

Effect of latent heat release on mountain waves in a sheared mean flow

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सारा — ऊँचाई के साथ पवन गति में परिवर्तन की धारणा के आधार पर पर्वत के पवनविमुख क्षेत्र में, पर्वतीय तरंगों पर वर्षा के कारण गुप्त उष्मा निकलने के प्रभाव का अध्ययन किया गया। विश्लेषण हेतु, पेशावर के वास्तविक आँकड़ों के आधार पर, एक ही स्तर पर विचार किया गया। प्रस्तुत अध्ययन के लिए मध्यम तल पर उष्मन के एक पतले तल को चुना गया। एक गैर-अलस्थैतिक मामले के लिए यह ज्ञात हुआ कि गैर-वर्षण के मामले में जब पर्वत के पवनविमुख/पवनविमुख क्षेत्र में संतुलित उष्मन/शीतलन होता है तब उष्मन का प्रभाव नगण्य होता है। तथापि वर्षण के मामले में, उष्मन के तल से कुछ ऊपर, पवनविमुख क्षेत्र में उद्योमुखी विस्थापन स्वाभाविक है। उत्पापित तापीय बल के कारण उत्पन्न और धरातल से परावर्तित तरंगों और उर्ध्वप्रवाही धारा का आपसी टकराव इस प्रक्रिया की विशेषता है। गैर-वर्षण मामले की तुलना में पर्वत के पवनविमुख क्षेत्र में तरंगीय आयाम में वृद्धि पाई गई। यह भी ज्ञात हुआ कि उष्मन का तल जितना अधिक ऊँचा होगा, प्रेरित विक्रोम का आयाम उतना ही कम होगा। सामान्यतः 4.5 कि० मी० (धरातल से ऊपर) का तल उष्मन से सर्वाधिक प्रभावित होता है।

प्रस्तुत अध्ययन में विस्तृत और उथले पर्वतीय घू-खंडों में अपरूपण (शियर्ड) प्रवाह के लिए उष्मन के तीन विभिन्न तलों हेतु जलस्थैतिक बल प्रस्तुत किए गए हैं। प्रवाही रेखाएँ खींची गईं। गैर-अपरूपण मामले की तुलना में यह अनुमान लगाया जा सकता है कि अपरूपण प्रभाव तापीय बल के प्रभाव के प्रतिकूल हैं।

ABSTRACT. The effect of latent heat release on windward side of the mountain, due to precipitation, over the mountain waves has been studied assuming wind speed changing with respect to height. A single profile based on actual Peshawar data has been considered for the analysis. A thin level of heating has been chosen at medium level for the purpose of study. For non-hydrostatic case it is observed that in non-precipitation case when balanced heating/cooling takes place on the windward/leeward side of the mountain the effect of heating is negligibly small. However, for precipitation case downward displacement on the windward side, just above the level of heating, is obvious. Interference with the upstream current by the waves, produced due to elevated thermal forcing and reflected from the ground surface is attributed to this phenomenon. Increase in the wave amplitude on the lee-side of the mountain as compared to non-precipitating case is also found. It is also revealed that higher the level of heating, lesser the amplitude of the induced disturbance. 4.5 km agl is the level which is maximum affected by heating in general.

For large and shallow mountainous terrains, hydrostatic solutions have been produced for three different levels of heating for sheared flow. Streamlines have been drawn. On comparison with no shear case, it may be inferred that shear effect is opposite to that due to thermal forcing.

Key words — Hydrostatic, Non-hydrostatic, Heating, Cooling, Mountain wave, Precipitation, Upstream.

1. Introduction

The orography over an area influences the distribution of rainfall in that region. The investigations carried out by Bonacina (1945), Bergeron (1960), Smith (1979) and Browning (1980) illustrate these details. The orographic influence is generally caused through forced uplift and thermal effects. However, the total modifications to the incident airflow by the mountains are not yet understood in its entirety, so as to explain the preferred regions of increased rainfall in a hilly region or in a valley. Enormous amount of latent heat is absorbed from the atmosphere in the evaporation process of the

clouds causing cooling. Similarly, enormous heat is released in the atmosphere during the condensation/precipitation process in the atmosphere causing heating. Both these processes cause significant difference in the stratified structure of the atmospheric layers. Effect of cooling induced on the cloud top and its influence on lee-waves has been studied by Kumar and Scorer (1993). It has been observed by them that evaporation/radiational cooling significantly changes the shape of the lee-wave and causes steepening of crests and shallowing of troughs. The study of effect of heating due to condensation has also been attempted in the past. Investigators in their theoretical attempts have

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combined latent heat with orographic forcing (Sarker 1966, 1967, Raymond 1972, Gocho 1978, Barcilon *et al.* 1980). Though these studies illustrate the various mathematical methods to quantify the latent heat released and the orographic forcing, the final results are not borne out by the observed rainfall data. The main difficulty, however, remains to be the parameterisation of the quantum of latent heat released. All these studies assume that the condensation arises from smooth orderly ascent of the type resolved by their equations. They also have not considered the excess of heating that occurs over the cooling when liquid water is precipitated (Barcilon *et al.* 1980).

The "smooth ascent hypothesis" may be more appropriate in selected mid-latitude belts during winter periods wherein shallow convection, embedded within frontal clouds, gets modified by orographic forcing (Browning *et al.* 1974, Browning 1980, Marwitz 1980). On the contrary, tropics exhibit vast regions with strong winds impinging on mountain ranges giving rise to closely packed strong convection triggered by mountain forcing in an unstable atmosphere. In any case, if the mountain height is taken as a measure of air ascent, it will be an underestimate of the condensation taking place in the actual convection on the mountainous region.

Conscious of the constraints of the "smooth ascent hypothesis", Smith and Lin (1982) adopted a semi-empirical approach, based on the distribution of the rainfall that may be available from actual data. The rainfall realised is considered as a "measure of the condensation aloft" and the effect of the latent heat thus released on the stratified air stream is then calculated. The vertical motion caused by the thermal forcing can be individually quantified. This added to the orographic forcing, provides the total effect of mountain on orographic rainfall features. They established that in hydrostatic flow, the phase relationship between the heating rate and the induced vertical displacement is found to be negative. Taking periodical heating and cooling rate in an unbounded fluid with heating layer in the centre, the response of heating function resulted in vertically propagating waves such that the vertical displacement at the heating level was found to be exactly out of phase to the heating rate. This process can be modified only to a limited extent by the presence of a rigid lower boundary or by an increased depth of heating region. The elevated heating in their case seemed to produce vertically propagating waves whose amplitude,

relative to mountain waves, was expressed by the parameter $\frac{gQb}{c_p \bar{T} \bar{U}^3 NH}$. For typical wind speeds and rainfall rates the thermally generated waves in their case seemed to equal or exceed the orographically generated waves.

The phase-relationship between the heating function and the induced vertical motion is especially important in the wave-CISK (Conditional Instability of the Second Kind) mechanism where the heat source is due to the condensation of water vapour in rising air induced by wave motion. Downward displacement in the vicinity of the heating may tend to limit the amount of condensation/precipitation which could occur in a standing orographic cloud aloft, unless the condensation is occurring in small-scale convection of a sort which is not strongly suppressed by the broad scale (horizontally 10-100 km) descent. Heating at certain special levels can produce upward air motion at some distance at and near the level of heating. Under marginal stability conditions, this could trigger deep cumulus convections and enhance the condensation and precipitation on the windward side of the slope.

Attempt of Smith and Lin (1982), however, was confined only to the hydrostatic atmosphere when the horizontal length scale is much larger than the vertical one. This type of atmospheric situation is quite not typical one and hence in order to include the realistic case we need to investigate their results for non-hydrostatic, atmosphere with respect to lee-waves (Wurtele 1953, Palm 1955). Sheared mean flow is another general feature of the atmosphere. Hence any attempt to extend the study for actual atmospheric case has to include these two features. In this paper, therefore, an attempt is made to evaluate the response of the sheared mean flow to the latent heat released in both the cases, *viz.*, hydrostatic and non-hydrostatic types of flow. A single layer two-dimensional model with exponential *l*-profile is considered in the analysis.

2. Formulation and solution

It is assumed that basic mean flow is increasing with height, *i.e.*, $\bar{U} = \bar{U}(z)$, with positive wind shear, thus ruling out any critical level aloft. Each dependent variable on the lee-side may be

represented as the sum of a basic mean value and a perturbation term as follows:

$$\begin{aligned} u(x, z) &= \bar{U}(z) + u'(x, z) \\ w(x, z) &= 0 + w'(x, z) \\ \rho(x, z) &= \bar{\rho}(z) + \rho'(x, z) \\ p(x, z) &= \bar{p}(z) + p'(x, z) \\ T(x, z) &= \bar{T}(z) + T'(x, z) \end{aligned} \quad (1)$$

All the variables are defined in Appendix I. Following Smith and Lin (1982), the equation for the vertical velocity of steady, two-dimensional, non-rotating, incompressible fluid, obeying Boussinesq approximation is,

$$\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2} + l^2(z) w = \frac{gH}{c_p \bar{T} \bar{U}^2} \left[\frac{\bar{\rho}(z)}{\rho_0} \right]^{1/2} \quad (2)$$

where,

$$l^2(z) = \frac{g\beta}{\bar{U}^2} - \frac{\bar{U}_{zz}}{\bar{U}} + \frac{S\bar{U}_z}{\bar{U}} - \frac{S^2}{4} + \frac{1}{2} S_z \quad (3)$$

Smooth orderly ascent hypothesis ($H \approx w$) is not strictly applicable (Browning 1980, Marwitz 1980) here. In fact experimental evidence presented by Hill *et al.* (1981) strongly supports the view that orographic rain is largely a low-level phenomenon and washout of cloud droplets is likely to be an important mechanism for generating the rain. For the mathematical formulation of heating function in the atmosphere we need to consider two different physical situations as follows:

- (i) when cloud forms on the windward side of the hill and dissipates due to downward motion on the leeward side, and
- (ii) when cloud precipitates on the windward side and the latent heat released exceeds evaporative cooling on the leeward side of the mountain.

