagating above the shear zone. Thus the saturated layer behaves differently on the waves of Mode III than on Modes II and IV. Therefore, in the case with $d=-5.0$, the growth rates are seen to increase substantially while the position of the critical level is affected once again by the different geometry, with corresponding variations in the phase velocities of the disturbances. The introduction of the moist layer above the point of inflection, unlike for the other Modes II and IV, is stabilizing. What effect does the moist layer position just below to the inflection point, on the unstable Mode III ? As shown by Jacovides (1988), the saturated layer behaves as a solid boundary, so that, the ground seems to reach the origin and the unstable waves of type III Mode, which radiate energy upward, dissappear. This is in accordance with the results reported by Lalas and Einaudi (1976).

TABLE 1

Characteristics of the unstable modes of gravity waves generated by shear flow, as predicted by the present model, the model used by Wang *et al.* (in parenthesis) along with observed values of 6 March 1979

	Horizontal phase velocities (m/s)		Horizontal wavelengths (km)		Richardson number
Below the inflection point					
Mode I	8	(10)	3.5-6.7	(2.6-4.2)	0.25
Mode II	10-12	—	9.3-12	—	0.1
Mode III	12-14	(11-12)	11.5-19	(6.3-14.5)	0.14
Mode IV	—		—		—
Observed	13		12-19		0.22
Above the inflection point					
Mode II	10.8-14		8-11.5		0.1
Mode III	11.5-16		12-21.5		0.14
Mode IV	12.2-17		14-23		0.2

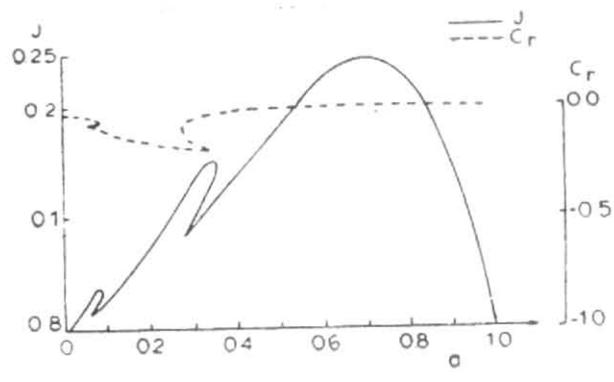

Fig. 9. Neutral stability boundary plots in the (a, J) plane projection for normalized layer thickness $y_m = 2.0$, normalized distance $d = 0.0$ and Brunt-Vaisala frequency $N_m = 0.00236$ s^{-1} which corresponds to a specific humidity of 0.435 gm/kg

3.2. *Effect of the stability variation δ, on the waves*

If one compares the two cases (a) and (b) for all figures the influence of the stability reduction is easily discernable. As $\delta = N_m/N$, is decreased from 0.46 to 0.24 the growth rates increases, the critical levels tend toward the inflection point and the phase velocities toward the mean shear value while smaller wavenumbers become unstable. Negative values of δ (*i.e.*, $N_m < 0$) are omitted because convective activity will then be triggered and the applicability of the present analysis will be in doubt. The above results referred to the case with $d = 5.0$ and for waves of Modes II and IV. For the case with $d = -5.0$ the growth rates decrease substantially when δ is decreased. However, the stability reduction is more important when $d = 0.0$, especially on the waves with greater Richardson numbers.

Thus, if the water vapour increases so that δ becomes less than a critical value of 0.18 there is an increase of growth rates as one could expect but also in this case, there is evidence of change in the model structure. Therefore a singular neutral mode analysis has been carried out for this case to establish the stability boundary and the results are shown in Fig. 9. The shape of the neutral curve is very similar to the one obtained by Lalas and Einaudi (1976) in their attempt to include an elevation inversion in the overall profile of temperature.

