Mausam, (1988), **39**, **1**, 83-86

551.553.21 : 551.577 : 519

A study of monsoon daily rainfall at Maithon by Markovian model and information theory

G. C. BASU

Meteorological Office, Gaya Aerodrome

(*Received 6 March 1986*)

सार — 1968–1982 की अवधि के दौरान मैथन में मानसून दैनिक वर्षा का सांख्यिकीय अध्ययन एक पद संक्रमणीय प्रायिकता आव्यूह (मैट्रिक्स) (प्रसंभाव्य आव्यूह) के माध्यम से मारकोवियन निदर्श द्वारा किया गया है, जिसे विभिन्न वर्षा अभिलक्षणों के लिए एक संक्रमणीय अवस्था से दूसरी अवस्था द्वारा प्राप्त किया गया। "सूचना सिद्धांत" की जानकारी द्वारा परिभाषा के अनुसार मानसून वर्षा अभिलक्षणों (जैसे गैर-वर्षा, हल्की वर्षा, सामान्य वर्षा, अधिक वर्षा, बहुत अधिक वर्षा के दिनों की संक्रमणीय अवस्था की अनिश्चितता या अनियमितता) का अध्ययन किया गया। मैथन में मानसून के दौरान विभिन्न राज्यों की अनुकूल और प्रतिकूल अवस्था का भी अतिरिक्तता परीक्षण द्वारा अध्ययन किया गया है।

ABSTRACT. A statistical study of monsoon daily rainfall at Maithon during the period 1968-1982, is carried out by Markovian model through one-step transitional probability matrix (stochastic matrix), obtained from one transitional state to other for different rainfall characteristics. By the knowledge of the 'Information Theory', the uncertainty (or disorderness) of the transitional system of the monsoon rainfall characteristics (such as non-rainy days, light rains, moderate rains, heavy rains & very heavy rains) according to definition are studied. The favourable or unfavourable condition of different states during the monsoon period at Maithon are also studied by the redundancy test.

1. Introduction

Gabriel & Neumann (1962) and Medhi (1976) have studied the probabilities of occurrences of dry & wet days only by fitting a Markov-chain model and a geometric spell distribution. In this paper the nature of the day's rainfall at Maithon dam, categorised into five classes according to definitions below, has been studied by a Markovian model from one transitional state to another and using also information theory.

The data of daily rainfall used are those recorded at Maithon dam site which is an important place in the Damodar valley area. The inflow of water into the reservoir at Maithon depends on the monsoon rainfall as 90% of the annual rainfall is received there during the monsoon period. The variation of monsoon rainfall (June-September) over Damodar valley area studied by Basu (1981) gives an idea of the yearly deviation of monsoon rainfall from the mean at Maithon.

2. Methodology

2.1. A one-step 5×5 transitional probability matrix from one transitional state to another has been found for each of the rainy months June to October from the frequency of occurrence of daily rainfall amount at Maithon. The disorderness (entropy) of such transitional probability is tested from the redundancy of the system.

Domenico (1972) suggested the test of uncertainty of transitional probabilities of all states of occurrence for the purpose of determining favourable or unfavourableness of the system through redundancy method. It is obtained by the average of all entropy values weighted in accordance with the probabilities of occurrence of the individual states. He chose the Markovian model to test whether the individual state of occurrence depends on previous states of occurrence.

2.2. The total probabilistic picture of possible changes that occur during a transition from one state of occurrence to another, is given by one-step transitional matrix:

$$\pi = | P_{ij} | = \begin{vmatrix} P_{00} \cdots\cdots\cdots\cdots P_{04} \\ P_{10} \cdots\cdots\cdots\cdots P_{14} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ P_{40} \cdots\cdots\cdots\cdots P_{44} \end{vmatrix}$$

P_{ij} $(i, j = 0, 1, 2, 3, 4)$ is the transitional probability from the jth state of occurrence to the ith with $\sum_{j=0}^{4} P_{ij} = 1$, for each i.

2.3. A mathematical model of Shannon serves as a measure of uncertainty of the transitional probability (P_{ij}) of the above system and is :

$$H_i = - \sum_{j=0}^{4} P_{ij} \log_{10} P_{ij}, \text{ for each } i,$$

TABLE 1

Conditional one-step transition probability matrices for each monsoon month at Maithon by Markovian model

June

$$P_{ij} = \begin{vmatrix} .76 & .07 & .13 & .03 & .01 \\ .55 & .21 & .18 & .06 & .00 \\ .60 & .12 & .21 & .07 & .00 \\ .47 & .12 & .18 & .23 & .00 \\ .00 & .50 & .00 & .50 & .00 \end{vmatrix}$$

July

$$P_{ij} = \begin{vmatrix} .58 & .16 & .19 & .07 & .00 \\ .42 & .23 & .25 & .10 & .00 \\ .31 & .24 & .33 & .12 & .00 \\ .38 & .29 & .15 & .18 & .00 \\ .00 & .00 & 1.00 & .00 & .00 \end{vmatrix}$$