In the first case one may simply attempt solution of Eqn. (2) with heating/cooling expressed as a function of x . Second case has been discussed by Smith and Lin (1982) in detail. When condensed water falls from cloud the atmosphere will receive net amount of heating and hence the heating function [e.g., $H(x, z) = Q \delta(x) \delta(z - z_H)$], which allows a net heating at any level, causes perturbation vertical velocity to decay downstream as proportional to $1/x$ and vertical displacement grows as $\log(x)$. This

implies that net heating does not produce localized disturbance. In such a case slow process of radiative cooling might cause decay of vertical displacement. To facilitate entire process, they correctly choose a prescribed weak and widely distributed cooling function added to the local heating function. This not only permits us to simplify our analysis under the assumption that the net heating at each level is zero,

$$\text{i.e.,} \quad \int_{-\infty}^{\infty} q(x) dx = 0 \quad (4)$$

but also avoids the inclusion of three dimensions and Coriolis force for the purpose of compensatory cooling. We, therefore, proceed to investigate solution of Eqn. (2) with the help of separable heating function,

$$H(x, z) = Q q(x) f(z) \quad (5)$$

where, $f(z)$ is normalized according to

$$\int_0^{\infty} f(z) dz = 1 \quad (6)$$

$$\text{so that, } \bar{\rho} \cdot \int_0^{\infty} \dot{H}(x, z) dz = \bar{\rho} Q q(x) \quad (7)$$

represent the total power added to the vertical column of atmosphere. We, therefore, consider below the following two heating functions for the precipitation and non-precipitation cases respectively:

Non-precipitating case.

$$q(x) = b \frac{d}{dx} \left(\frac{b^2}{b^2 + x^2} \right) = - \frac{2b^3 x}{(x^2 + b^2)^2} \quad (8)$$

Precipitating case:

$$q(x) = \frac{b_1^2}{x^2 + b_1^2} - \frac{b_1 b_2}{x^2 + b_2^2} \quad (9)$$

The method of solution is to first find the response to a heating distribution which is concentrated at z_H according to the equation.

$$\dot{H}(x, z) = Q q(x) \delta(z - z_H) \quad (10)$$

and then to integrate with respect to z_H with the weighted function $f(z)$. Substituting Eqn. (10) in

Eqn. (2) and then integrating from just below to just above the level $z = z_H$, we get

$$\Delta w_z = \rho^{1/2}(z_H) Q q(x) g / (c_p \bar{T} \bar{U}^2 \rho_0^{1/2}) \quad (11)$$

Also w continuity at level $z = z_H$ gives the difference of w from just below to just above as zero, i.e., $\Delta w = 0$. (12)

Away from the heating layer, Eqn. (2) reduces to the equation,

$$w_{xx} + w_{zz} + l^2(z) w = 0 \quad (13)$$

Solution of Eqn. (11) - (13) is given below when depth of the heating region ($2d$) is small. The details are given in Appendix II.

Case I: For non-precipitating case, when heating function is given by Eqn. (8) and if,

$$\eta_1 = \text{Re} \int_0^\infty \frac{\bar{U}(0) h a J_\nu(\mu) e^{-ka+ikx}}{\bar{U}(z) J_\nu(\mu_0)} dk$$

$$\eta_2 = \text{Re} \int_0^\infty \frac{g Q b^2 \pi \rho^{1/2}(z_H) J_\nu(\mu) J_\nu(\mu_H) Y_\nu(\mu_0) e^{-kb+ikx}}{2c_p \bar{T} \bar{U}_H^2 \alpha \rho_0^{1/2} \bar{U}(z) J_\nu(\mu_0)} dk$$

$$\eta_3 = \text{Re} \int_0^\infty \frac{g Q b^2 \pi \rho^{1/2}(z_H) J_\nu(\mu_H) Y_\nu(\mu) e^{-kb+ikx}}{2c_p \bar{T} \bar{U}_H^2 \alpha \rho_0^{1/2} \bar{U}(z)} dk$$

$$\eta_4 = \text{Re} \int_0^\infty \frac{g Q b^2 \pi \rho^{1/2}(z_H) Y_\nu(\mu_H) J_\nu(\mu) e^{-kb+ikx}}{2c_p \bar{T} \bar{U}_H^2 \alpha \rho_0^{1/2} \bar{U}(z)} dk \quad (14)$$

then

$$\eta = \eta_1 + \eta_2 - \eta_3 \quad \text{for } z < z_H \quad (15)$$

$$\eta = \eta_1 + \eta_2 - \eta_4 \quad \text{for } z > z_H \quad (16)$$

The pressure distribution at the ground level can be computed by Bernoulli's equation (Scorer 1975).

$$p(x, 0) = \rho \bar{U}^2(0) \left. \frac{\partial \eta}{\partial z} \right|_{z=0} \quad (17)$$

$$= \rho \bar{U}^2(0) \left. \frac{\partial}{\partial z} \{ \eta_1 + (\eta_2 - \eta_3) \} \right|_{z=0} \quad (18)$$

Case II: For precipitating case when heating function is given by Eqn. (9) and assuming latent

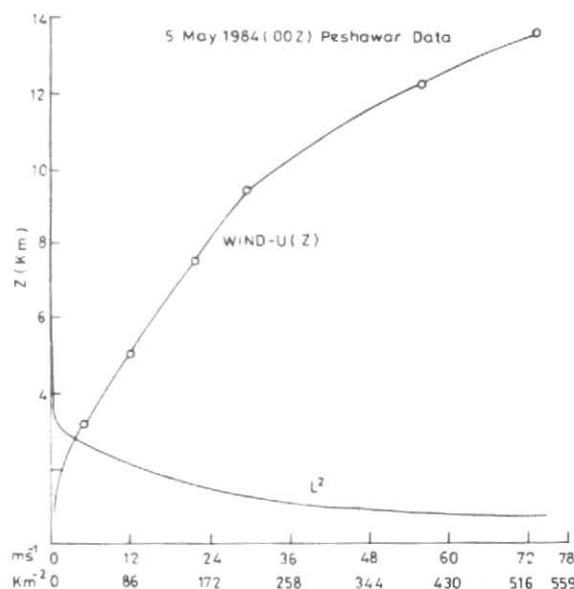


Fig. 1. Wind and l^2 -profile for 5 May 1984 (0000 UTC) over Peshawar (Pakistan)

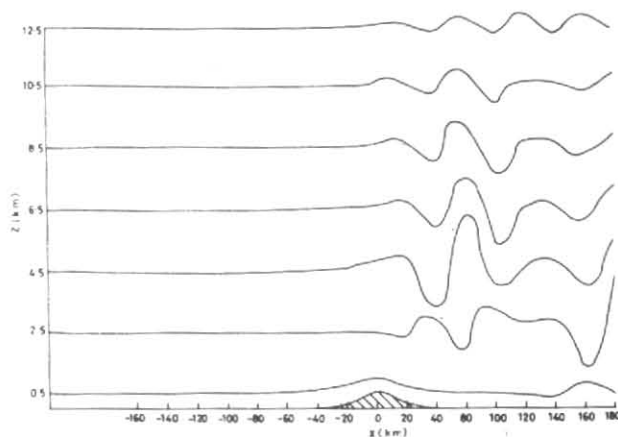


Fig. 2. $h = 0.5$ km; $a = 20.0$ km. Streamlines of flow over a bell-shaped mountain with shear. Solution given by Doos (1961)

