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Letters to the Editor

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DRYSPELL PROBABILITY BY MARKOV CHAIN MODEL AT MADRAS DURING THE NORTHEAST MONSOON MONTHS

1. Markov chain model gives the basic probable representation for spells (wet and dry) distribution and also makes it possible to derive several other properties of rainfall occurrence patterns. This model has found wide application. Gabriel and Neumann (1962) studied the sequences in daily rainfall occurrence at Tel Aviv and found them to be well described by a Markov chain model. Other studies include those by Hopkins and Robillard (1964), Pattison (1965), Maunder *et al.* (1971), Chowdhury *et al.* (1979), Agnihotri *et al.* (1984). Kapoor and Sastri (1970) described a method to evaluate frequencies of continuous days with rainfall above or below a chosen threshold value. Pandharinath (1991) applied Markov chain model to know the sequence of dry and wet periods over Andhra Pradesh during the monsoon months which is useful for proper agricultural planning. In this investigation an attempt has been made to apply a first order Markov chain model to daily rainfall data at Madras during northeast monsoon months.

2. In this study it is assumed that the probability of a day being wet/dry depends only on the event (wet/dry) of the previous day and is independent of the farther preceding days. Such a model is the Markov chain model of first order. Conditional probabilities P_1 and P_0 are the parameters of this model where.

$$P_1 = P(W/W) \qquad (1)$$

P_1 is the probability of occurrence of a wet day when the preceding day is wet and

$$P_0 = P(W/D) \qquad (2)$$

P_0 is the probability of occurrence of a wet day when the preceding day is dry. The probability of dryspell of length 'n' days is given by

$$P_0 (1 - P_0)^{n-1}$$

The probability of dry sequences $> n$ is

$$(1 - P_0)^n$$

2.1. Daily rainfall data for the northeast monsoon months (October to December) at Madras (13°N. 80°E) for 30 years (1961-1990) have been utilized. A day was called wet day when it received total rainfall of at least 5 mm/day otherwise it was called a dry day. Plant pathologists are of the view that this threshold provides the crop with significant amount of moisture (Rao and Reddy 1990). Victor and Sastri (1979) also chose somewhat higher threshold value than the average Potential Evapotranspiration (PET) while studying daily rainfall over Delhi.

2.2. In cases where a dry spell overlaps between two adjoining months, the days of the spell belonging to the first month were not assigned to the adjoining month, but the spell was treated as ending on the last day of the first month and rest of the days of spell were treated as a spell belonging to the next month. The probabilities of different lengths of dry spells have been calculated using the expression $P_0(1 - P_0)^{n-1}$. The expected frequencies of dry spells have been calculated by multiplying these probabilities with the total number of dry days in a month. The Chi-square test was applied to test the adequacy of the fit by comparing estimated and observed frequencies. The expected length of runs (dry/wet), length of a weather cycle, *i.e.*, a wet spell followed by a dry spell or *vice versa*, were obtained from Victor and Sastry (1979). Stationary probabilities for the occurrence of dry and wet days (π_1 and π_2) have been worked out using the following two equations :

$$\pi_1 = \frac{1 - P_1}{1 + P_0 - P_1} \qquad (3)$$

$$\pi_2 = \frac{P_0}{1 + P_0 - P_1} \qquad (4)$$

3. Probability P_d of dry day, and the conditional probabilities P_1 and P_0 and other properties are

TABLE 1

Different parameters and properties of Markov chain probability model

	Conditional probabilities		Probability of a dry day	Expected length of runs		Dry-wet cycle	Expected number of days		S.D. of dry or wet days	Stationary probability	
	P_1	P_0	P_d	Dry	Wet		Dry	Wet		π_1	π_2
October	0.55	0.16	0.14	6.3	2.2	8.5	22.9	8.1	3.7	0.74	0.26
November	0.64	0.14	0.13	7.0	2.8	9.8	21.4	8.6	1.5	0.71	0.29
December	0.57	0.08	0.10	13.0	2.3	15.4	26.3	4.7	1.2	0.85	0.15

presented in Table 1. Perusal of the data in Table 1 revealed that compared to the probability of occurrence of wet day when the previous day is dry, *i.e.*, P_0, the probability of occurrence of wet day when the previous day is wet (P_1) is higher. It is also important to notice that there is a higher probability (0.57) of wet day being followed by a wet day in December than that in October (0.55) though October receives more rain than December.

3.1. Table 1 also revealed that on an average the wet spell lasts for about 2-3 days for all the three months whereas the dry spell lasts for about 6-7 days in October and November (active northeast monsoon months) and is about 13 days in December. The value of π_1 (stationary probability for the occurrence of dry day) is least in the month of November followed by October and December, and value of π_2 (stationary probability for the occurrence of wet day) is maximum in November followed by October and December. This indicates that during the northeast monsoon period over Madras, November is the most humid month followed by October and December.

3.2. Table 2 contains the expected and observed frequencies of the sequence of dry days for October, November and December. It is seen that the differences are not significant at 5% level for all the 3 months; in other words it means that the fit is good.

3.3. The probabilities may also be expressed in terms of an average recurrence interval or the return period (Weiss 1964). Table 3 represents the length (in days) of dry sequences corresponding to a given return period for rainfall less than 5 mm/day. Perusal of the data shows that only once in two years, on an average, Madras experiences a dry sequence of 11 days in October for which precipitation does not exceed 5 mm/day. The data also show that on an average dry sequences of longer length,

TABLE 2

Expected (E) and observed (O) frequency of dry spells

Spell length (days)	October		November		December	
	O	E	O	E	O	E
1	27	21	27	17	12	7
2	18	18	18	15	6	6
3	14	15	14	13	10	6
4	12	13	10	11	6	5
5	12	11	8	9	4	5
6	10	9	7	8	4	4
7	8	8	6	7	5	4
8	4	6	3	6	6	4
9	5	5	4	5	4	4
10	1	4	3	4	3	3
11	6	4	3	4	2	3
12	3	3	1	3	2	3
13	4	3	2	3	2	3
14	2	2	3	2	2	2
15	2	2	1	2	3	2
16	1	2	1	2	3	2
17	2	1	2	1	3	2
18	0	1	1	1	3	2
19	1	1	2	1	1	2
20	0	1	0	1	1	1
21	0	1	0	1	1	1
22	1	1	2	1	1	1
23	1	0	1	1	1	1
24	0	0	0	0	1	1
25	0	0	0	0	0	1
26	0	0	1	0	2	1
27	0	0	0	0	0	1
28	0	0	0	0	0	1
29	0	0	0	0	0	1
30	0	0	0	0	0	1
31	0	0	0	0	1	1
χ^2	2.85		9.31		6.17	

TABLE 3

Dry sequence length n (days) corresponding to given return period T_d for rainfall < 5 mm/day

T_d (years)	October	November	December
50	30	*	*
25	26	28	*
10	21	22	*
5	17	18	29
2	11	12	17

* — n exceeds S.

S — Number of days in the sub-interval for which the sequences are counted.

for all the return periods, occur in December compared to October or November.

4. From the above study it can be inferred that during the northeast monsoon months the dry run lasts for about 6-7 days during October and November whereas for December it is about 13 days. Dry sequences of longer lengths occur in December compared to October or November.

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