The influence of stability reduction on the waves of type Mode III, is illustrated in Figs. 7 and 8. Comparing the cases (a) and (b) is easily shown the effect of stability reduction on these waves depending on the propagation characteristics and the moist layer position. This reduction resulted in an increase of the range of the unstable wavenumbers by incorporating smaller values of a. For the case of layer position above the point of inflection the stability reduction is more important, since for a critical value of $\delta = 0.17$ the unstable waves of Mode III, disappear. For the case with $d = 0.0$ the unstable waves of Mode III seem to merge into one continuum of modes (*i.e.*, Mode I).

The present analysis reveals the effects of moisture on a broaden spectrum of gravity modes. In this way, the practical implementation of this analysis is that, it allows the stability of any atmospheric profile to be tested as the one presented in the next section.

4. Model application to the atmosphere

The simple model of condensation effects on gravity waves presented in this analysis could be used to explain the conditions under which some mesoscale phenomena are organised and triggered. The ranges of the parameters considered in the previous sections have been chosen keeping in mind representative atmospheric situations.

An interesting physical situation to which our analysis can be applied is the formation mechanism of the so-called wavelike rainbands. In this way we are concerned with rainbands similar to these described by Wang *et al.* (1983). They reported that the vertical velocity profiles of the horizontal component perpendicular to the wavelike rainbands is similar to a hyperbolic tangent one (*see* Fig. 18 of Wang *et al.* 1983). It must be noted here that, although these types of rainbands are rare, some of them have a horizontal scale commensurate with gravity waves.

Assuming now that the unstable waves of Modes I, II, III and IV, with a equal to a_{max}, are excited, let us see how they compare with observations. In Table 1, we reproduced the cases mentioned by Wang *et al.* (1983) along with their calculated and observed efficient wavelengths and phase velocities of our most unstable waves. In the present model, wavelengths and phase velocities have been calculated by assuming a moist region above and below the origin, as opposed to an atmosphere saturated throughout, as assumed by Wang *et al.* (1983). The relevant physical parameters have been taken from their reported data (*i. e.*, width of the shear layer, Richardson numbers, height of inflection point, etc).

The results of the present model agree approximately with those of Wang *et al.* (1983) for Modes I and II, with the present results covering wider wavelength and phase velocity range. Mode IV, which was not investigated by Wang *et al.* (1983) though possesses phase velocity as well as wavelengths that are quite close to the observed ones. However, it can be seen that the type III Mode agrees best with the observed wavelengths of the wavelike rainbands. Given the uncertainties in the experimental data and the departures of the actual velocity and density profiles from the simple configuration utilised, the agreement seems to be good enough to suggest that instabilities of the gravity type can be associated with the formation mechanism of the wavelike rainbands. It is evident, however that, the lack of experimental data together with the coarsenes of the model do not permit a definite identification of the gravity waves as the cause of the rainband structure.

5. Conclusions

It has been shown that the presence of a moist layer at different levels, in respect of the point of inflection and the ground, can significantly affect the stability characteristics of internal gravity waves. The presence of such effective layer is shown to stabilise short wavelengths and destabilise substantially longer wavelengths. Also it is found that moisture-saturation has a destabilising effect on the resonant modes which contain long wavelengths. Additionally, the propagation characteristics of the waves may be altered, even no convection is triggered by ducting phenomena generated by the wave itself, through the induced changes of the vertical structure of the atmosphere *via* the presence of the effective moist region.

The characteristics of internal gravity waves in actual atmospheric flows, once the velocity and temperature profiles are known, can be readily obtained by the present model. Also, the waves studied in this work, which are excited in an almost saturated shear flow, are bound to play an important role in determining possible new generation mechanisms for various phenomena which are organised and triggered with spatial and temporal scales well within the gravity wave domain (Wang *et al.* 1983). Thus, the present analysis, reveals some new important results which are associated with rainband structure, since only Mode III provides a consistent interpretation of the observed waves in such environment (Testud *et al.* 1980). However, incorporation of several other important features such as more realistic distributions of temperature, nonlinear effects, etc. which were not taken into account here, is essential.

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