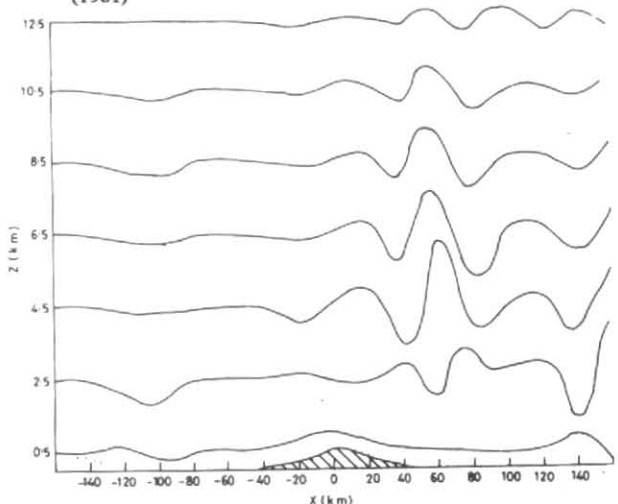


Fig. 3. $b = 20.0$ km; $z_H = 2.5$ km; $h = 0.5$ km; $a = 20.0$ km; $Q = 1107 \text{ w m kg}^{-1}$. Streamlines of flow as a result of Eqn. (15)

heat to be centred at a distance c from the mountain top and if

$$\begin{aligned}\eta_2^c &= \text{Im} \int_0^\infty \frac{\pi g Q b_1 \rho^{1/2}(z_H) (e^{-kb_1} - e^{-kb_2}) Y_{\sqrt{\mu_0}} J_{\sqrt{\mu_H}} J_{\sqrt{\mu}} e^{ik(x+c)}}{2ac_p \bar{T} \bar{U}_H^2 \rho_0^{1/2} k \bar{U}(z) J_{\sqrt{\mu_0}}} dk \\ \eta_3^c &= \text{Im} \int_0^\infty \frac{\pi g Q b_1 \rho^{1/2}(z_H) (e^{-kb_1} - e^{-kb_2}) J_{\sqrt{\mu_H}} Y_{\sqrt{\mu}} e^{ik(x+c)}}{2ac_p \bar{T} \bar{U}_H^2 \rho_0^{1/2} k \bar{U}(z)} dk \\ \eta_4^c &= \text{Im} \int_0^\infty \frac{\pi g Q b_1 \rho^{1/2}(z_H) (e^{-kb_1} - e^{-kb_2}) J_{\sqrt{\mu}} Y_{\sqrt{\mu_H}} e^{ik(x+c)}}{2ac_p \bar{T} \bar{U}_H^2 \rho_0^{1/2} k \bar{U}(z)} dk\end{aligned}\quad (19)$$

then,

$$\begin{aligned}\eta &= \eta_1 + \eta_2^c - \eta_3^c \quad \text{for } z < z_H \\ \eta &= \eta_1 + \eta_2^c - \eta_4^c \quad \text{for } z > z_H\end{aligned}\quad (20)$$

The results are expressible only as lengthy integrals. The most important property of these integrals is that they include lee-wave resonances — the contribution of positive wind shear and non-hydrostatic effects guarantee them. This also refers to the findings of Wurtele (1953). The pressure distribution at the ground is given by

$$p(x, 0) = -\bar{\rho} \bar{U}^2(0) \frac{\partial}{\partial z} \{ \eta_1 + \eta_2^c - \eta_3^c \}_{z=0} \quad (21)$$

3. Discussion

Total reflection caused by elevated thermal forcing due to rigid lid ($w = 0$ at $z = 0$) below raises the issue of constructive or destructive wave interference. This makes the flow field more complex. Fig. 2 shows the simple case of wave occurrence for a flow past small bell-shaped mountain (Doos 1961, Wurtele 1953). To include the normal atmospheric shear, actual data have been taken from Peshawar for 5 May 1984 (Fig. 1), which lies on the foothills, on the west end of Jammu & Kashmir.

Fig. 3 indicates the shape of the wave with balanced heating and cooling produced on either side of the hill. A thin level of heating is chosen at 2.5 km which is normally the level of medium level stratified clouds, which do occur in stable atmospheric layers. In all, 3 wave numbers ($k = 0.06994204, 2.37675103, 4.62572730$) have been considered. Subsequent wave numbers are appreciably large to give any significant effect. A comparison of Figs. 2-3 does not indicate any significant change in

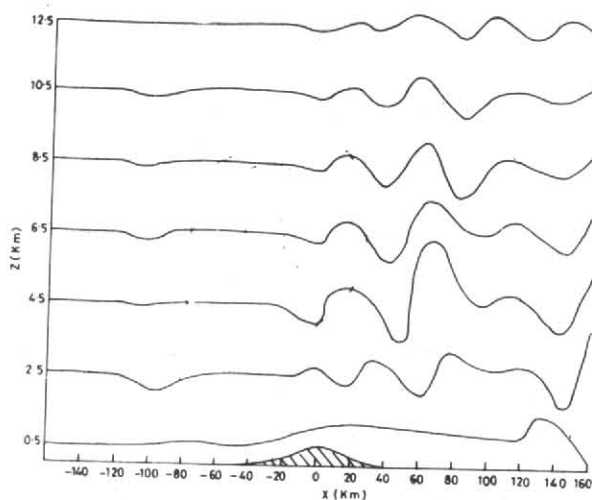


Fig. 4. $b_1 = 20.0$ km; $b_2 = 100.0$ km; $z_H = 2.5$ km; $c = 20.0$ km; $Q = 1107$ wm kg^{-1} ; $h = 0.5$ km; $a = 20.0$ km. Streamlines of flow as a result of Eqn. (20)

the streamline pattern, implying that for balanced heating and cooling on either side of the hill top the diabatic effect is negligibly small, though not zero in our computation. Figs. 4-5 show effects of isolated heating due to precipitation 20 and 40 km away from the hill top. Significant difference may be noticed in the case of no precipitation and that of precipitation in the streamline patterns on either side of the mountain top. Downward displacement on the windward side just above the level of heating is obvious. Wave interference due to reflection from the ground may be noted at 100 km upstream. Although details depend on the precise nature of heating function, it may be noted that such downward displacement occurs, in the present case, not at the place of isolated heating but slightly above it. May be the increase in buoyancy associated with the heated zone combined with the orographic effect restricts downward displacement at the region of latent heat released due to precipitation. Upward propagation

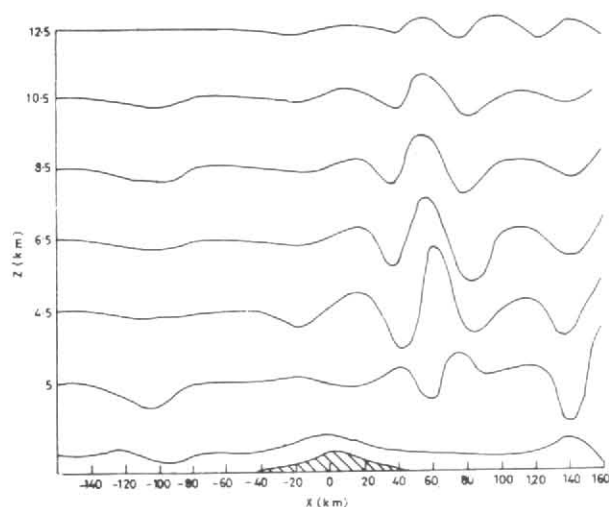
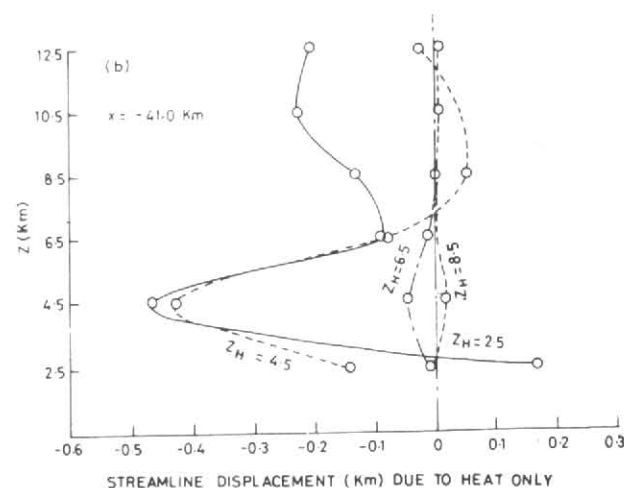
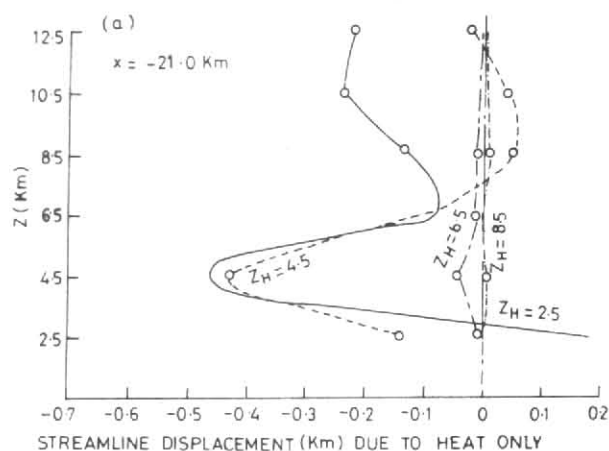


Fig. 5. $b_1 = 20.0$ km; $b_2 = 100.0$ km; $z_H = 2.5$ km; $c = 40.0$ km; $Q = 1107 \text{ w m kg}^{-1}$; $h = 0.5$ km; $a = 20.0$ km. Streamlines of flow as a result of Eqn. (20)

of disturbance is apparent with the phase of the wave tilting upstream. Increase in the wave amplitude on the lee-side of the mountain as compared to the case of no precipitation can also be noticed. Disturbance induced by heating is more prominently marked when isolated centre of heating is further upstream ($c = 40$ km). Upstream disturbance due to wave reflection is more pronounced at 100 km. This case is more realistic as maximum condensation rate often occurs far upstream of the mountain.