August

$$P_{ij} = \begin{vmatrix} .62 & .16 & .17 & .04 & .01 \\ .41 & .21 & .31 & .07 & .00 \\ .41 & .18 & .34 & 07 & .00 \\ .30 & .25 & .25 & .15 & .05 \\ .50 & .00 & .00 & .50 & .00 \end{vmatrix}$$

September

$$P_{ij} = \begin{vmatrix} .74 & .09 & .14 & .03 & .00 \\ .44 & .17 & .31 & .08 & .00 \\ .38 & .17 & .32 & .13 & .00 \\ .48 & .09 & .33 & .05 & .05 \\ .00 & .00 & .50 & .50 & .00 \end{vmatrix}$$

October

$$P_{ij} = \begin{vmatrix} .89 & .04 & .05 & .01 & .01 \\ .76 & .06 & .12 & .06 & .00 \\ .63 & .14 & .18 & .05 & .00 \\ .29 & .14 & .43 & .14 & .00 \\ .00 & 1.00 & .00 & .00 & .00 \end{vmatrix}$$

P_{ij} $(i, j = 0, 1, 2, 3, 4)$ = transitional probability of the ith state from the jth state.
where, '0' indicates 'non-rainy days'; '1' 'light rain' '2' 'moderate rain', '3' 'heavy rain' and '4' 'very heavy rain'

TABLE 2

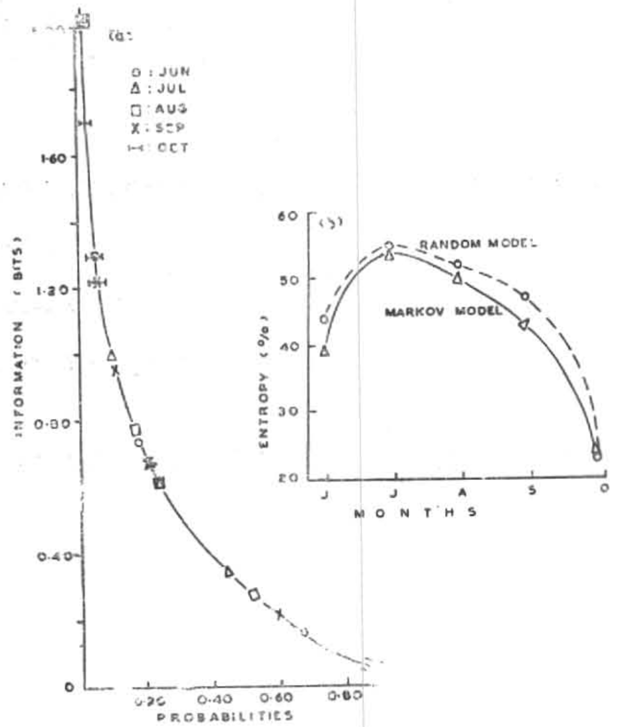
Probability and corresponding information (in bits) for different individual state of occurrence for each of the monsoon months at Maithon

Probability of occurrences (P_i)		Information in bits $I(P_i) = \log P_i$ June	July	August	September	October
Non-rainy day	$P_0/I(P_0)$	.67/.17	.45/.35	.52/.28	.61/.21	.86/.07
Light rain	$P_1/I(P_1)$	.10/1.00	.21/.68	.17/.77	.11/.96	.05/1.30
Moderate rain	$P_2/I(P_2)$	.18/.74	.24/.62	.24/.62	.21/.68	.06/1.22
Heavy rain	$P_3/I(P_3)$	.05/1.30	.10/1.00	.06/1.22	.06/1.22	.02/1.70
Very heavy rain	$P_4/I(P_4)$	.01/2.00	.00/-	.01/2.00	.01/2.00	.00/-
$-\Sigma P_i \log P$		.44	.55	.52	.47	.23

TABLE 3

Measure of entropy and redundancy test statistics during the months of monsoon period at Maithon

	June	July	August	September	October
Non-rainy day (H_0)	.35	.48	.47	.36	.21
Light rain (H_1)	.48	.56	.54	.54	.34
Moderate rain (H_2)	.46	.58	.53	.56	.44
Heavy rain (H_3)	.54	.57	.65	.54	.56
Very heavy rain (H_4)	.30	.00	.30	.30	.00
Weighted entropy, $H = \sum_{i=0}^{4} P_i H_i$	.39	.54	.50	.43	.24
Redundancy, R (in per cent)	44	23	29	39	66
Measure of entropy $M = (-\Sigma P_i \log P_i) - H$	+.05	+.01	+.02	+.04	—.01



Figs. 1 (a & b). Probabilities of occurrence against information in bits for differet categories in each of the month (Jun-Oct) and (b) distribution of uncertainty of the Markovian and random models

where, H_i denotes the entropy of the ith state of occurrence ; $i = 0, 1, 2, 3, 4$.