Figs. 6 (a & b) indicate the sensitivity of wave phase and its amplitude with respect to the level of heat. They indicate the streamline displacement contributed only by isolated heating centred 21 and 41 km upwind from the mountain top respectively. Keeping in view the damping effect upstream of the flow, these diagrams are drawn at a point which is not too far from the mountain top and not too close to the zone of heat. We, therefore, considered vertical cross section of atmosphere 20 km ahead of the isolated heating centre. It may be observed that in the present non-hydrostatic case with the variation of the level of heating, the phase of the wave is continuously affected and higher the level of heating, lesser is the amplitude of the induced disturbance. It may be noted that maximum and minimum of the wave amplitudes for all the levels of heating function are interestingly confined to the same medium level layer ($z = 4.5$ km). This level remains unaltered with the horizontal shift of the heating function from 21 to 41 km away from the mountain peak. Perhaps, it may be attributed to the interference between the upward moving wave from the

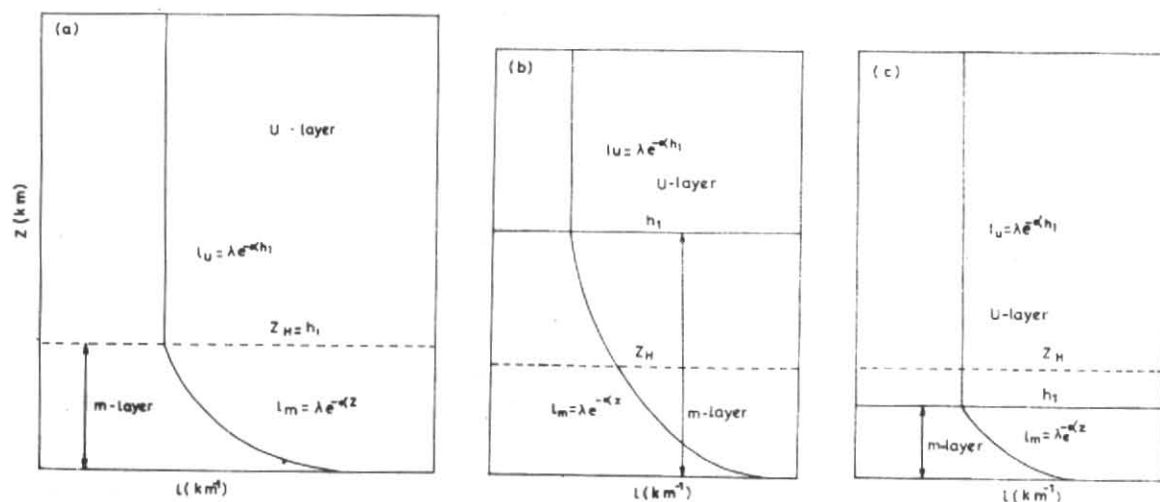


Figs. 6 (a & b). Streamline displacement contributed by isolated heating at (a) 21.0 km and (b) 41.0 km upwind at various levels

heating level and the reflected wave from the mountain surface. Fixed dimension of the mountain, possibly, produces such a resonance level. As in hydrostatic case, interaction with the direct upward and downward travelling waves and the reflected wave from the ground gives complex situation of destructive and constructive interference at places. As a result, for a medium level heating ($z_H = 2.5$), which may be caused by thin layer of altostratus cloud, constructive interaction causes small positive displacement of streamline even at the level of heating. It may also be noted that the shift in the level of heating from 2.5 to 4.5 km changes the final phase by almost $\pi/2$.

4. Hydrostatic approximation

In the non-hydrostatic case all the expressions have to be put in integral form. For flow past large



Figs. 7 (a-c). The assumed vertical structure of Scorer's l -parameter when (a) $h_1 = z_H$, (b) $h_1 > z_H$ and (c) $h_1 < z_H$

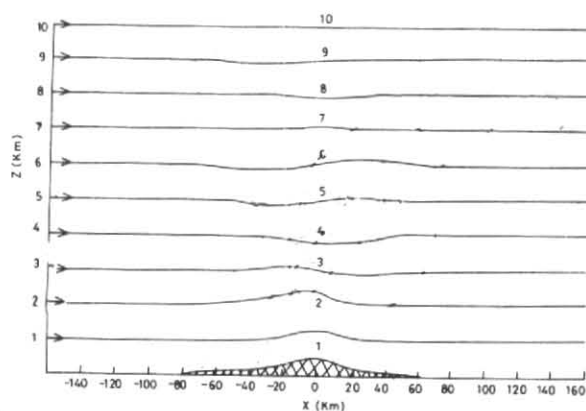


Fig. 8. $\bar{U}(10) = 10$ m/s; $\bar{U}(2.0) = 15$ m/s; $z_H = 2.0$ km; $b = 20.0$ km; $Q = 1107$ wm kg^{-1} ; $a = 20.0$ km; $h = 0.5$ km. Hydrostatic flow with combined thermal and orographic forcing. Heating occurs over the windward side and cooling over the leeward side; Eqns. 27 (a-b)

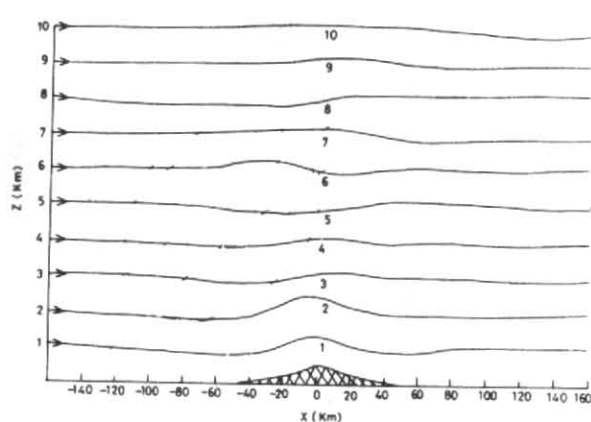


Fig. 9. $\bar{U}(0) = 10$ m/s; $\bar{U}(2.0) = 15$ m/s; $z_H = 2.0$ km; $b_1 = 20.0$ km; $b_2 = 100.00$ km; $c = 40.0$ km; $Q = 1107$ wm kg^{-1} ; $a = 20.0$ km; $h = 0.5$ km. Hydrostatic flow with combined thermal and orographic forcing. Isolated heating is specified over the windward side; Eqns. 27 (c-d)

mountainous terrain, the horizontal length scale is much larger than the vertical scale (i.e., $a \gg h$). We can, therefore, apply the hydrostatic approximation ($k^2 \ll l^2$) to a fair degree of accuracy (Smith and Lin 1982). Study of these hydrostatic waves is more relevant for the Himalayan region where large scale mountains (half width ' a ' and vertical height is of the order of 50 to 200 km and 3.0 to 6.0 km respectively) are common features. Also quite often, the Scorer's parameter satisfies the condition $\{k^2 \ll l^2\}$ for the atmosphere in the Himalayan region. For such a study, assuming Scorer's parameter of the form

$$l_m = \lambda e^{\alpha z}, \quad z \leq h_1 \quad (22a)$$

and
$$l_u = \lambda e^{\alpha h_1}, \quad z > h_1 \quad (22b)$$

the following three cases can be considered :

(i) $h_1 = z_H$ [Fig. 7 (a)] (23a)

(ii) $h_1 > z_H$ [Fig. 7 (b)] (23b)

(iii) $h_1 < z_H$ [Fig. 7 (c)] (23c)

The method of solution for case (i) is presented in this study. Solutions for cases (ii) and (iii) are similar but lengthy. Hence only the final results for them are presented hereunder.

Case (i): $h_1 = z_H$

Assuming that $l^2 \gg k^2$, we can find the solution in the two layers in a straight forward manner.

$$\tilde{w}_1 = C_1 J_0(\mu) + D_1 Y_0(\mu), \quad z \leq h_1, \text{ i.e., in } m\text{-layer} \quad (24a) \quad (ii) \Delta \tilde{w}_z = \tilde{H}$$

$$\tilde{w}_2 = C_2 e^{i l_u z} + D_2 e^{-i l_u z}, \quad z > h_1, \text{ i.e., in } U\text{-layer} \quad (24b)$$

\tilde{w}_1 and \tilde{w}_2 are the \tilde{w} — values in m and U -layer respectively and $\mu = \frac{\lambda e^{\alpha z}}{|\alpha|}$ and $l_u = \lambda e^{\alpha h_1}$. C_1, C_2 and D_1, D_2 are constants to be determined by the boundary and interface conditions. For upward travelling waves we must have $D_2 = 0$.

At $z = h_1$

$$(i) \Delta \tilde{w} = 0, \text{ i.e.,}$$

$$C_1 J_0(\mu_{h_1}) + D_1 Y_0(\mu_{h_1}) = C_2 e^{i l_u h_1} \quad (25a)$$

$$\text{where, } \tilde{H} = \frac{1}{\pi} \int_{-\infty}^{\infty} H e^{-ikx} dk;$$

$$\text{where, } H = \frac{\rho(z_H)}{\rho_0} \frac{Q g q(x)}{c_p T U_H^2}$$

or

$$C_1 \alpha \mu_{h_1} J_0'(\mu_{h_1}) + D_1 \alpha \mu_{h_1} Y_0'(\mu_{h_1}) = i l_u C_2 e^{i l_u h_1} + \tilde{H} \quad (25b)$$

Also applying lower boundary condition, we get,

$$C_1 J_0(\mu_0) + D_1 Y_0(\mu_0) = i k \bar{U}(0) a h e^{-a|k|} \quad (25c)$$

where

$$\mu_{h_1} = \mu \Big|_{z=h_1}, \quad \mu_0 = \mu \Big|_{z=0}$$

Solving (25a), (25b) and (25c), we get,

$$\begin{aligned} C_1 &= \frac{C_2 \{ \alpha \mu_{h_1} Y_0'(\mu_{h_1}) e^{i l_u h_1} - i l_u e^{i l_u h_1} Y_0(\mu_{h_1}) \} - \tilde{H} Y_0(\mu_{h_1})}{\alpha} \\ &= \frac{C_2 \bar{C}_1 - \tilde{H} Y_0(\mu_{h_1})}{\alpha} \text{ (say)} \end{aligned} \quad (26a)$$

$$\begin{aligned} D_1 &= \frac{C_2 \{ \alpha \mu_{h_1} J_0'(\mu_{h_1}) e^{i l_u h_1} - i l_u e^{i l_u h_1} J_0(\mu_{h_1}) \} - \tilde{H} Y_0(\mu_{h_1})}{\alpha} \\ &= \frac{C_2 \bar{D}_1 - \tilde{H} Y_0(\mu_{h_1})}{\alpha} \text{ (say)} \end{aligned} \quad (26b)$$

$$C_2 = \frac{i k \bar{U}(0) a h e^{-a|k|} e^{-i l_u h_1} + \tilde{H} \{ Y_0(\mu_{h_1}) J_0(\mu_0) - J_0(\mu_{h_1}) Y_0(\mu_0) \} e^{-i l_u h_1}}{\{ \alpha \mu_{h_1} Y_0'(\mu_{h_1}) - i l_u Y_0(\mu_{h_1}) \} J_0(\mu_0) - \{ \alpha \mu_{h_1} J_0'(\mu_{h_1}) - i l_u J_0(\mu_{h_1}) \} Y_0(\mu_0)} \quad (26c)$$

Substituting Eqns. (26a), (26b) & (26c) in Eqns. (24a) & (24b) and taking inverse Fourier transform, we get w_1 and w_2 values for m and U -layers respectively. Vertical displacement of streamlines are, then obtained, with the help of (AII.14). We present hereunder the final results for this case. If,

$$P = l_u \{ Y_0'(\mu_{h_1}) J_0(\mu_0) - J_0(\mu_{h_1}) Y_0(\mu_0) \},$$

$$M = l_u \{ J_0(\mu_{h_1}) Y_0(\mu_0) - Y_0(\mu_{h_1}) J_0(\mu_0) \},$$

$$N = \mu_{h_1} \{ Y_0'(\mu_{h_1}) J_0(\mu) - J_0(\mu_{h_1}) Y_0(\mu) \},$$

$$V = \frac{l_u}{\alpha} \{ J_0(\mu_{h_1}) Y_0(\mu) - Y_0(\mu_{h_1}) J_0(\mu) \},$$

$$R = \{ J_0(\mu_{h_1}) Y_0(\mu) - Y_0(\mu_{h_1}) J_0(\mu) \},$$

$$S_1 = P \cos\{l_u(z-h_1)\} + M \sin\{l_u(z-h_1)\},$$

$$S_2 = P \sin\{l_u(z-h_1)\} - M \cos\{l_u(z-h_1)\},$$

$$T_1 = (PN + MV),$$

$$T_2 = (MN - PV),$$

$$\text{and } T_3 = (P^2 + M^2)$$

then expressions for the displacements in the two cases of heat functions are given below:

(i) No precipitation case [Eqn. (8)]

$$\eta = \frac{gQMb^3}{\alpha l_u c_p \bar{T} \bar{U}_H^3 (x^2 + b^2)} + \left\{ \frac{T_2 x + aT_1}{T_3(x^2 + a^2)} \right\} \alpha ah - \frac{gQb^2 M}{l_u c_p \bar{T} \bar{U}_H^3 T_3} \left\{ \frac{T_2 x + bT_1}{(x^2 + b^2)} \right\} \quad z \leq z_H \quad (27a)$$

$$\eta = \frac{\alpha ah(S_1 a - S_2 x)}{T_3(x^2 + a^2)} - \frac{b^2 gQM}{l_u c_p \bar{T} \bar{U}_H^3 T_3} \frac{bS_1 - xS_2}{(x^2 + b^2)} \quad z > z_H \quad (27b)$$

(ii) Precipitation case [Eqn. (9)]

$$\begin{aligned} \eta = & -\frac{gQMb_1}{l_u c_p \bar{T} \bar{U}_H^3} b_1 \tan^{-1} \left\{ \left(\frac{x}{b_1} \right) - b_2 \tan^{-1} \left(\frac{x}{b_2} \right) \right\} + \left\{ \frac{T_2 x^c + T_1 a}{T_3(x^c + a^2)} \right\} \alpha ah \\ & + \frac{gQMb_1}{l_u c_p T_3 \bar{T} \bar{U}_H^3} \left[\left\{ T_1 \left(b_1 \tan^{-1} \left(\frac{x^c}{b_1} \right) - b_2 \tan^{-1} \left(\frac{x^c}{b_2} \right) \right) \right\} \right. \\ & \left. + \frac{T_2}{2} \{ b_1 \log(x^c + b_1^2) - b_2 \log(x^c + b_2^2) \} \right] \quad z \leq z_H \end{aligned} \quad (27c)$$

where, $x^c = x + c$

$$\begin{aligned} \eta = & \frac{\alpha ah(aS_1 - xS_2)}{T_3(x^c + a^2)} - \frac{gQM}{l_u c_p \bar{T} \bar{U}_H^3 T_3} \left[\left\{ b_1 S_1 \left(b_1 \tan^{-1} \left(\frac{x^c}{b_1} \right) - b_2 \tan^{-1} \left(\frac{x^c}{b_2} \right) \right) \right\} \right. \\ & \left. + \frac{b_1 S_2}{2} \{ b_2 \log(x^c + b_2^2) - b_1 \log(x^c + b_1^2) \} \right] \quad z > z_H, \text{ where } x^c = x + c \end{aligned} \quad (27d)$$

Case (ii) $h_1 < z_H$

$$\tilde{w}_3 = E_3 e^{il_u z} + F_3 e^{-il_u z} \quad z > z_H \quad (28c)$$

Following the procedure adopted above, we obtain,

$F_3 = 0$ for upward travelling waves only.

Here,

$$\tilde{w}_1 = E_1 J_0(\mu) + F_1 Y_0(\mu) \quad z \leq h_1 \quad (28a)$$

$$\tilde{w}_2 = E_2 e^{il_u z} + F_1 e^{-il_u z} \quad z_H \geq z > h_1 \quad (28b)$$

$$\frac{E_1}{E_{11}} = \frac{-F_1}{F_{11}} = \frac{E_2}{E_{22}} = \frac{-F_2}{F_{22}} = \frac{E_3}{E_{33}} = \frac{-1}{XX} \quad (29a)$$

where,

$$\begin{aligned} XX = & J_0(\mu_0) \left[-2il_u \alpha \mu_{h_1} Y_0(\mu_{h_1}) e^{il_u h_1} - Y_0(\mu_{h_1}) l_u^2 \{ e^{il_u h_1} (e^{-2il_u z_H} + 1) \right. \\ & \left. + e^{il_u h_1} (e^{2il_u z_H} - 1) \} \right] \sigma Y_0(\mu_0) \left[J_0(\mu_{h_1}) l_u^2 \{ e^{-il_u h_1} (1 - e^{2il_u z_H}) \right. \\ & \left. - e^{il_u h_1} (1 + e^{-2il_u z_H}) \} - i\alpha \mu_{h_1} J_0(\mu_{h_1}) l_u \{ e^{il_u h_1} (1 + e^{-2il_u z_H}) \right. \\ & \left. - e^{-il_u h_1} (1 + e^{il_u z_H}) \} \right] \end{aligned} \quad (29b)$$

$$E_{11} = 2i Y_0(\mu_0) \tilde{H} l_u e^{i l_u z_H} - ik \bar{U}(0) a h e^{-a|k|} [2i l_u e^{i l_u h_1} \{ Y_0(\mu_{h_1}) - \alpha \mu_{h_1} Y_0(\mu_{h_1}) \}] \quad (29c)$$

$$F_{11} = J_0(\mu_0) \tilde{H} e^{i l_u z_H} 2i l_u - ik \bar{U}(0) a h e^{-a|k|} [-\alpha \mu_{h_1} J_0(\mu_{h_1}) + i l_u J_0(\mu_{h_1})] 2i l_u e^{i l_u h_1} \quad (29d)$$

$$E_{22} = -J_0(\mu_0) [\{ \alpha \mu_{h_1} Y_0(\mu_{h_1}) + i l_u Y_0(\mu_{h_1}) \} \tilde{H} e^{i l_u (z_H - h_1)}] - Y_0(\mu_0) [\{ \alpha \mu_{h_1} J_0(\mu_{h_1}) + i l_u J_0(\mu_{h_1}) \} \tilde{H} e^{-i l_u (h_1 + z_H)}] - k \bar{U}(0) a h e^{-a|k|} 2l_u \alpha \quad (29e)$$

$$F_{22} = J_0(\mu_0) \tilde{H} e^{-i l_u z_H} \{ Y_0(\mu_{h_1}) i l_u - \mu_{h_1} \alpha Y_0(\mu_{h_1}) \} e^{i l_u h_1} - Y_0(\mu_0) \tilde{H} e^{i l_u (z_H + h_1)} \{ i l_u J_0(\mu_{h_1}) - \mu_{h_1} \alpha J_0(\mu_{h_1}) \} \quad (29f)$$

$$E_{33} = -J_0(\mu_0) \tilde{H} [-\alpha \mu_{h_1} Y_0(\mu_{h_1}) \{ e^{-i l_u (z_H - h_1)} - e^{i l_u (z_H - h_1)} \} + Y_0(\mu_{h_1}) \{ i l_u (e^{-i l_u (z_H - h_1)} + e^{i l_u (z_H - h_1)}) \}] - 2k \bar{U}(0) a h e^{-a|k|} \alpha l_u + Y_0(\mu_0) \tilde{H} (e^{i l_u (z_H - h_1)}) + e^{-i l_u (z_H - h_1)} \{ (-\alpha \mu_{h_1} J_0(\mu_{h_1}) + i l_u J_0(\mu_{h_1})) \} \quad (29g)$$