2.4. The entropy H of the Markovian system of the transitional probability matrix (π) obtained from the probability of individual state of occurrence P_i and weighted by H_i is :

$$H = \sum_i P_i H_i$$

where, P_i is the probability of the ith state of occurrence.

A measure of uncertainty M of the random model obtained from the individual states of occurrence over the Markovian model in the ststem is :

$$M = (-\sum_i P_i \log_{10} P_i) - H$$

$$= -\sum_i P_i \log_{10} P_i - \sum_i P_i H_i$$

2.5. As the probability P_i of the individual state of occurrence of categorised rainfall is taken as random, its distribution is given by using Shannon's formula for 'Information' (in bits) of the system in the form :

$$I(P_i) = -\log_{10} P_i$$

The redundancy of the state of occurrence R is obtained as the difference from 1 of the ratio of the weighted entropy value H to the maximum possible entropy ($H_{max} = \log 5$).

$$R = 1 - H/H_{max}$$

This redundancy value is used to determine the favourable or unfavourableness of the system. When the redundancy value R tends to 1, the system tends to maximum favourable condition, *i.e.,* almost certain.

3. Computational procedure

3.1. The nature of day's rainfall has been categorised according to amount.

Category No. (rainy day)	*Daily rainfall amount (mm)*
0 Non-rainy day	0-2.4
1 Light rain	2.5.-7.5
2 Moderate rain	7.6-34.9
3 Heavy rain	35-124.9
4 Very heavy rain	greater than 125

The conditional transitional probabilities P_{ij} are calculated by dividing the frequencies of one state of occurrence for the day followed from another state of occurrence of the previous day f_{ij} by the frequencies of the state f_i, *i.e.,* $P_{ij}=f_{ij}/f_i$ and are arranged in the form of a matrix for each monsoon month (Table 1).

3.2. The weighted entropy values (Table 3) for each of the months are calculated as a sum of the entropy values of all the different categories with the random probabilities of the corresponding states (Table 2).

4. Discussion

4.1. *Markovian model and redundancy test*

The weighted entropy values over the transitional probabilities during the beginning of June and in the closing month, October, at Maithon are less in comparison with the other monsoon months whereas, during the active monsoon months, July and August, these values are higher (Table 3).

4.2. *Measure of uncertainty*

The distribution of uncertainty of the Markovian & the random models is shown in Fig. 1(b). The difference in uncertainty between the Markovian model and the random model at Maithon lies between +5% and—1%, the differences are small, almost negligible (Table 3).

4.3. The study of the logarithmic probability of the individual occurrences gives the information of the system in bits (Table 2). The probability of occurrence P_i is plotted against $I(P_i)$ information in bits for the different categories in each of the months and the envelope of these points is shown as the curve in Fig. 1(a). The curve has a shape somewhat similar to a rectangular hyperbola, asymptotic to the axis of information bits.

5. Conclusion and remarks

From Table 2, it is evident that the degree of certainty increases with the increase of information bits. During the monsoon period at Maithon, the degree of certainty of heavy rainfall and very heavy rainfall is more than the other states of occurrence though the values of the probability of occurrence are very low. The occurrence of non-rainy days at Maithon during June and October is more probable whereas during July and August, it is less.

The present study has been carried out for the dam site at Maithon only. The applicability of this method to various other places/areas requires detailed study after carefully examining the quantitative values of certainty and redundancy.

Acknowledgements

The author would like to express his deep gratitude to Dr. S.R. Khamrui, Professor of Mathematics, Dept. Jadavpur University, Calcutta for his valuable suggestion and generous help. The author wishes to express his grateful thanks to Shri U.S. Das, Director, Meteorological Centre, Patna and to Shri P.K. Bhattacharya, a retired Meteorologist, India Met. Dep. & at present, a senior research associate, Basic Research in monsoon dynamics at Jadavpur University, Calcutta for their advice & encouragement throughout this work. The author also expresses his thanks to the Dy. Director General of Meteorology (H), Regional Meteorological Centre, Calcutta and to the Meteorologist Incharge, Flood Met. Office, Asansol for perusing the study and to the referee for his valuable suggestions.

References

Basu, G.C., 1981, Indian Met. Soc., *Vayu Mandal*, **11**, 3 & 4, p. 52.

Clarke, A.B. and Disney, R.L., 1970, Probability and Random Processes for Engineers and Scientists, 4, 125.

Domenico, P.A., 1972, *Concepts & Models in Groundwater Hydrology*, **2**-57.

Gabriel, K.R. and Neumann, J., 1962, *Concepts and Model in Groundwater Hydrology*, 88, 90.

Gnadenko, B.V., 1969, *The Theory of Probability*, **3**, 110.

Medhi, J., 1976, *Indian J. Met. Hydrol. Geophys.*, **27**, p. 431.