Case (iii) $h_1 > z_H$

In this case the solutions are,

$$\tilde{w}_1 = G_1 J_0(\mu) + H_1 Y_0(\mu) \quad z < z_H \quad (30a)$$

$$\tilde{w}_2 = G_2 J_0(\mu) + H_2 Y_0(\mu) \quad h_1 > z \geq z_H \quad (30b)$$

$$\tilde{w}_3 = G_3 e^{i l_u z} + H_3 e^{-i l_u z} \quad z \geq h_1 \quad (30c)$$

$H_3 = 0$ for upward propagating waves only.

Here,

$$\frac{G_1}{G_{11}} = \frac{-H_1}{H_{11}} = \frac{G_2}{G_{22}} = \frac{-H_2}{H_{22}} = \frac{G_3}{G_{33}} = \frac{-1}{YY} \quad (31a)$$

where,

$$G_{11} = -ik U(0) a h e^{-a|k|} [Y_0'(\mu_{z_H}) \{ \alpha \mu_{h_1} e^{i l_u h_1} \{ J_0(\mu_{z_H}) Y_0(\mu_{h_1}) - J_0(\mu_{h_1}) Y_0(\mu_{z_H}) \} - i l_u e^{i l_u h_1} [J_0(\mu_{z_H}) Y_0(\mu_{h_1}) - J_0(\mu_{h_1}) Y_0(\mu_{z_H})] - Y_0(\mu_{z_H}) \{ e^{i l_u h_1} \alpha \mu_{h_1} \{ J_0(\mu_{z_H}) Y_0(\mu_{h_1}) - J_0(\mu_{h_1}) Y_0(\mu_{z_H}) \} - i l_u e^{i l_u h_1} \{ J_0(\mu_{z_H}) Y_0(\mu_{h_1}) - J_0(\mu_{h_1}) Y_0(\mu_{z_H}) \} \}]]$$

$$\begin{aligned}
& - \frac{\tilde{H}}{\alpha \mu_{zH}} Y_0(\mu_0) \left[\{ e^{i\mu_{zH} h_1} \propto \mu_{h_1} \{ J_0(\mu_{zH}) Y_0(\mu_{h_1}) - J_0(\mu_{h_1}) Y_0(\mu_{zH}) \} \right. \\
& \left. - i l_u e^{i\mu_{zH} h_1} \{ J_0(\mu_{zH}) Y_0(\mu_{h_1}) - J_0(\mu_{h_1}) Y_0(\mu_{zH}) \} \right] \quad (31b)
\end{aligned}$$

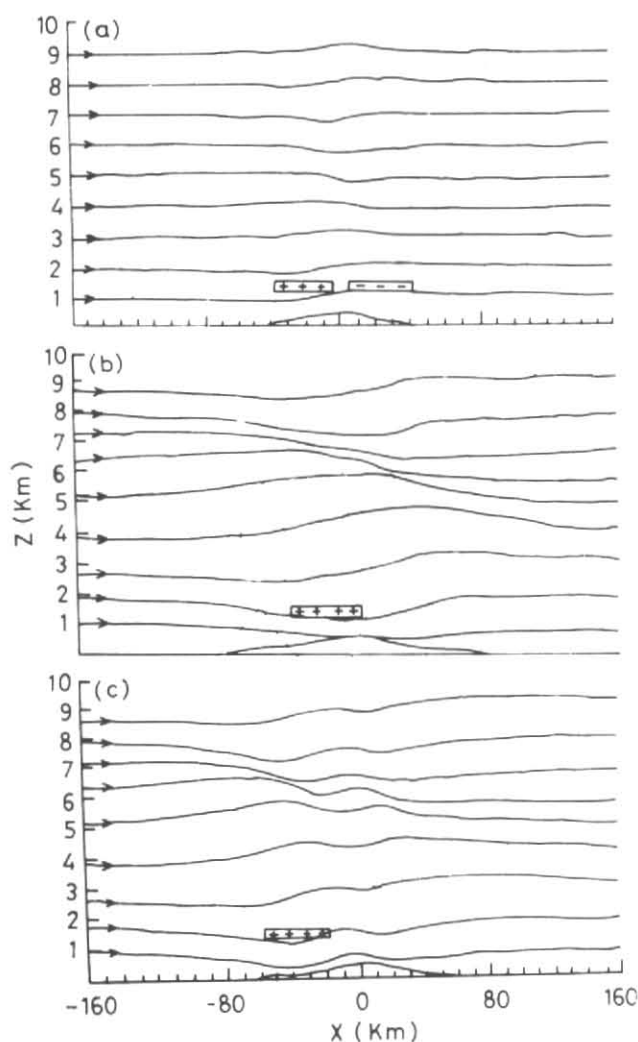
$$\begin{aligned}
H_{11} &= \frac{-ik\bar{U}(0) ah e^{-a|k|} \alpha \mu_{h_1}}{\mu_{zH}} \{ J_0'(\mu_{h_1}) - i l_u J_0(\mu_{h_1}) \} e^{i\mu_{zH} h_1} \\
& - \frac{\tilde{H} J_0(\mu_0) e^{i\mu_{zH} h_1}}{\alpha \mu_{zH}} \left[\alpha \mu_{h_1} \{ J_0(\mu_{zH}) Y_0(\mu_{h_1}) - J_0(\mu_{h_1}) Y_0(\mu_{zH}) \} \right. \\
& \left. - i l_u e^{i\mu_{zH} h_1} \{ J_0(\mu_{zH}) Y_0(\mu_{h_1}) - Y_0(\mu_{zH}) J_0(\mu_{h_1}) \} \right] \quad (31c)
\end{aligned}$$

$$\begin{aligned}
G_{22} &= - \frac{ik\bar{U}(0) ah e^{-a|k|}}{\mu_{zH}} \{ e^{i\mu_{zH} h_1} \alpha \mu_{h_1} Y_0'(\mu_{h_1}) - i l_u e^{i\mu_{zH} h_1} Y_0(\mu_{h_1}) \} \\
& - \frac{\tilde{H}}{\alpha \mu_{zH}} \left[\{ J_0(\mu_{zH}) Y_0(\mu_0) - J_0(\mu_0) Y_0(\mu_{zH}) \} \{ \alpha \mu_{h_1} Y_0'(\mu_{h_1}) \right. \\
& \left. - i l_u Y_0(\mu_{h_1}) \} e^{i\mu_{zH} h_1} \right] \quad (31d)
\end{aligned}$$

$$\begin{aligned}
H_{22} &= - ik\bar{U}(0) ah e^{-a|k|} (\alpha J_0'(\mu_{h_1}) - \frac{i l_u J_0(\mu_{h_1})}{\mu_{zH}}) e^{i\mu_{zH} h_1} \\
& - \frac{\tilde{H}}{\alpha \mu_{zH}} \{ J_0(\mu_{zH}) Y_0(\mu_0) - J_0(\mu_0) Y_0(\mu_{zH}) \} \{ \alpha \mu_{h_1} J_0'(\mu_{h_1}) \\
& - i l_u J_0(\mu_{h_1}) \} e^{i\mu_{zH} h_1} \quad (31e)
\end{aligned}$$

$$G_{33} = - \frac{ik\bar{U}(0) ah e^{-a|k|}}{\mu_{zH}} - \frac{\tilde{H}}{z_H} \{ Y_0(\mu_0) J_0'(\mu_{zH}) - J_0(\mu_0) Y_0'(\mu_{zH}) \} \quad (31f)$$

$$\begin{aligned}
YY &= e^{i\mu_{zH} h_1} \left[J_0(\mu_0) \{ Y_0'(\mu_{zH}) \{ J_0(\mu_{zH}) \{ \alpha \mu_{h_1} Y_0'(\mu_{h_1}) \right. \\
& \left. - i l_u Y_0(\mu_{h_1}) \} - Y_0(\mu_{zH}) \{ \alpha \mu_{h_1} J_0'(\mu_{h_1}) - i l_u J_0(\mu_{h_1}) \} \} \right. \\
& \left. - Y_0(\mu_{zH}) \{ J_0'(\mu_{zH}) \{ (\alpha \mu_{h_1}) Y_0'(\mu_{h_1}) - Y_0(\mu_{h_1}) i l_u \} \right. \\
& \left. - Y_0(\mu_{zH}) \{ \alpha \mu_{h_1} J_0'(\mu_{h_1}) - i l_u J_0(\mu_{h_1}) \} \} \right] \\
& - e^{i\mu_{zH} h_1} \left[Y_0(\mu_0) \{ J_0'(\mu_{zH}) \{ J_0(\mu_{zH}) \& \alpha \mu_{h_1} Y_0'(\mu_{h_1}) \right. \\
& \left. - i l_u Y_0(\mu_{h_1}) \} - Y_0(\mu_{zH}) \{ \alpha \mu_{h_1} J_0'(\mu_{h_1}) - i l_u J_0(\mu_{h_1}) \} \} \right. \\
& \left. - J_0(\mu_{zH}) \{ J_0'(\mu_{zH}) \{ \alpha \mu_{h_1} Y_0'(\mu_{h_1}) - i l_u Y_0(\mu_{h_1}) \} \right. \\
& \left. - Y_0(\mu_{zH}) \{ \alpha \mu_{h_1} J_0'(\mu_{h_1}) - J_0(\mu_{h_1}) i l_u \} \} \right] \quad (31g)
\end{aligned}$$



Figs. 10 (a-c). Hydrostatic flow with combined thermal and orographic forcing (Smith and Lin 1982) with (a) $Q = 1107 \text{ w m kg}^{-1}$; $b = 20.0 \text{ km}$; $U = 10 \text{ m/s}$; $N = 0.01 \text{ sec}^{-1}$; $z_H = 1.5 \text{ km}$; $h = 0.5 \text{ km}$; $a = 20.0 \text{ km}$, (b) $Q = 1107 \text{ w m kg}^{-1}$; $b_1 = 20 \text{ km}$; $b_2 = 100 \text{ km}$; $U = 10 \text{ m/s}$; $N = 0.01 \text{ sec}^{-1}$; $c = 20 \text{ km}$; $z_H = 1.5 \text{ km}$; $h = 0.5 \text{ km}$; $a = 20 \text{ km}$ and (c) Similar to Fig. (b) but with the isolated heating centred further upstream ($c = 40 \text{ km}$).

Figs. 7 (a-c) show various possibilities of atmospheric situations for hydrostatic approximation. For case (i), (when $h_1 = z_H$), streamlines have been plotted in Fig. 8 for balanced heating and cooling, and in Fig. 9 for isolated heating centred at 40 km away from the mountain peak. Heat is applied at 2.0 km level. Effect of shear seems to dampen the thermal forcing effect in general. A comparison with Smith and Lin (1982) case of uniform l -profile [Figs. 10 (a-c)] with our case of exponentially decreasing l -profile in the lower layer does not show significant change in the non-precipitating case. However,

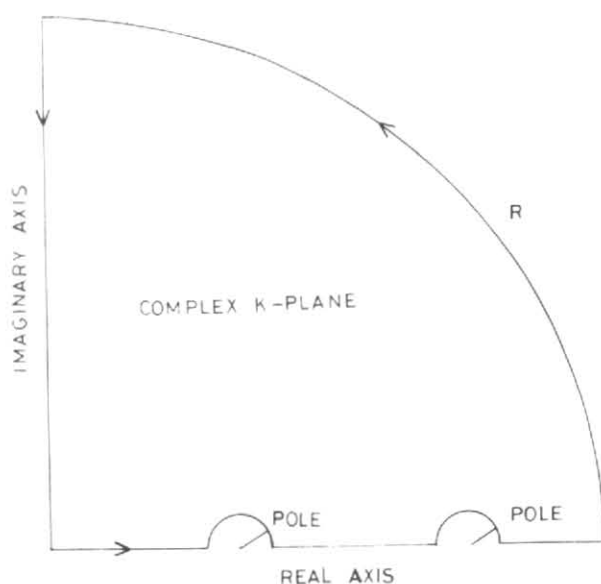


Fig. 11. Contour for the evaluation of integrals in the η_1, η_2, η_p in complex k -plane

marked depleted displacement values can be observed in the case of isolated heating introduced ahead of the mountain. It may, therefore, be suggested that the shear effect acts opposite to the thermal forcing. We also observe a much weakened thermal forcing effect even above the heating level, in our case, in the region of uniform l -profile. Presence of shear at the heating level reduces the effect of thermal forcing at that level itself. This, in turn, leads to dampening (amplitudes) of hydrostatic waves at all levels. The amplitudes of all hydrostatic waves are, thus, less pronounced compared to the case of Smith and Lin (1982). This also leads to comparatively lesser propagation of energy at higher levels.

APPENDIX I

List of symbols

- | | |
|--------|---|
| a | half width of mountain |
| b | horizontal scale of heating function Eqn. (8) |
| b_1 | width of the heating part of Eqn. (9) |
| b_2 | width of the cooling part of Eqn. (9) |
| c | upstream displacement of heating function |
| c_p | specific heat capacity at constant pressure |
| d | half depth of heating layer |
| $f(z)$ | vertical distribution of heating |

g	gravitational acceleration	z_H	heating level
\dot{H}	heating rate (W kg^{-1})	α	exponent parameter for l
$h(x)$	mountain profile	β	$\frac{1}{\theta} \frac{\partial \theta}{\partial z}$
h	mountain height	θ	potential temperature
h_1	height of the layer with exponentially decreasing l -profile [Figs. 7 (a-c)]	δ	Dirac delta function
$J_\nu(\mu)$	Bessel function of first kind of order ν and argument μ ; μ' represents differential w.r.t. z .	λ	value of l at $z = 0$ for exponentially decreasing profile
k	horizontal wave number	μ	$\frac{l}{ \alpha }$
l	Scorer parameter	μ_0	$(\mu)_{z=0}$
l_u	constant l above $z = h_1$ (hydrostatic case)	μ_H	$(\mu)_{z=z_H}$
l_m	exponentially decreasing within h_1 , $l_m = \lambda e^{\alpha z}$ ($\alpha < 0$)	μ_{h_1}	$(\mu)_{z=h_1}$
N	Brunt-Vaisala frequency	Δ	difference
p	down stream pressure	$\bar{\rho}$	incoming density
\bar{p}	incoming pressure	ρ	down stream density
p'	perturbation component of p	$\bar{\rho}_0$	surface incoming density
Q	amplitude of heating function	$\rho(z_H)$	air density at heating level $z = z_H$
$q(x)$	horizontal distribution of heating	ν	$\frac{k}{ \alpha }$
S	$\frac{d}{dz} \ln \bar{\rho}(z)$	η	vertical displacement
S_z	$\frac{\partial S}{\partial z}$	η_p	vertical displacement components ($p = 1, 2, 3$ or 4).
\bar{T}	incoming temperature		
T	down stream temperature		
T'	perturbation component of T		
\bar{U}	incoming velocity (suffix z with \bar{U} denotes differentiation with respect to z)		
$\bar{U}(0)$	incoming velocity at $z = 0$		
\bar{U}_H	incoming velocity at $z = z_H$		
u	down stream velocity		
u'	perturbation component of u		
w	vertical velocity		
w'	perturbation component of w		
x	down stream coordinate		
$Y_\nu(\mu)$	Bessel function of second kind of order ν and argument μ ; μ' represents differential w.r.t. z .		
z	vertical coordinate		

APPENDIX II

First let us consider heating function of the form given by Eqn. 8 with the heat added at a certain height z_H .

$$H = -Q \frac{2b^3x}{(x^2 + b^2)^2} \delta(z - z_H)$$

The governing equation can be written as

$$w_{xx} + w_{zz} + l^2w = -\frac{gQ}{c_p T \bar{U}^2} \frac{2b^3x}{(x^2 + b^2)^2} \delta(z - z_H) \left\{ \frac{\rho(z)}{\rho_0} \right\}^{1/2} \quad (\text{AII.1})$$

Let $w(k, z)$ be the one-sided Fourier transform of $w(x, z)$ in x , i.e.,

$$\tilde{w}(k, z) = \frac{1}{\pi} \int_{-\infty}^{\infty} w(x, z) e^{-ikx} dx$$

$$w(x, z) = \operatorname{Re} \int_0^{\infty} w(k, z) e^{ikx} dk$$

Then the Fourier transform of (AII.1) becomes.

$$\tilde{w}_{zz} + (l^2 - k^2) \tilde{w} = \frac{\rho^{1/2}(z_H) igQ b^2 k}{\rho_0^{1/2} c_p \bar{T} \bar{U}^2} e^{-b|k|} \delta(z - z_H) \quad (\text{AII.2})$$

For $z \neq z_H$ (AII.2) becomes

$$\tilde{w}_{zz} + (l^2 - k^2) \tilde{w} = 0 \quad (\text{AII.3})$$

For Scorer's parameter varying exponentially with height we have,

$$l(z) = \lambda e^{\alpha z} \quad (\alpha < 0)$$

Now if we substitute

$$\nu = \frac{k}{|\alpha|} \quad \text{and} \quad \mu = \frac{l(z)}{|\alpha|}$$

then (AII.3) becomes

$$\frac{d^2 \tilde{w}}{d\mu^2} + \frac{1}{\mu} \frac{d\tilde{w}}{d\mu} + \left(1 - \frac{\nu^2}{\mu^2}\right) \tilde{w} = 0 \quad (\text{AII.4})$$

General solution of (AII.4) is

$$\tilde{w} = A_1 J_\nu(\mu) + B_1 Y_\nu(\mu) \quad \text{for } z < z_H \quad (\text{AII.5a})$$

$$\tilde{w} = A_2 J_\nu(\mu) + B_2 Y_\nu(\mu) \quad \text{for } z > z_H \quad (\text{AII.5b})$$

where, A_1, A_2 and B_1, B_2 are the unknown constants, to be determined.

Boundary Conditions

(a) At the ground, the flow is assumed to follow the terrain, thus

$$\frac{w}{u} = \frac{w'}{U+u'} = \frac{d}{dx} h(x) \quad (\text{AII.6})$$

where $h(x)$ represents the terrain profile. For a small amplitude topography and disturbance (AII.6) can be simplified as

$$w = \bar{U} \frac{d}{dx} h(x) \quad \text{at } z = 0 \quad (\text{AII.7})$$

Since $h(x)$ is assumed to be a bell shaped function

$$h(x) = \frac{ha^2}{(x^2 + a^2)} \quad (\text{AII.8})$$

substitute (AII.8) into (AII.7) and take the Fourier transform. Then

$$\tilde{w}(k, 0) = ik \bar{U}(0) ha e^{-a|k|} \quad \text{at } z = 0 \quad (\text{AII.9})$$

(b) Radiation condition prescribes us that

$$\text{as } z \rightarrow \infty, w(x, z) \rightarrow \epsilon e^{i\xi z}$$

$$\text{where } \xi = (l^2 - k^2)^{1/2}$$

$$\text{and } 0 < \epsilon \ll 1;$$

$$\text{for } \alpha < 0 \text{ as } z \rightarrow \infty, \xi \text{ is imaginary. Hence as } z \rightarrow \infty, \tilde{w} \rightarrow 0 \quad (\text{AII.10})$$

(c) Matching condition at the level $z = z_H$ is given by Eqns. 11-12 along the heating level z_H .

Taking the Fourier transform of Eqns. 11 & 12,

$$\Delta \tilde{w}_z = \frac{\rho^{1/2}(z_H) gQq(x)}{\rho_0^{1/2} c_p T U^2} \quad (\text{AII.11})$$

$$\Delta \tilde{w} = 0 \quad (\text{AII.12})$$

we solve Eqns. (AII.5a) and (AII.5b) and assuming only upward propagation of energy, we get, using the lower and upper boundary conditions only

$$\tilde{w} = A_1 \left\{ J_\nu(\mu) - \frac{Y_\nu(\mu)}{Y_\nu(\mu_0)} J_\nu(\mu_0) \right\} + \frac{ik \bar{U}(0) ha e^{-a|k|} Y_\nu(\mu)}{Y_\nu(\mu_0)} \quad (\text{AII.13a})$$

$$\text{for } z < z_H$$

$$\tilde{w} = A_2 J_\nu(\mu) \quad \text{for } z \geq z_H. \quad (\text{AII.13b})$$

To obtain, A_1 and A_2 we apply matching conditions (AII.11) and (AII.12). Thus,

$$A_1 = \frac{ik \bar{U}(0) ha e^{-a|k|}}{J_\nu(\mu_0)} + \frac{igQ b^2 k e^{-b|k|} \pi Y_\nu(\mu_0)}{2c_p \bar{T} \bar{U}_H^2 J_\nu(\mu_0)} \left(\frac{\rho(z_H)^{1/2}}{\rho_0} \right) J_\nu(\mu_0)$$

$$A_2 = \frac{ik \bar{U}(0) ha e^{-a|k|}}{J_\nu(\mu_0)} - \frac{igQ b^2 k e^{-b|k|} \pi \rho^{1/2}(z_H)}{2c_p \bar{T} \bar{U}_H^2 J_\nu(\mu_0) \rho^{1/2}} \{ Y_\nu(\mu_H) J_\nu(\mu_0) - J_\nu(\mu_H) Y_\nu(\mu_0) \}$$

We substitute the values of A_1 and A_2 into Eqns. (AII.13a) and (AII.13b) respectively and take the inverse Fourier transform. Thus,

$$w(x, z) = w_1 + w_2 - w_3 \quad z < z_H$$

$$w(x, z) = w_1 + w_2 - w_4 \quad z \geq z_H$$

where,

$$w_1 = \operatorname{Re} \int_0^{\infty} \frac{ik\bar{U}(0)ha e^{-ak} J_\nu(\mu) e^{ikx}}{J_\nu(\mu_0)} dk$$

$$w_2 = \operatorname{Re} \int_0^{\infty} \frac{igQb^2k e^{-bk} \pi Y_\nu(\mu_0) J_\nu(\mu_H) J_\nu(\mu) \rho^{1/2}(z_H) e^{ikx}}{2c_p \bar{T} \bar{U}_H^2 \alpha J_\nu(\mu_0) \rho_0^{1/2}} dk$$

$$w_3 = \operatorname{Re} \int_0^{\infty} \frac{igQb^2k e^{-bk} \pi J_\nu(\mu_H) Y_\nu(\mu) \rho^{1/2}(z_H) e^{ikx}}{2c_p \bar{T} \bar{U}_H^2 \alpha \rho_0^{1/2}} dk$$

$$w_4 = \operatorname{Re} \int_0^{\infty} \frac{igQb^2k e^{-bk} \pi Y_\nu(\mu_H) J_\nu(\mu) \rho^{1/2}(z_H) e^{ikx}}{2c_p \bar{T} \bar{U}_H^2 \alpha \rho_0^{1/2}} dk$$

Similarly, if we take precipitation case when $q(x)$ is given by Eqn. (9),

Then

$$w(x, z) = w_1 + w_2 - w_3, \quad z < z_H$$

$$w(x, z) = w_1 + w_2 - w_4, \quad z > z_H$$

where,

$$w_1 = \operatorname{Re} \int_0^{\infty} \frac{ik\bar{U}(0)ha e^{-ak} J_\nu(\mu) e^{ikx}}{J_\nu(\mu_0)} dk$$

$$w_2 = \operatorname{Re} \int_0^{\infty} \frac{\rho^{1/2}(z_H) \pi gQb (e^{-b_1k} - e^{-b_2k}) Y_\nu(\mu_0) J_\nu(\mu_H) J_\nu(\mu) e^{ikx}}{\rho_0^{1/2} c_p \bar{T} \bar{U}_H^2 2 \alpha J_\nu(\mu_0)} dk$$

$$w_3 = \operatorname{Re} \int_0^{\infty} \frac{\rho^{1/2}(z_H) \pi gQb_1 (e^{-b_1k} - e^{-b_2k}) J_\nu(\mu_H) Y_\nu(\mu) e^{ikx}}{\rho_0^{1/2} c_p \bar{T} \bar{U}_H^2 2 \alpha} dk$$

$$w_4 = \operatorname{Re} \int_0^{\infty} \frac{\rho^{1/2}(z_H) \pi gQb_1 (e^{-b_1k} - e^{-b_2k}) J_\nu(\mu) Y_\nu(\mu) e^{ikx}}{\rho_0^{1/2} c_p \bar{T} \bar{U}_H^2 2 \alpha} dk$$

The relation for vertical displacement can be obtained as $w = \bar{U} \frac{\partial z}{\partial x}$, so

$$\eta(x, z) = \frac{1}{\bar{U}} \int_{-\infty}^x w(x, z) dx \quad (\text{AII.14})$$

Now let us consider the case when heat is uniformly added to a layer of $z = z_H - d$ to $z = z_H + d$, that is

$$\dot{H} = Q \frac{-2b^3x}{(x^2 + b^2)^2}, \quad z_H - d < z < z_H + d,$$

$$\dot{H} = 0 \quad \text{elsewhere.}$$

The solution can be obtained by superposition of heating term. It gives us, for Eqn. (8)

$$\eta(x, z) = \eta_1 + \int_{z_H-d}^{z_H+d} (\eta_2 - \eta_3) dh, \quad z < z_H - d$$

$$= \eta_1 + \int_{z_H-d}^z (\eta_2 - \eta_4) dh$$

$$+ \int_z^{z_H+d} (\eta_2 - \eta_3) dh, \quad z_H - d \leq z \leq z_H + d$$

$$= \eta_1 + \int_{z_H-d}^{z_H+d} (\eta_2 - \eta_4) dh, \quad z > z_H + d$$

for Eqn. (9)

$$\eta(x, z) = \eta_1 + \int_{z_H-d}^{z_H+d} (\eta_2^0 - \eta_3^0) dh, \quad z < z_H - d$$

$$\begin{aligned}
 &= \eta_1 + \int_{z_H-d}^z (\eta_2^0 - \eta_4^0) dh \\
 &+ \int_z^{z_H+d} (\eta_2^0 - \eta_3^0) dh, \quad z_H-d < z < z_H+d \\
 &= \eta_1 + \int_{z_H-d}^{z_H+d} (\eta_2^0 - \eta_4^0) dh, \quad z > z_H+d
 \end{aligned}$$

where,

$$\eta_p^0 = \eta_p^c \quad \text{for } c = 0 \quad (p = 2, 3, 4).$$

Evaluation of the integrals η_1, η_2 are to be made with the help of contour integral in complex k -plane (Fig. 11).

Since the zero of $J_\nu(\mu)$ for real argument, are real we make indentation around the poles on real axis and evaluate the integral by computing residues of poles on the real axis added with the values of the integrals along the imaginary k -axis. $J_\nu(\mu)$ has no zero on the imaginary k -axis and values of the integrals along the arc tend to zero for large k . η_3, η_4 are evaluated numerically using Gauss-Laguerre quadrature formula.

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